



TUGAS AKHIR - SF 141501

**ALGORITMA DEUTSCH-JOZSA PADA Kuantum
Komputer Sistem NMR (*Nuclear Magnetic
Resonance*) 4 Qubit**

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NRP 1112 100 060

Dosen Pembimbing
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Jurusan Fisika
Fakultas Matematika dan Ilmu Pengetahuan Alam
Institut Teknologi Sepuluh Nopember
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FINAL PROJECT - SF 141501

**DEUTSCH-JOZSA ALGORITHM IN QANTUM
COMPUTER WITH NMR (Nuclear
Resonance) 4 QUBITS SYSTEM**

Bayu Dwi Hatmoko
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Advisor
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Institute of Technology Sepuluh Nopember
Surabaya 2017

**ALGORTIMA DEUSCH JOZSA PADA KOMPUTER
KUANTUM SISTEM NMR (*Nuclear Magnetic
Resonance*) 4 QUBIT**

TUGAS AKHIR

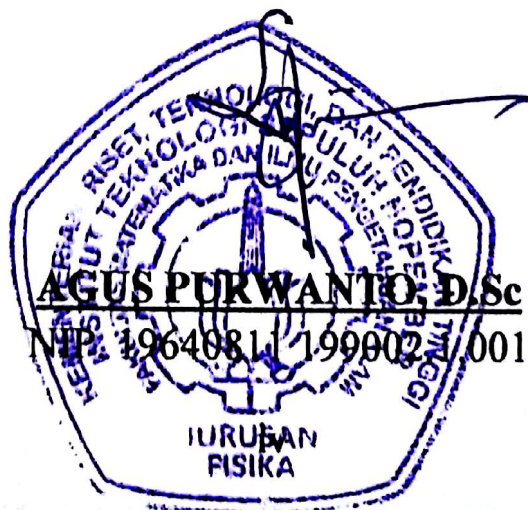
**Diajukan Untuk Memenuhi Salah Satu Syarat
Memperoleh Gelar Sarjana Sains
pada
Bidang Fisika Teori
Program Studi S-1 Jurusan Fisika
Fakultas Matematika dan Ilmu Pengetahuan Alam
Institut Teknologi Sepuluh Nopember**

Oleh:

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**Disetujui oleh Pembimbing Tugas Akhir
Surabaya, Januari 2017**

Pembimbing



ALGORITMA DEUTSCH-JOZSA PADA KOMPUTER KUANTUM SISTEM NMR (*Nuclear Magnetic Resonance*) 4 QUBIT

Nama : Bayu Dwi Hatmoko
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Jurusan : Fisika, FMIPA-ITS
Pembimbing : Agus Purwanto, D.Sc

Abstrak

Gagasan tentang komputer kuantum dikenalkan oleh Yuri Marin pada tahun 1980, kemudian disusul ilmuwan Paul Benioff pada tahun 1981 dan disusul lagi oleh Richard feynman pada tahun 1982, komputer kuantum sendiri merupakan teknologi masa depan komputasi yang dapat menggantikan komputer saat ini. Komputer merupakan piranti yang dapat menghitung berdasarkan logika analitis yang dimiliki oleh rangkaian-rangkaian transistor logika. Menurut hukum Moore jumlah transistor pada mikroprosesor terus meningkat dua kali lipat setiap dua tahun. Berdasarkan hal itu para ahli memperkirakan akan menemukan sirkuit pada mikroprosesor yang diukur pada skala atom, dari sinilah kemudian munculah gagasan yang memanfaatkan kuantitas atom dan molekul untuk melakukan pengolahan data dan memori, yang kemudian disebut sebagai komputer kuantum. Kemudian Deutsc membuat rancangan algoritma kuantum yaitu algoritma Deutsch pada sistem dua qubit. (Deustch, 1985).Komputer kuantum saat ini masih dalam prototype 10 qubit, namun dalam Tugas akhir ini akan di bahas secara matematis untuk NMR 4 qubit.

Kata kunci : *Algoritma Deutsch-Jozsa, Kuantum Komputer, NMR, qubit*

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***DEUTSCH-JOZSA ALGORITHM IN QUANTUM
COMPUTER WITH NMR (Nuclear Magnetic Resonance) 4
QUBITS SYSTEM***

Name : Bayu Dwi Hatmoko
NRP : 1112100060
Major : Physics, FMIPA-ITS
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Abstrak

The idea of a quantum computer introduced by Yuri Marin in 1980, scientist Paul Benioff followed in 1981 and was followed again by Richard Feynman in 1982, quantum computer is the future of computing technology that can replace today's computers. The computer is a tool that can calculate based on the analytical logic used by transistor logic circuits. According to Moore's law continues the number of transistors on a microprocessor has doubled every two years. Based on that scientist predict will find circuits on a microprocessor measured on the atomic scale, from here then comes the idea of utilizing the quantity of atoms and molecules to perform memory and data processing, which is then referred to as quantum computers. Then Deutsch make an Algorithm that is Deutsch quantum algorithm which is an algorithm on a two qubit system. (Deustch, 1985). A quantum computer is still in the prototype 10 qubits, but the final project will be discussed mathematically for NMR four qubits.

Keywords : Deutsch-Jozsa algorithm, NMR, Quantum Computers , qubit

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KATA PENGANTAR

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“Algoritma Deutsch-Jozsa pada NMR (Nuclear Magnetic Resonance) 4 Qubit”.

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Surabaya, Januari 2017

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BAB I

PENDAHULUAN

1.1 Latar Belakang

Gagasan tentang komputer kuantum dikenalkan oleh Yuri Marin pada tahun 1980, kemudian disusul ilmuwan Paul Benioff pada tahun 1981 dan disusul lagi oleh Richard Feynman pada tahun 1982, komputer kuantum sendiri merupakan teknologi masa depan komputasi yang dapat menggantikan komputer saat ini. Komputer merupakan piranti yang dapat menghitung berdasarkan logika analitis yang dimiliki oleh rangkaian-rangkaian transistor logika. Menurut hukum Moore jumlah transistor pada mikroprosesor terus meningkat dua kali lipat setiap dua tahun. Berdasarkan hal itu para ahli memperkirakan akan menemukan sirkuit pada mikroprosesor yang diukur pada skala atom, dari sinilah kemudian munculah gagasan yang memanfaatkan kuantitas atom dan molekul untuk melakukan pengolahan data dan memori, yang kemudian disebut sebagai komputer kuantum. Kemudian pada tahun 1985, David Deutsch menyatakan bahwa setiap komputer secara umum dapat meniru setiap sistem, sehingga permodelan komputasi yang lebih umum sangat diharapkan untuk menggambarkan komputer umum. Akhirnya Deutsch mengambil sistem-sistem fisika yaitu permasalahan fisika kuantum untuk membuat permodelan komputer kuantum yang diharapkan mampu menyelesaikan persoalan-persoalan komputasi secara efisien dimana belum memiliki penyelesaian yang efisien di komputer klasik. Kemudian Deutsch membuat rancangan algoritma kuantum yaitu algoritma Deutsch pada sistem dua qubit. (Deutsch, 1985).

NMR (Nuclear Magnetic Resonance) merupakan sistem

kantum dua keadaan yang mampu menerapkan algoritma Deutsch, algorima Deutsch-Jozsa dan beberapa algoritma kuntum yang laian. NMR memiliki teori yang paling mapan dari realisasi fisis yang berkaitan dengan komputer kuantum. Qubit pada realisasi sistem ini merupakan inti yang berspin $1/2$. Molekul yang berisi beberapa inti disebut sebagai "Quantum Register" dimana sistem NMR dibuat dari jumlah molekul makroskopik dalam keadaan kesetimbangan termal. Komputer kuantum saat ini masih dalam prototype 10 qubit, namun dalam jurnal-jurnal belum menerangkan perhitungan matematis secara terperinci. Sehubungan dengan hal tersebut, maka ada peluang yang besar untuk mengkaji lebih mendalam dari perhitungan matematis yang lebih terperinci untuk sistem kuantum NMR ini.

1.2 Perumusan Masalah

Dalam tugas akhir ini permasalahan yang akan dibahas adalah bagaimana pengaplikasian algoritma Deutsch-Josza 4 qubit secara matematis.

1.3 Batasan Masalah

Pada tugas akhir ini permasalahan hanya dibatasi pada sistem fisis NMR.

1.4 Tujuan

Dalam Tugas Akhir ini bertujuan untuk menganalisa grafik hasil penerapan Algoritma Deutsch-Josza pada Kuantum komputer sistem NMR 4 qubit.

BAB II

TINTJAUAN PUSTAKA

2.1 Komputer Klasik

2.1.1 Bilangan Biner

Bilangan biner adalah bilangan yang berbasis 2 keadaan yaitu 0 dan 1, maka dengan demikian apabila dipandang sebagai bilangan desimal, bilangan biner adalah bilangan yang berbasis 2^n , jadi untuk mengkonversikan bilangan biner ke bilangan desimal atau sebaliknya maka dapat dibuat jumlahan deret, sebagai berikut:

Misalkan kita memiliki angka biner 100110

Tabel 2.1 Bilangan biner







Basis	2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Biner					1	0	0	1	1	0
Desimal					32	0	0	4	2	0

maka bilangan biner 100110 merupakan representasi bilangan desimal 38, begitu seterusnya, apabila kita meninjau alat pengukur jarak odometer yang berbasis lima digit maka kita dapatkan

2.1.2 Gerbang Klasik

Komputer bekerja berdasarkan gerbang-gerbang logika yang telah disusun sedemikian rupa dalam transistor-transistor logika, berikut adalah gerbang-gerbang logika dalam komputer klasik:

Tabel 2.2 Gerbang Komputer klasik

AND		$F = A \cdot B$ or $F = AB$	<table><tr><th>A</th><th>B</th><th>F</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table>	A	B	F	0	0	0	0	1	0	1	0	0	1	1	1
A	B	F																
0	0	0																
0	1	0																
1	0	0																
1	1	1																
OR		$F = A + B$	<table><tr><th>A</th><th>B</th><th>F</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table>	A	B	F	0	0	0	0	1	1	1	0	1	1	1	1
A	B	F																
0	0	0																
0	1	1																
1	0	1																
1	1	1																
NOT		$F = \overline{A}$ or $F = A'$	<table><tr><th>A</th><th>F</th></tr><tr><td>0</td><td>1</td></tr><tr><td>1</td><td>0</td></tr></table>	A	F	0	1	1	0									
A	F																	
0	1																	
1	0																	
NAND		$F = \overline{AB}$	<table><tr><th>A</th><th>B</th><th>F</th></tr><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table>	A	B	F	0	0	1	0	1	1	1	0	1	1	1	0
A	B	F																
0	0	1																
0	1	1																
1	0	1																
1	1	0																
NOR		$F = \overline{A + B}$	<table><tr><th>A</th><th>B</th><th>F</th></tr><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table>	A	B	F	0	0	1	0	1	0	1	0	0	1	1	0
A	B	F																
0	0	1																
0	1	0																
1	0	0																
1	1	0																
XOR		$F = A \oplus B$	<table><tr><th>A</th><th>B</th><th>F</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table>	A	B	F	0	0	0	0	1	1	1	0	1	1	1	0
A	B	F																
0	0	0																
0	1	1																
1	0	1																
1	1	0																

2.2 Komputer kuantum

2.2.1 Qubit

Didalam logika komputer klasik dikenal istilah ketukan, ketukan ini berkaitan dengan kemampuan hitung komputer, ketukan dalam proses perhitungan ini disebut sebagai bit, untuk saat ini komputer klasik sudah mencapai 64 bit artinya dalam satu kali hitung komputer dapat menghitung 64 masukan yang berbeda. Namun bit dalam istilah komputer kuantum disebut sebagai qubit (quantum bit) yang dinyatakan dalam basis keadaan berikut:

$$|0\rangle \equiv |+\rangle \equiv \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad (1)$$

untuk keadaan spin up, sedangkan untuk keadaan spin down dinyatakan dengan ket berikut:

$$|1\rangle \equiv |-\rangle \equiv \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (2)$$

dan untuk keadaan yang menyatakan solusi secara lengkap (mengkombinasikan kedua keadaan spin up dan spin down) dinyatakan sebagai berikut:

$$|\psi\rangle = \alpha|+\rangle + \beta|-\rangle \quad (3)$$

dimana koefisien alfa dan beta merupakan koefisien kompleks yang memenuhi orthonormalitas berikut:

$$\langle\psi|\psi\rangle = |\alpha|^2 + |\beta|^2 = 1 \quad (4)$$

sedangkan $|+\rangle$ dan $|-\rangle$ merupakan vektor basis orthonormal dari keadaan $|\psi\rangle$

2.2.2 Gerbang Kuantum

Dalam komputer klasik untuk memproses suatu data maka diperlukan gerbang-gerbang logika seperti gerbang AND, OR, NOT dan lain sebagainya, begitu juga dalam komputer kuantum, komputer kuantum memiliki gerbang logika yang disebut sebagai gerbang kuantum. Gerbang kuantum ini digunakan untuk memproses data kuantum agar dapat membuat informasi yang utuh. Apabila gerbang kuantum dikenai operator uniter maka gerbang kuantum tersebut berevolusi sesuai persamaan berikut:

$$|\psi\rangle \rightarrow |\psi'\rangle = U|\psi\rangle \quad (5)$$

berdasarkan jumlah qubitnya gerbang kuantum dibagi menjadi 2 yaitu gerbang qubit tunggal dan gerbang 2 qubit, berikut adalah uraian dari masing-masing gerbang:

2.2.2.1 Gerbang Qubit Tunggal

Ada beberapa gerbang logika dalam gerbang qubit tunggal yaitu sebagai berikut: gerbang Z, gerbang NOT dan operator Hadamard. Berikut adalah penjelasan dari masing-masing gerbang:

Gerbang Z

Gerbang Z bekerja dalam logika persamaan berikut:

$$\begin{aligned} Z|+\rangle &= |+\rangle \\ Z|-\rangle &= -|-\rangle \end{aligned} \tag{6}$$

dikarenakan $|\psi\rangle = \alpha|+\rangle + \beta|-\rangle$, maka

$$\begin{aligned} Z|\psi\rangle &= \alpha Z|+\rangle + \beta Z|-\rangle \\ &= \alpha|+\rangle - \beta|-\rangle \end{aligned} \tag{7}$$

maka dengan demikian operator Z merupakan operator uniter yaitu sebagai berikut:

$$Z = \begin{bmatrix} \langle +|Z|+\rangle & \langle +|Z|-\rangle \\ \langle -|Z|+\rangle & \langle -|Z|-\rangle \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{8}$$



Gambar 2.1 Gerbang Z

Gerbang NOT

Gerbang logika NOT memiliki simbol X, yang bekerja sesuai dengan persamaan:

$$\begin{aligned} X|+\rangle &= |-\rangle \\ X|-\rangle &= |+\rangle \end{aligned} \quad (9)$$

dikarenakan $|\psi\rangle = \alpha|+\rangle + \beta|-\rangle$, maka

$$\begin{aligned} X|\psi\rangle &= \alpha X|+\rangle + \beta X|-\rangle \\ &= \alpha|-\rangle - \beta|+\rangle \end{aligned} \quad (10)$$

maka dengan demikian operator X merupakan operator uniter yaitu sebagai berikut:

$$X = \begin{bmatrix} \langle +|X|+\rangle & \langle +|X|-\rangle \\ \langle -|X|+\rangle & \langle -|X|-\rangle \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad (11)$$

Operator Hadamard

Operator Hadamard memiliki simbol H yang bekerja sesuai dengan persamaan berikut:



Gambar 2.2 Gerbang hadamard

$$\begin{aligned}
H|+\rangle &= \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle) \\
H|-\rangle &= \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle)
\end{aligned} \tag{12}$$

dikarenakan $|\psi\rangle = \alpha|+\rangle + \beta|-\rangle$, maka

$$\begin{aligned}
H|\psi\rangle &= \alpha H|+\rangle + \beta H|-\rangle \\
&= \alpha \left[\frac{1}{\sqrt{2}}(|+\rangle + |-\rangle) \right] + \beta \left[\frac{1}{\sqrt{2}}(|+\rangle - |-\rangle) \right] \\
&= \frac{1}{\sqrt{2}}[\alpha + \beta]|+\rangle - \frac{1}{\sqrt{2}}\beta[\alpha - \beta]|-\rangle
\end{aligned} \tag{13}$$

maka dengan demikian operator H merupakan operator uniter yaitu sebagai berikut:

$$X = \begin{bmatrix} \langle +|H|+ \rangle & \langle +|H| - \rangle \\ \langle -|H|+ \rangle & \langle -|H| - \rangle \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{14}$$

2.2.2.2 Gerbang 2 Qubit

Ada beberapa operator yang bekerja pada sistem kuantum 2 qubit salah satunya adalah SWAP (exchange) dan operator CNOT. Berikut adalah penjelasan dari masing-masing gerbang:

Operator SWAP

Operator SWAP berfungsi untuk menukarkan keadaan qubit ke-0 dengan keadaan qubit ke-1 dengan memenuhi persamaan berikut:

$$S|xy\rangle = |yx\rangle \tag{15}$$

maka didapatkan keadaan lengkapnya sebagai berikut:

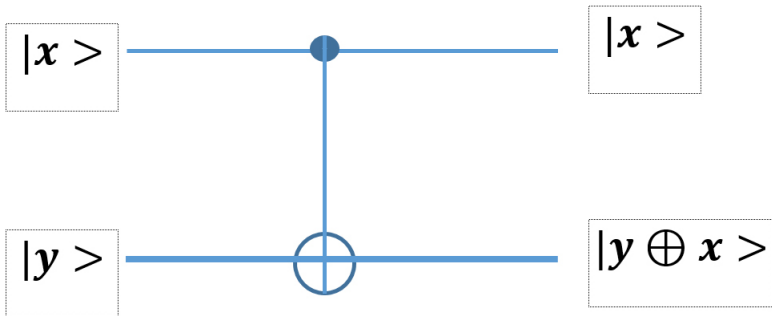
$$\begin{aligned}
 S|00\rangle &= |00\rangle \\
 S|01\rangle &= |01\rangle \\
 S|10\rangle &= |10\rangle \\
 S|11\rangle &= |11\rangle
 \end{aligned} \tag{16}$$

yang memiliki operator uniter berikut

$$\begin{aligned}
 S &= \begin{bmatrix} \langle 00|S|00\rangle & \langle 00|S|01\rangle & \langle 00|S|10\rangle & \langle 00|S|11\rangle \\ \langle 01|S|00\rangle & \langle 01|S|01\rangle & \langle 01|S|10\rangle & \langle 01|S|11\rangle \\ \langle 10|S|00\rangle & \langle 10|S|01\rangle & \langle 10|S|10\rangle & \langle 10|S|11\rangle \\ \langle 11|S|00\rangle & \langle 11|S|01\rangle & \langle 11|S|10\rangle & \langle 11|S|11\rangle \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
 \end{aligned} \tag{17}$$

Operator CNOT (Controlled NOT)

Berikut adalah diagram operator CNOT:



Gambar 2.3 Gerbang UCNOT

Operator CNOT disimbolkan dengna huruf C, yang bekerja sesuai dengan persamaan berikut:

$$C_{10}|xy\rangle = |xy \oplus x\rangle \quad (18)$$

$$C_{10}|00\rangle = |00\rangle$$

$$C_{10}|01\rangle = |01\rangle$$

$$C_{10}|10\rangle = |10\rangle$$

$$C_{10}|11\rangle = |11\rangle$$

$$C_{01}|xy\rangle = |xy \oplus x\rangle \quad (19)$$

$$C_{01}|00\rangle = |00\rangle$$

$$C_{01}|01\rangle = |11\rangle$$

$$C_{01}|10\rangle = |10\rangle$$

$$C_{01}|11\rangle = |01\rangle$$

sehingga didapatkan operator uniternya sebagai berikut:

$$\begin{aligned} C_{10} &= \begin{bmatrix} \langle 00|C_{10}|00\rangle & \langle 00|C_{10}|01\rangle & \langle 00|C_{10}|10\rangle & \langle 00|C_{10}|11\rangle \\ \langle 01|C_{10}|00\rangle & \langle 01|C_{10}|01\rangle & \langle 01|C_{10}|10\rangle & \langle 01|C_{10}|11\rangle \\ \langle 10|C_{10}|00\rangle & \langle 10|C_{10}|01\rangle & \langle 10|C_{10}|10\rangle & \langle 10|C_{10}|11\rangle \\ \langle 11|C_{10}|00\rangle & \langle 11|C_{10}|01\rangle & \langle 11|C_{10}|10\rangle & \langle 11|C_{10}|11\rangle \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{aligned} \quad (20)$$

$$C_{01} = \begin{bmatrix} \langle 00|C_{01}|00\rangle & \langle 00|C_{01}|01\rangle & \langle 00|C_{01}|10\rangle & \langle 00|C_{01}|11\rangle \\ \langle 01|C_{01}|00\rangle & \langle 01|C_{01}|01\rangle & \langle 01|C_{01}|10\rangle & \langle 01|C_{01}|11\rangle \\ \langle 10|C_{01}|00\rangle & \langle 10|C_{01}|01\rangle & \langle 10|C_{01}|10\rangle & \langle 10|C_{01}|11\rangle \\ \langle 11|C_{01}|00\rangle & \langle 11|C_{01}|01\rangle & \langle 11|C_{01}|10\rangle & \langle 11|C_{01}|11\rangle \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad (21)$$

2.3 Sitem Dua Keadaan

Komputer klasik menggunakan komponen terpadu (IC) yang terdiri dari ribuan transistor yang berfungsi sebagai saklar logika dengan dua keadaan, yaitu saklar terbuka atau saklar tertutup, maka dengan demikian komputer klasik memang sudah dirancang untuk bekerja dalam prinsip dua keadaan. Didalam memori komputer terdapat magnetik yang berisi ribuan register yang bekerja atas dasar dua keadaan yaitu fluks magnetik yang searah dengan jarum jam memiliki register (0) dan yang berlawanan dengan arah jarum jam memiliki register (1), sedangkan saklar elektronik yang bekerja dalam komputer juga berdasarkan dua keadaan yaitu saklar terbuka (0) dan saklar tertutup (1). Setiap sistem kuantum dua keadaan disebut sebagai qubit. Berikut adalah penerapan fisis dari qubit yang telah berhasil :

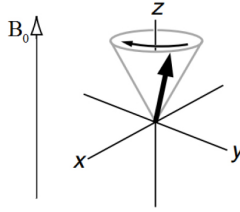
Tabel 2.3 Realiasi Fisis Kuantum Komputer

No	Sistem Fisis	Sistem dua keadaan	keadaan	
			$ 0\rangle$	$ 1\rangle$
1	NMR	spin inti atom	up	down
2	trapped ion, atom neutral dalam potensial optik, rongga QED dengan atom-atom	keadaan ion atau atom	Keadaan dasar	Keadaan terkesitasi
3	elektron	spin elektron	up	down
		Banyak elektron	Tidak ada elektron	ada elektron Tunggal
4	keadaan koheren cahaya	squeezed light	amplitude squeezed light	phase squeezed light
5	kisi optik	spin atom	up	down
6	josephson junction	Qubit bermuatan superkonduktor	Tidak bermuatan	bermuatan
		Qubit fluks superkonduktor	Arus searah jarum jam	Arus berlawanan arah jarum jam
		Qubit fase superkonduktor	Keadaan dasar	Keadaan eksitasi pertama
7	Pasangan kuantum dot bermuatan tunggal	Lokalisasi elektron	Elektron disebelah kiri dot	Elektron disebelah kanan dot
8	quantum dot	dot spin	down	up
9	foton	Pengkodean polarisasi	horizontal	vertikal
		Jumlah foton	vakum	Keadaan tunggal
		time-bin encoding	Lebih cepat	Lebih lambat

2.4 NMR (Nuclear Magnetic Resonance)

2.4.1 Presesi Larmor di dalam Medan Magnet Konstan

Apabila ada momen magnetik yang ditempatkan dalam ruang dimana terdapat medan magnet luar yang konstan yang telah ditentukan sebagai arahsumbu-z, sesuai dengan gambar 2.1 yaitu sebagai berikut:



Gambar 2.4 Presesi Larmor

maka momen magnetik tersebut akan mengalami interaksi dengan medan magnet luar sehingga mengalami presesi larmor. Dimana besarnya medan magnet luar sebesar:

$$\vec{B} = B_0 \hat{k} \quad (22)$$

Energi interaksi momen magnetik dengan medan luar dinyatakan oleh hamiltonian sebagai berikut:

$$H = -\vec{\mu} \cdot \vec{B} = -(\mu_0 \sigma_z \hbar) = -\mu_0 \sigma_0 B_0 \quad (23)$$

dimana σ_z merupakan matriks Pauli. Berikut adalah matriks Pauli:

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad (24)$$

apabila ditentukan bahwa

$$-\mu_0 \cdot B_0 = \frac{\hbar\omega_0}{2} \quad (25)$$

maka hamiltoniannya dapat dinyatakan sebagai berikut:

$$H = \frac{\hbar\omega_0}{2}\sigma_z \quad (26)$$

kemudian apabila hamiltonian bekerja pada vektor keadaan $|\pm\rangle$

$$H|\pm\rangle = \frac{\hbar\omega_0}{2}\sigma_z|\pm\rangle = \pm\frac{\hbar\omega_0}{2}|\pm\rangle \quad (27)$$

maka didapatkan bahwa medan luar yang konstan menyebabkan adanya dua tingkat energi pada sistem dengan kondisi awal dari vektor keadaan dinyatakan sebagai berikut:

$$|\psi(0)\rangle = \alpha|+\rangle + \beta|-\rangle \quad (28)$$

dengan α dan β merupakan koefisien kompleks yang memenuhi kaidah $|\alpha|^2 + |\beta|^2 = 1$ yang berevolusi terhadap waktu menjadi :

$$|\psi(t)\rangle = \alpha e^{\frac{-i\omega_0 t}{2}}|+\rangle + \beta e^{\frac{i\omega_0 t}{2}}|-\rangle \quad (29)$$

maka didapatkan nilai ekspektasi momen magnetik sebagai berikut:

$$\langle\mu\rangle = \mu_0\langle\psi(t)|(\sigma_x\hat{i} + \sigma_y\hat{j} + \sigma_z\hat{k})|\psi(t)\rangle \quad (30)$$

dimana $\vec{\sigma} = \sigma_x\hat{i} + \sigma_y\hat{j} + \sigma_z\hat{k}$, maka didapatkan

$$\langle\vec{\mu}\rangle = \mu_0\langle\psi(t)|\sigma_x\hat{i} + \sigma_y\hat{j} + \sigma_z\hat{k}|\psi(t)\rangle \quad (31)$$

maka nilai ekspektasi momen magnetik setiap sumbu adalah sebagai berikut:

$$\begin{aligned} \langle\vec{\mu}_0\rangle &= \mu_0\langle\psi(t)|\sigma_x|\psi(t)\rangle \\ &= \mu_0(\langle+|\alpha^*e^{\frac{-i\omega_0 t}{2}} + \langle-|\beta^*e^{\frac{i\omega_0 t}{2}})\sigma_x(\alpha^*e^{\frac{i\omega_0 t}{2}}|+\rangle + \beta^*e^{\frac{-i\omega_0 t}{2}}|-\rangle) \end{aligned}$$

mengingat bahwa $\sigma_x|\pm\rangle = |\mp\rangle$, maka didapatkan bahwa

$$\begin{aligned}
&= \mu_0(\langle +|\alpha^*e^{\frac{-i\omega_0 t}{2}} + \langle -|\beta^*e^{\frac{i\omega_0 t}{2}})(\alpha^*e^{\frac{i\omega_0 t}{2}}|-\rangle + \beta^*e^{\frac{-i\omega_0 t}{2}}|+\rangle) \\
&= \mu_0(\alpha^*\alpha\langle +|-\rangle + \alpha^*\beta e^{-i\omega_0 t}\langle +|+\rangle + \beta^*\alpha e^{i\omega_0 t}\langle -|-\rangle + \beta^*\beta\langle -|+\rangle) \\
&= \mu_0(\alpha^*\beta e^{-i\omega_0 t} + \beta^*\alpha e^{i\omega_0 t})
\end{aligned}$$

kemudian kita misalkan bahwa $\alpha = r_0 e^{i\theta_0}$, $\beta = r_1 e^{i\theta_1}$, $\psi = \theta_1 = \theta_2$, maka didapatkan

$$\langle \bar{\mu}_x \rangle = 2\mu_0 r_0 r_1 \cos(\omega_0 t + \psi) \quad (32)$$

kemudian komponen ekspektasi nilai momen magnetik terhadap sumbu-y adalah sebagai berikut:

$$\begin{aligned}
\langle \bar{\mu}_y \rangle &= \mu_0 \langle \psi(t) | \sigma_y | \psi(t) \rangle \\
&= \mu_0(\langle +|\alpha^*e^{\frac{-i\omega_0 t}{2}} + \langle -|\beta^*e^{\frac{i\omega_0 t}{2}}) \sigma_y (\alpha^*e^{\frac{i\omega_0 t}{2}}|+\rangle + \beta^*e^{\frac{-i\omega_0 t}{2}}|-\rangle)
\end{aligned}$$

mengingat bahwa $\sigma_y|\pm\rangle = \pm i|\mp\rangle$, maka didapatkan bahwa

$$\begin{aligned}
&= i\mu_0(\langle +|\alpha^*e^{\frac{-i\omega_0 t}{2}} + \langle -|\beta^*e^{\frac{i\omega_0 t}{2}})(\alpha^*e^{\frac{i\omega_0 t}{2}}|-\rangle - \beta^*e^{\frac{-i\omega_0 t}{2}}|+\rangle) \\
&= i\mu_0(\alpha^*\alpha\langle +|-\rangle - \alpha^*\beta e^{-i\omega_0 t}\langle +|+\rangle + \beta^*\alpha e^{i\omega_0 t}\langle -|-\rangle - \beta^*\beta\langle -|+\rangle) \\
&= i\mu_0(-\alpha^*\beta e^{-i\omega_0 t} + \beta^*\alpha e^{i\omega_0 t})
\end{aligned}$$

kemudian kita misalkan bahwa $\alpha = r_0 e^{i\theta_0}$, $\beta = r_1 e^{i\theta_1}$, $\psi = \theta_1 = \theta_2$, maka didapatkan

$$\langle \bar{\mu}_y \rangle = -2\mu_0 r_0 r_1 \sin(\omega_0 t + \psi) \quad (33)$$

kemudian komponen ekspektasi nilai momen magnetik terhadap sumbu-z adalah sebagai berikut:

$$\langle \bar{\mu}_z \rangle = \mu_0 \langle \psi(t) | \sigma_z | \psi(t) \rangle$$

$$= \mu_0(\langle + | \alpha^* e^{\frac{-i\omega_0 t}{2}} + \langle - | \beta^* e^{\frac{i\omega_0 t}{2}}) \sigma_z (\alpha^* e^{\frac{i\omega_0 t}{2}} | + \rangle + \beta^* e^{\frac{-i\omega_0 t}{2}} | - \rangle)$$

mengingat bahwa $\sigma_z |\pm\rangle = \pm |\pm\rangle$, maka didapatkan bahwa

$$\begin{aligned} &= \mu_0(\langle + | \alpha^* e^{\frac{-i\omega_0 t}{2}} + \langle - | \beta^* e^{\frac{i\omega_0 t}{2}}) (\alpha^* e^{\frac{i\omega_0 t}{2}} | + \rangle - \beta^* e^{\frac{-i\omega_0 t}{2}} | - \rangle) \\ &= \mu_0(\alpha^* \alpha \langle + | + \rangle + \beta^* \beta \langle - | - \rangle) \\ &= \mu_0(|\alpha|^2 + |\beta|^2) \end{aligned}$$

maka didapatkan

$$\langle \bar{\mu}_z \rangle = \mu_0(|\alpha|^2 + |\beta|^2) \quad (34)$$

2.4.2 Teknik Rabi

Apabila sistem momen magnetik dipengaruhi oleh medan magnet bergantung waktu yang tegak lurus dengan medan magnet statis, katakanlah medan magnet yang bergantung waktu adalah sebidang dengan bidang-xy, yang besarnya :

$$\bar{B}_1 = B_1 \cos \omega t \hat{i} + B_1 \sin \omega t \hat{j} \quad (35)$$

maka besar medan magnet total adalah sebagai berikut:

$$\bar{B}_1 = B_1 \cos \omega t \hat{i} + B_1 \sin \omega t \hat{j} + B_0 \hat{k} \quad (36)$$

dengan demikian hamiltonian interaksinya adalah sebagai berikut:

$$\begin{aligned} H &= \bar{\mu} \cdot \bar{B} \\ H &= \bar{\mu}_0(\sigma_x \hat{i} + \sigma_y \hat{j} + \sigma_z \hat{k}) \cdot (B_1 \cos \omega t \hat{i} + B_1 \sin \omega t \hat{j} + B_0 \hat{k}) \end{aligned}$$

maka didapatkan:

$$H = -\mu_0 B_1 \cos \omega t \sigma_x - B_1 \mu_0 \sin \omega t \sigma_y - \mu_0 B_0 \sigma_z \quad (37)$$

dalam keadaan sembarang setiap keadaan diberikan oleh bentuk umum

$$|\psi(t)\rangle = \alpha(t)|+\rangle + \beta(t)|-\rangle \quad (38)$$

persamaan schrodinger untuk keadaan tersebut adalah

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = H |\psi(t)\rangle \quad (39)$$

maka dapat kita selesaikan terlebih dahulu secara terpisah yaitu ruas kiri diselesaikan dulu kemudian ruas kanan kita selesaikan, berikut adalah penyelesaian ruas kiri:

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = i\hbar \dot{\alpha}(t)|+\rangle + i\hbar \dot{\beta}(t)|-\rangle \quad (40)$$

dan berikut dalah penyelesaian ruas kanannya

$$\begin{aligned} H|\psi(t)\rangle &= (-\mu_0 B_1 \cos \omega t \sigma_x - B_1 \mu_0 \sin \omega t \sigma_y - \mu_0 B_0 \sigma_z)(\alpha(t)|+\rangle + \beta(t)|-\rangle) \\ &= -\mu_0 B_1 \cos \omega t \sigma_x (\alpha(t)|+\rangle + \beta(t)|-\rangle) \\ &\quad - B_1 \mu_0 \sin \omega t \sigma_y (\alpha(t)|+\rangle + \beta(t)|-\rangle) - \mu_0 B_0 \sigma_z (\alpha(t)|+\rangle + \beta(t)|-\rangle) \\ &= -\mu_0 B_1 \cos \omega t (\alpha(t)|+\rangle + \beta(t)|-\rangle) \\ &\quad - i B_1 \mu_0 \sin \omega t (\alpha(t)|+\rangle - \beta(t)|-\rangle) - \mu_0 B_0 (\alpha(t)|+\rangle - \beta(t)|-\rangle) \\ &= (-\mu_0 B_1 \cos \omega t \alpha(t) - i B_1 \mu_0 \sin \omega t \alpha(t) + \mu_0 B_0)|-\rangle \\ &\quad + (-\mu_0 B_1 \cos \omega t \beta(t) + i B_1 \mu_0 \sin \omega t \beta(t) - \mu_0 B_0)|+\rangle \\ &= -\mu_0 (B_1 \alpha(t) e^{i\omega t} - B_0)|-\rangle - \mu_0 (B_1(t) e^{-i\omega t} + B_0)|+\rangle \quad (41) \end{aligned}$$

maka ketika kita samakan antara persamaan 2.19 (ruas kiri) dengan 2.20 (ruas kanan) maka didapatkan bahwa koefisien masing-masing keadaan.

untuk keadaan $|+\rangle$

$$\begin{aligned} i\hbar\dot{\alpha}(t) &= -\mu_0(B_1\beta(t)e^{-i\omega t} + \mu_0 B_0\alpha(t)) \\ i\dot{\alpha}(t) &= \frac{-\mu_0 B_1}{\hbar}\beta(t)e^{-i\omega t} - \frac{\mu_0 B_0}{\hbar}\alpha(t) \end{aligned}$$

kita ingat kembali persamaan 2.4, maka kita dapatkan bahwa

$$i\dot{\alpha}(t) = \frac{\omega_1}{2}\beta(t)e^{-i\omega t} - \frac{\omega_0}{2}\alpha(t) \quad (42)$$

untuk keadaan $|-\rangle$

$$\begin{aligned} i\hbar\dot{\beta}(t) &= -\mu_0(\beta_1\alpha(t)e^{-\omega t} + \mu_0 B_0\beta(t)) \\ i\dot{\beta}(t) &= \frac{-\mu_0 B_1}{\hbar}\alpha(t)e^{-i\omega t} - \frac{\mu_0 B_0}{\hbar}\beta(t) \end{aligned}$$

kita ingat kembali persamaan 2.4, maka kita dapatkan bahwa

$$i\dot{\beta}(t) = \frac{\omega_1}{2}\alpha(t)e^{-i\omega t} - \frac{\omega_0}{2}\beta(t) \quad (43)$$

untuk mempermudah perhitungan maka dilakukan perubahan koefisien kompleks yaitu sebagai berikut:

$$\begin{aligned} \dot{\alpha}(t) &= \dot{\alpha}(t)e^{\frac{i\omega t}{2}} \\ \dot{\beta}(t) &= \dot{\beta}(t)e^{\frac{-i\omega t}{2}} \end{aligned}$$

maka apabila kita turunkan koefisien tersebut satu kali terhadap waktu diperoleh

$$\begin{aligned} \dot{\alpha}(t) &= \dot{\alpha}(t)e^{\frac{i\omega t}{2}} + \frac{i\omega}{2}\alpha(t)e^{\frac{i\omega t}{2}} \\ \dot{\beta}(t) &= \dot{\beta}(t)e^{\frac{-i\omega t}{2}} - \frac{i\omega}{2}\beta(t)e^{\frac{-i\omega t}{2}} \end{aligned} \quad (44)$$

kemudian kita substitusikan persamaan 2.21 dan 2.22 ke persamaan 2.23, maka didapatkan hasil sebagai berikut:

$$\begin{aligned}\dot{\alpha}(t) &= \frac{\omega_1}{2i}\beta(t) + \frac{\omega_0 - \omega}{2i}\alpha(t) \\ \dot{\beta}(t) &= \frac{\omega_1}{2i}\alpha(t) - \frac{\omega_0 - \omega}{2i}\beta(t)\end{aligned}\quad (45)$$

maka kita dapatkan hamiltonian dalam bentuk matriks yaitu sebagai berikut:

$$\begin{aligned}H|\psi(t)\rangle &= i\hbar \frac{d}{dt}|\psi(t)\rangle \\ &= i\hbar \begin{bmatrix} \dot{\alpha}(t) \\ \dot{\beta}(t) \end{bmatrix} \\ &= i\hbar \begin{bmatrix} \frac{\omega_1}{2i}\beta(t) + \frac{\omega_0 - \omega}{2i}\alpha(t) \\ \frac{\omega_1}{2i}\alpha(t) - \frac{\omega_0 - \omega}{2i}\beta(t) \end{bmatrix} \\ &= \frac{i\hbar}{2i} \begin{bmatrix} \omega_0 - \omega & \omega_1 \\ \omega_1 & -(\omega_0 - \omega) \end{bmatrix} \begin{bmatrix} \alpha(t) \\ \beta(t) \end{bmatrix}\end{aligned}$$

sehingga diperoleh bentuk hamiltonian sebagai berikut:

$$\begin{aligned}H &= \frac{\hbar}{2} \begin{bmatrix} \omega_0 - \omega & \omega_1 \\ \omega_1 & -(\omega_0 - \omega) \end{bmatrix} \\ &= \frac{\hbar}{2} \begin{bmatrix} \omega_0 - \omega & 0 \\ 0 & -(\omega_0 - \omega) \end{bmatrix} + \frac{\hbar}{2} \begin{bmatrix} 0 & \omega_1 \\ \omega_1 & 0 \end{bmatrix}\end{aligned}$$

sehingga diperoleh

$$H = \frac{\hbar}{2}(\omega_0 - \omega) \sigma_z + \frac{\hbar}{2}\omega_1\sigma_x \quad (46)$$

kemudian kita turunkan koefisien kompleks α dan β satu kali lagi terhadap waktu maka kita dapatkan

$$\ddot{\alpha}(t) = -\frac{\omega_1}{2i}\beta(t) + \frac{\omega_0 - \omega}{2i}\dot{\alpha}(t)$$

$$\ddot{\beta}(t) = \frac{\omega_1}{2i}\alpha(t) - \frac{\omega_0 - \omega}{2i}\dot{\beta}(t) \quad (47)$$

kemudian apabila kita substitusikan persamaan 2.24 ke persamaan 2.26 maka kita dapatkan

$$\begin{aligned} \ddot{\alpha}(t) &= -\frac{1}{4}[(\omega_0 - \omega)^2 + \omega_1^2]\alpha(t) \\ \ddot{\beta}(t) &= -\frac{1}{4}[(\omega_0 - \omega)^2 + \omega_1^2]\beta(t) \end{aligned} \quad (48)$$

kita definisikan bahwa $\omega^2 = [(\omega_0 - \omega)^2 + \omega_1^2]$, maka diperoleh

$$\begin{aligned} \ddot{\alpha}(t) + \left(\frac{\omega}{2}\right)^2\alpha(t) &= 0 \\ \ddot{\beta}(t) + \left(\frac{\omega}{2}\right)^2\beta(t) &= 0 \end{aligned} \quad (49)$$

sebagaimana kita tahu bahwa solusi dari persamaan 2.28 merupakan kombinasi *sin* dan *cos*, yaitu sebagai berikut:

$$\begin{aligned} \alpha(t) &= A_1 \sin \frac{\omega t}{2} + A_2 \cos \frac{\omega t}{2} \\ \beta(t) &= A_3 \sin \frac{\omega t}{2} + A_4 \cos \frac{\omega t}{2} \end{aligned} \quad (50)$$

dimana A_1, A_2, A_3, A_4 merupakan konstanta, misalkan dalam keadaan spin up memiliki kondisi sebagai berikut:

$$\alpha(0) = 1 \text{ dan } \beta(0) = 0$$

maka kita dapatkan

$$\begin{aligned} \alpha(0) &= A_1 \sin \frac{0 \cdot t}{2} + A_2 \cos \frac{0 \cdot t}{2} \\ 1 &= A_1 \sin 0 + A_2 \cos 0 \\ 1 &= A_2 \end{aligned}$$

dan

$$\begin{aligned}\beta(0) &= A_3 \sin \frac{0.t}{2} + A_4 \cos \frac{0.t}{2} \\ 1 &= A_3 \sin 0 + A_4 \cos 0 \\ 1 &= A_4\end{aligned}$$

sehingga persamaan 2.29 menjadi berikut

$$\begin{aligned}\alpha(t) &= A_1 \sin \frac{\omega t}{2} + \cos \frac{\omega t}{2} \\ \beta(t) &= A_3 \sin \frac{\omega t}{2}\end{aligned}\tag{51}$$

untuk mendapatkan A_1 dan A_3 maka dipakai syarat orthonormalitas yaitu sebagai berikut:

$$\begin{aligned}1 &= \langle \psi(t) | \psi(t) \rangle \\ 1 &= (\langle + | \alpha^*(t) e^{-i\frac{\omega_0 t}{2}} + \langle - | \beta^*(t) e^{i\frac{\omega_0 t}{2}}) (\alpha(t) e^{-i\frac{\omega_0 t}{2}} | + \rangle + \beta(t) e^{i\frac{\omega_0 t}{2}} | - \rangle) \\ 1 &= \alpha^*(t) \alpha(t) + \beta^*(t) \beta(t) \\ 1 &= (A_1^* \sin \frac{\omega t}{2} + \cos \frac{\omega t}{2}) (A_1 \sin \frac{\omega t}{2} + \cos \frac{\omega t}{2}) + A_3^* A_3 \sin^2 \frac{\Omega t}{2} \\ 1 &= (|A_1|^2 + |A_3|^2) \sin^2 \frac{\Omega t}{2} + \cos^2 \frac{\Omega t}{2} + (A_1 + A_1^*) \sin \frac{\Omega t}{2} \cos \frac{\Omega t}{2}\end{aligned}$$

maka dapat kita selesaikan terlebih dahulu yang bagian tidak konstan harus dibuat nol, yaitu sebagai berikut:

$$(A_1 + A_1^*) \sin \frac{\Omega t}{2} \cos \frac{\Omega t}{2} = 0 \quad A_1 + A_1^* = 0$$

sehingga

$$A_1^* = -A_1$$

hal ini dapat terpenuhi jika A_1 merupakan perkalian bilangan imajiner dengan bagian imajiner suatu bilangan kompleks yang bagian rielnnya bernilai nol (imaginer murni).

$$A_1 = iA_5 \quad (52)$$

Kemudian yang bagian konstan dapat kita selesaikan, yaitu sebagai berikut

$$1 = (|A_1|^2 + |A_3|^2) \sin^2 \frac{\Omega t}{2} + \cos^2 \frac{\Omega t}{2}$$

maka kita dapat menggunakan kaidah $\sin^2 \theta + \cos^2 \theta = 1$, maka dengan nilai dari

$$|A_1|^2 + |A_3|^2 = 1 \quad (53)$$

persamaan 2.32 dapat dinyatakan dalam bentuk $(\frac{\omega - \omega_0}{\Omega})^2 + (\frac{\omega_1}{\Omega})^2$ dan dipenuhi oleh A_1 dan A_3 yaitu

$$\begin{aligned} A_1 &= i \frac{\omega - \omega_0}{\Omega} \\ A_3 &= -i \frac{\omega_1}{\Omega} \end{aligned} \quad (54)$$

berdasarkan persamaan 2.31 dengan 2.32 maka didapatkan keadaan awal spin adalah sebagai berikut:

$$\begin{aligned} \alpha(t) &= i \frac{\omega - \omega_0}{\Omega} \sin \frac{\Omega t}{2} + \cos \frac{\Omega t}{2} \\ \beta(t) &= -i \frac{\omega_1}{\Omega} \sin \frac{\Omega t}{2} \end{aligned} \quad (55)$$

maka dengan demikian diperoleh set lengkap keadaan adalah sebagai berikut:

$$\begin{aligned} |\psi\rangle &= i \frac{\omega - \omega_0}{\Omega} \sin \frac{\Omega t}{2} + \cos \frac{\Omega t}{2} |0\rangle \\ &\quad - i \frac{\omega_1}{\Omega} \sin \frac{\Omega t}{2} |1\rangle \end{aligned} \quad (56)$$

apabila kita ingin mengukur spin dari keadaan negatif $|-\rangle \equiv |1\rangle$ maka

$$\langle 1|\psi\rangle = -i\frac{\omega_1}{\Omega} \sin \frac{\Omega t}{2} \quad (57)$$

dan apabila dihitung probabilitas spin flip dari keadaan up ke keadaan down maka diperoleh :

$$\begin{aligned} P_{+\rightarrow-} &= |\langle 1|\psi\rangle|^2 \\ &= \left(\frac{\omega_1}{\Omega}\right)^2 \sin^2 \frac{\Omega t}{2} \end{aligned} \quad (58)$$

berdasarkan pers. (58) maka dapat diambil suatu keadaan khusus yaitu sebagai berikut:

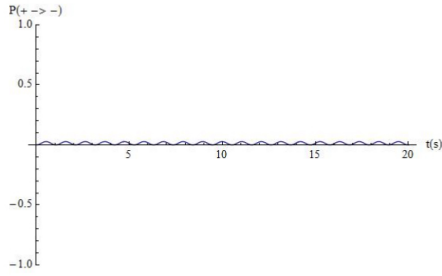
1. apabila $|\omega_0 - \omega_1| \gg \omega$ maka didapatkan spin flip dari keadaan up ke keadaan down memiliki probabilitas yang kecil, artinya sebagian besar masih tetap pada keadaan semula. (gambar a)
2. apabila $\omega = \omega_0$ maka pada waktu tertentu nilai probabilitas spin flip dari keadaan up ke keadaan down dapat bernilai 1, berikut adalah fungsi waktu untuk spin flip keadaan up ke keadaan down

$$t_n = \frac{(2n+1)\pi}{\omega_1} \quad (59)$$

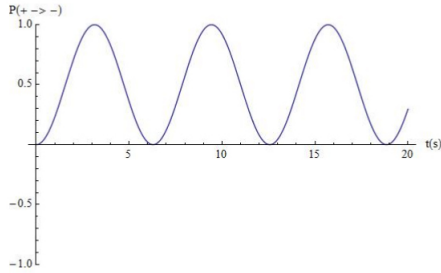
dengan $n=0,1,2,..$ (gambar b)

3. apabila $|\omega_o - \omega_1| \approx \omega$ maka spin flip dari keadaan up ke keadaan down memiliki probabilitas kurang dari 0.5 (gambar c)

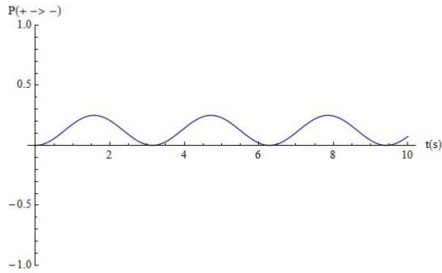
berikut adalah grafik probabilitas spin flip dari keadaan up ke keadaan down dari kondisi-kondisi diatas:



Gambar 2.4a Probabilitas untuk $|\omega_0 - \omega_1| \gg \omega$



Gambar 2.4b Probabilitas untuk $\omega = \omega_0$



Gambar 2.4c Probabilitas untuk $|\omega_o - \omega_1| \approx \omega$

BAB III

ALGORITMA DEUTSCH DAN DUETSCH JOSZA

sebelum membahas tentang algoritma Deutsch dan Algoritma Deutsch-Josza diperlukan untuk membahas terlebih dahulu tentang register, yaitu sebagai berikut:

3.1 Register

3.1.1 Register Qubit Tunggal

Pada algoritma Deutsch register yang diperlukan adalah register qubit tunggal dikarenakan Algoritma Deutsch berfungsi untuk memetakan input qubit tunggal ke output qubit tunggal. Maka registernya adalah sebagai berikut: $|0\rangle$ dan $|1\rangle$ sedangkan set lengkap dari keadaan adalah sebagai berikut:

$$|\psi\rangle = a|0\rangle + b|1\rangle \quad (60)$$

3.1.2 Register Qubit Jamak

Sebelum membahas tentang register pada qubit jamak maka diperlukan terlebih dahulu pemahaman tentang keadaan terbelit dan paralelisme kuantum. **Keadaan terbelit dan**

Paralelisme Kuantum

Keadaan Terbelit

keadaan terbelit kuantum adalah fungsi keadaan yang tidak dapat di pisah menjadi perkalian direct product:

$$|\psi\rangle \neq |\psi_1\rangle \otimes |\psi_2\rangle \quad (61)$$

contoh

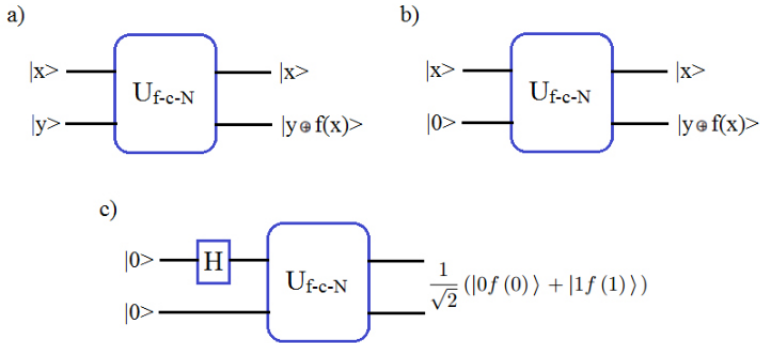
$$|\psi\rangle = \frac{1}{\sqrt{2}} [|00\rangle + |11\rangle] \quad (62)$$

Paralelisme Kuantum

Diberikan suatu fungsi $f(x)$ yang membangun suatu sirkuit kuantum U_{f-c-N} yang bekerja sesuai dengan persamaan berikut:

$$|xy\rangle \rightarrow |xy \oplus f(x)\rangle \quad (63)$$

berikut adalah beberapa contoh keadaan paralelisme kuantum:



Gambar 3.1 Paralelisme Kuantum

berdasarkan gambar diagram diatas maka dapat diketahui bahwa sirkuit memiliki masukan $|x\rangle$ dan $|y\rangle$ dan memiliki keluaran $|xy \oplus f(x)\rangle$ (gambar a), untuk (gambar b) masukan $|y\rangle = |0\rangle$ maka sirkuit akan memiliki keluaran $|xf(x)\rangle$ sedangkan pada (gambar c) sirkuit memiliki kedua masukan $|0\rangle$ namun salah satunya melewati gerbang hadamar sehingga keluaran sirkuit adalah $\frac{1}{\sqrt{2}}(|0f(0)\rangle + |1f(1)\rangle)$

3.1.2a Register 2 Qubit

dalam register 2 qubit masukan merupakan superposisi keadaan yaitu berupa direct product:

$$|00\rangle = |0\rangle \otimes |0\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (64)$$

sedangkkn dalam 2 qubit berikut adalah kemungkinan keadaan yang terjadi $|00\rangle, |01\rangle, |10\rangle, |11\rangle$, secara lengkap dapat dituliskan dalam bentuk

$$|\psi\rangle = a_0 |00\rangle + a_1 |01\rangle + a_3 |10\rangle + a_4 |11\rangle \quad (65)$$

3.1.2b Register 3 Qubit

Register 3 qubit masukan keadaan berupa $|000\rangle, |0001\rangle, \dots, |111\rangle$ maka secara lengkap set keadaan dapat dituliskan sebagai berikut:

$$\begin{aligned} |\psi\rangle = & a_0 |000\rangle + a_1 |001\rangle + a_2 |010\rangle + a_3 |011\rangle \\ & + a_4 |100\rangle + a_5 |101\rangle + a_6 |110\rangle + a_7 |111\rangle \end{aligned} \quad (66)$$

3.1.2c Register 4 Qubit

Register 4 qubit juga terdiri dari 4 direct product dari keadaan yaitu sebagai berikut $|0000\rangle, |0001\rangle, \dots, |1111\rangle$ dengan demikian set lengkap dari fungsi keadaan untuk 4 qubit dapat ditulis sebagai berikut:

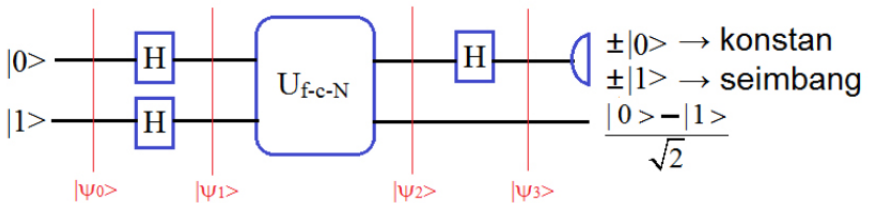
$$\begin{aligned} |\psi\rangle = & a_0 |0000\rangle + a_1 |0001\rangle + a_2 |0010\rangle + a_3 |0011\rangle \\ & + a_4 |0100\rangle + a_5 |0101\rangle + a_6 |0110\rangle + a_7 |0111\rangle \\ & + a_8 |1000\rangle + a_9 |1001\rangle + a_{10} |1010\rangle + a_{11} |1011\rangle \\ & + a_{12} |1100\rangle + a_{13} |1101\rangle + a_{14} |1110\rangle + a_{15} |1111\rangle \end{aligned} \quad (67)$$

3.2 Alogaritma Deutsch

Algoritma Deutsch merupakan algoritma kuantum yang pertama yang mana algoritma ini jauh lebih efisien daripada algoritma pada perhitungan klasik, Untuk memudahkan pemahaman maka kita misalkan $f : (0, 1) \rightarrow (0, 1)$ adalah fungsi biner maka fungsi tersebut hanya memiliki 4 kemungkinan yaitu :

$$\begin{aligned} f_1 : 0 \rightarrow 0, 1 \rightarrow 0 \\ f_2 : 0 \rightarrow 1, 1 \rightarrow 1 \\ f_3 : 0 \rightarrow 0, 1 \rightarrow 1 \\ f_4 : 0 \rightarrow 0, 1 \rightarrow 0 \end{aligned} \quad (68)$$

fungsi f_1 dan f_2 merupakan fungsi tetapan sedangkan f_3 dan f_4 merupakan fungsi setimbang. maka perlu dilakukan dua evaluasi untuk mengetahui bahwa fungsi f merupakan fungsi klasik atau fungsi setimbang.



Gambar 3.2 Skema Algoritma Deutsch

Algoritma kuantum yang paling sederhana untuk mengevaluasi fungsi f adalah algoritma Deutsch, sehingga dapat menentukan fungsi f merupakan tetapan atau setimbang, Evaluasi dapat dimulai dengan memasukkan qubit $|01\rangle$ Katakanlah $|\psi_{in} = |01\rangle$ merupakan keadaan masukan yang dikenai dengan operator Hadamard menghasilkan keadaan yang lain

katakanlah $|\psi_1\rangle$ Maka Nilainya adalah

$$\begin{aligned}
 |\psi_1\rangle &= H \otimes H |01\rangle \\
 &= H|0\rangle \otimes H|1\rangle \\
 &= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \\
 &= \frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle) \tag{69}
 \end{aligned}$$

kemudian kita dapat menerapkan operasi f pada keadaan $|\psi_1\rangle$ dalam suku operator uniter U_f

$$U_f|xy\rangle = |xy \otimes f(x)\rangle \tag{70}$$

maka operasi mendapatkan hasil

$$\begin{aligned}
 |\psi_2\rangle &= U_f|\psi_1\rangle \\
 &= \frac{1}{2}U_f(|00\rangle - |01\rangle + |10\rangle - |11\rangle)
 \end{aligned}$$

$$|\psi_2\rangle = \frac{1}{2}(|0\rangle|f(0)\rangle - |0\rangle|f(0)\rangle + |1\rangle + |1\rangle|f(1)\rangle - |1\rangle - f(1)\rangle) \tag{71}$$

dimana $-f(x)$ bukan merupakan $f(x)$

selanjutnya kita terapkan gerbang hadamard pada qubit pertama dan gerbang identitas pada qubit kedua

$$\begin{aligned}
|\psi_3\rangle &= H \otimes I |\psi_2\rangle \\
&= \frac{1}{2} H \otimes I (|0\rangle |f(0)\rangle - |0\rangle |f(1)\rangle + |1\rangle |f(1)\rangle - |1\rangle |-f(1)\rangle) \\
&= \frac{1}{2} (H |0\rangle \otimes I |f(0)\rangle - H |0\rangle \otimes I |-f(0)\rangle) \\
&\quad + \frac{1}{2} (H |1\rangle \otimes I |f(1)\rangle - H |1\rangle \otimes I |-f(1)\rangle) \\
&= \frac{1}{2\sqrt{2}} [(|0\rangle + |1\rangle) |f(0)\rangle - (|0\rangle + |1\rangle) |-f(0)\rangle] \\
&\quad + \frac{1}{2} [(|0\rangle - |1\rangle) |f(1)\rangle + (|0\rangle - |1\rangle) |-f(1)\rangle] \\
&= \frac{\sqrt{2}}{4} [(|0\rangle + |1\rangle) (|f(0)\rangle - |-f(0)\rangle)] \\
&\quad + \frac{1}{2} [(|0\rangle - |1\rangle) (|f(1)\rangle - |-f(1)\rangle)]
\end{aligned} \tag{72}$$

$|\psi_3\rangle$ merupakan output yang dapat mengetahui fungsi f itu konstan atau setimbang.

misalkan $f(0) = f(1)$ maka f konstan, sehingga:

$$\begin{aligned}
|\psi_3\rangle &= \frac{\sqrt{2}}{4} [(|0\rangle + |1\rangle) (|f(0)\rangle - |-f(0)\rangle)] \\
&\quad + \frac{\sqrt{2}}{4} [(|0\rangle - |1\rangle) (|f(1)\rangle - |-f(1)\rangle)] \\
&= \frac{\sqrt{2}}{4} (|0\rangle + |1\rangle + |0\rangle - |1\rangle) (|f(0)\rangle - |-f(0)\rangle) \tag{73} \\
&= \frac{\sqrt{2}}{4} 2 |0\rangle (|f(0)\rangle - |-f(0)\rangle) \\
&= \frac{1}{\sqrt{2}} |0\rangle (|f(0)\rangle - |-f(0)\rangle)
\end{aligned}$$

sedangkan untuk f merupakan fungsi setimbang dimana $f(0) = -f(1)$ dapat diuraikan sebagai berikut:

$$\begin{aligned}
|\psi_3\rangle &= \frac{\sqrt{2}}{4} [(|0\rangle + |1\rangle) (|f(0)\rangle - |-f(0)\rangle)] \\
&+ \frac{\sqrt{2}}{4} [(|0\rangle - |1\rangle) (|f(1)\rangle - |-f(1)\rangle)] \\
&= \frac{\sqrt{2}}{4} (|0\rangle + |1\rangle - (|0\rangle - |1\rangle)) (|f(0)\rangle - |-f(0)\rangle) \quad (74) \\
&= \frac{\sqrt{2}}{4} 2|1\rangle (|f(0)\rangle - |-f(0)\rangle) \\
&= \frac{\sqrt{2}}{2} |1\rangle (|f(0)\rangle - |-f(0)\rangle)
\end{aligned}$$

berikut adalah rincian luaran untuk setiap f : untuk $f_1(0) = 0$

$$\begin{aligned}
|\psi_{out}\rangle &= \frac{1}{\sqrt{2}} |0\rangle (|0\rangle - |1\rangle) \\
&= \frac{1}{\sqrt{2}} (|00\rangle - |01\rangle)
\end{aligned} \quad (75)$$

untuk $f_2(0) = 1$

$$\begin{aligned}
|\psi_{out}\rangle &= \frac{1}{\sqrt{2}} |1\rangle (|0\rangle - |0\rangle) \\
&= -\frac{1}{\sqrt{2}} (|00\rangle - |01\rangle)
\end{aligned} \quad (76)$$

maka didapatkan bahwa $f_2 = -f_1$ kemudian untuk fungsi yang setimbang didapatkan rincian sebagai berikut: untuk $f_3(0) = 0$

$$\begin{aligned}
|\psi_{out}\rangle &= \frac{1}{\sqrt{2}} |1\rangle (|0\rangle - |1\rangle) \\
&= \frac{1}{\sqrt{2}} (|10\rangle - |11\rangle)
\end{aligned} \quad (77)$$

untuk $f_4(0) = 1$

$$\begin{aligned} |\psi_{out}\rangle &= \frac{1}{\sqrt{2}} |1\rangle (|1\rangle - |0\rangle) \\ &= -\frac{1}{\sqrt{2}} (|10\rangle - |11\rangle) \end{aligned} \tag{78}$$

maka didapatkan bahwa $f_4 = -f_3$.

3.3 Algoritma Deutsch Josza pada sistem 2 qubit

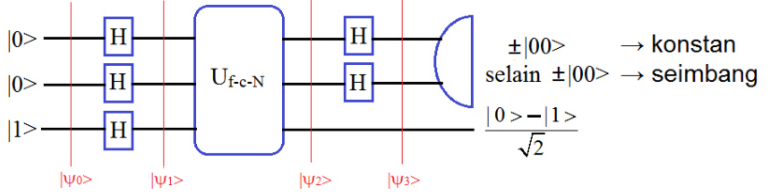
algoritma Deutsch-josza merupakan perluasan daripada algoritma Deutsch, yaitu dengan masukan n-qubit untuk memetakan pada qubit tunggal. misalkan kita memiliki suatu kotak hitam untuk 2 qubit maka kita mendapat masukan:

$$|\psi\rangle = |x_1x_0\rangle = |00\rangle + |01\rangle + |10\rangle + |11\rangle$$

dari masukan tersebut dapat dibuat tabel kemungkinan untuk fungsi-fungsi konstan dan setimbang yakni sebagai berikut:

fungsi	$ 00\rangle$	$ 01\rangle$	$ 10\rangle$	$ 11\rangle$
f_{k0}	0	0	0	0
f_{k1}	1	1	1	1
f_{s1}	1	1	0	0
f_{s2}	1	0	1	0
f_{s3}	1	0	0	1
f_{s4}	0	1	1	0
f_{s5}	0	1	0	1
f_{s6}	0	0	1	1

berikut adalah diagram algoritma Deutsch-Josza 2 qubit :



Gambar 3.3 Algoritma Deutsch-Josza 2 qubit

kemudian kemungkinan-kemungkinan fungsi konstan dan seimbang diatas di evaluasi sebagai berikut: maka kita definisikan terlebih dahulu untuk masukan, dengan syarat bahwa dalam algoritma Deutsch-josza diberikan qubit tambahan dengan keadaan $|1\rangle$ maka dengan demikian didapatkan :

$$|\psi_{in}\rangle = |0\rangle \otimes |0\rangle \otimes |1\rangle = |001\rangle \quad (79)$$

kemudian diaplikasikan transformasi walsh-Hadamard didapatkan:

$$\begin{aligned} |\psi_1\rangle &= H \otimes H \otimes H |\psi_{in}\rangle \\ &= H |0\rangle \otimes H |0\rangle \otimes H |1\rangle \\ &= \frac{1}{2^{\frac{3}{2}}} (|0\rangle + |1\rangle) (|0\rangle + |1\rangle) (|0\rangle - |1\rangle) \\ &= \frac{1}{2^{\frac{3}{2}}} [|000\rangle - |001\rangle + |010\rangle - |011\rangle] \\ &\quad + \frac{1}{2^{\frac{3}{2}}} [|100\rangle - |101\rangle + |110\rangle - |111\rangle] \end{aligned} \quad (80)$$

kemudian diterapakan u_f pada $|\psi_1\rangle$ daan didapatkan:

$$\begin{aligned}
|\psi_2\rangle &= U_f |\psi_1\rangle \\
&= \frac{1}{2^{\frac{3}{2}}} [U |000\rangle - U |001\rangle + U |010\rangle - U |011\rangle] \\
&+ \frac{1}{2^{\frac{3}{2}}} [U |100\rangle - U |101\rangle + U |110\rangle - U |111\rangle] \\
&= \frac{1}{2^{\frac{3}{2}}} (|000 \oplus f(00)\rangle - |001 \oplus f(00)\rangle) \\
&+ \frac{1}{2^{\frac{3}{2}}} (|010 \oplus f(01)\rangle - |011 \oplus f(01)\rangle) \\
&+ \frac{1}{2^{\frac{3}{2}}} (|100 \oplus f(10)\rangle - |101 \oplus f(10)\rangle) \\
&+ \frac{1}{2^{\frac{3}{2}}} (|110 \oplus f(11)\rangle - |111 \oplus f(11)\rangle) \\
&= \frac{1}{2^{\frac{3}{2}}} |00\rangle (|0 \oplus f(00)\rangle - |1 \oplus f(00)\rangle) \\
&+ \frac{1}{2^{\frac{3}{2}}} |01\rangle (|0 \oplus f(01)\rangle - |1 \oplus f(01)\rangle) \\
&+ \frac{1}{2^{\frac{3}{2}}} |10\rangle (|0 \oplus f(10)\rangle - |1 \oplus f(10)\rangle) \\
&+ \frac{1}{2^{\frac{3}{2}}} |11\rangle (|0 \oplus f(11)\rangle - |1 \oplus f(11)\rangle) \\
&= \frac{1}{2^{\frac{3}{2}}} |00\rangle (|f(00)\rangle - |-f(00)\rangle) \\
&+ \frac{1}{2^{\frac{3}{2}}} |01\rangle (|f(01)\rangle - |-f(01)\rangle) \\
&+ \frac{1}{2^{\frac{3}{2}}} |10\rangle (|f(10)\rangle - |-f(10)\rangle) \\
&+ \frac{1}{2^{\frac{3}{2}}} |11\rangle (|f(11)\rangle - |-f(11)\rangle)
\end{aligned} \tag{81}$$

kemudian diterapkan kembali transformasi walsh hadamard maka diperoleh:

$$\begin{aligned}
|\psi_3\rangle &= (H \otimes H \otimes I) |\psi_2\rangle \\
&= \frac{1}{2^{\frac{3}{2}}} H |0\rangle \otimes H |0\rangle \otimes I (|f(00)\rangle - |-f(00)\rangle) \\
&+ \frac{1}{2^{\frac{3}{2}}} H |0\rangle \otimes H |1\rangle \otimes I (|f(01)\rangle - |-f(01)\rangle) \\
&+ \frac{1}{2^{\frac{3}{2}}} H |1\rangle \otimes H |0\rangle \otimes I (|f(10)\rangle - |-f(10)\rangle) \\
&+ \frac{1}{2^{\frac{3}{2}}} |1\rangle \otimes H |1\rangle \otimes I (|f(11)\rangle - |-f(11)\rangle) \\
&= \frac{1}{2^{\frac{3}{2}}} (|0\rangle + |1\rangle) (|0\rangle + |1\rangle) (|f(00)\rangle - |-f(00)\rangle) \\
&+ \frac{1}{2^{\frac{3}{2}}} (|0\rangle + |1\rangle) (|0\rangle - |1\rangle) (|f(01)\rangle - |-f(01)\rangle) \\
&+ \frac{1}{2^{\frac{3}{2}}} (|0\rangle - |1\rangle) (|0\rangle + |1\rangle) (|f(10)\rangle - |-f(10)\rangle) \\
&+ \frac{1}{2^{\frac{3}{2}}} (|0\rangle - |1\rangle) (|0\rangle - |1\rangle) (|f(11)\rangle - |-f(11)\rangle) \\
&= \frac{1}{2^{\frac{5}{2}}} (|00\rangle + |01\rangle + |10\rangle + |11\rangle) (|f(00)\rangle - |-f(00)\rangle) \\
&+ \frac{1}{2^{\frac{5}{2}}} (|00\rangle - |01\rangle + |10\rangle - |11\rangle) (|f(01)\rangle - |-f(01)\rangle) \\
&+ \frac{1}{2^{\frac{5}{2}}} (|00\rangle + |01\rangle - |10\rangle - |11\rangle) (|f(10)\rangle - |-f(10)\rangle) \\
&+ \frac{1}{2^{\frac{5}{2}}} (|00\rangle - |01\rangle - |10\rangle + |11\rangle) (|f(11)\rangle - |-f(11)\rangle)
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{2^{\frac{5}{2}}} (|00\rangle [|f(00)\rangle - |-f(00)\rangle + |f(01)\rangle - |-f(01)\rangle]) \\
&+ \frac{1}{2^{\frac{5}{2}}} (|00\rangle [|f(10)\rangle - |-f(10)\rangle + |f(11)\rangle - |-f(11)\rangle]) \\
&+ \frac{1}{2^{\frac{5}{2}}} (|01\rangle [|f(00)\rangle - |-f(00)\rangle - |f(01)\rangle + |-f(01)\rangle]) \\
&+ \frac{1}{2^{\frac{5}{2}}} (|01\rangle [|f(10)\rangle - |-f(10)\rangle - |f(11)\rangle + |-f(11)\rangle]) \\
&+ \frac{1}{2^{\frac{5}{2}}} (|10\rangle [|f(00)\rangle - |-f(00)\rangle + |f(01)\rangle - |-f(01)\rangle]) \\
&- \frac{1}{2^{\frac{5}{2}}} (|10\rangle [|f(10)\rangle + |-f(10)\rangle - |f(11)\rangle + |-f(11)\rangle]) \\
&+ \frac{1}{2^{\frac{5}{2}}} (|11\rangle [|f(00)\rangle - |-f(00)\rangle - |f(01)\rangle + |-f(01)\rangle]) \\
&- \frac{1}{2^{\frac{5}{2}}} (|11\rangle [|f(10)\rangle + |-f(10)\rangle + |f(11)\rangle - |-f(11)\rangle])
\end{aligned} \tag{82}$$

kemudian dievaluasi untuk setiap fungsi konstan dan setimbang.

untuk fungsi setimbang maka kita dapat mengambil bahwa $f(00) = f(01) = f(10) = f(11)$ maka dengan demikian didapatkan persamaan $|\psi_3\rangle$ untuk fungsi setimbang sebagai

berikut:

$$\begin{aligned}
|\psi_3\rangle &= \frac{1}{2^{\frac{5}{2}}} (|00\rangle [|f(00)\rangle - |-f(00)\rangle + |f(00)\rangle - |-f(00)\rangle]) \\
&\quad \frac{1}{2^{\frac{5}{2}}} (|00\rangle [+|f(00)\rangle - |-f(00)\rangle + |f(00)\rangle - |-f(00)\rangle]) \\
&\quad + \frac{1}{2^{\frac{5}{2}}} (|01\rangle [|f(00)\rangle - |-f(00)\rangle - |f(00)\rangle + |-f(00)\rangle]) \\
&\quad + \frac{1}{2^{\frac{5}{2}}} (|01\rangle [|f(00)\rangle - |-f(00)\rangle - |f(00)\rangle + |-f(00)\rangle]) \\
&\quad + \frac{1}{2^{\frac{5}{2}}} (|10\rangle [|f(00)\rangle - |-f(00)\rangle + |f(00)\rangle - |-f(00)\rangle]) \\
&\quad - \frac{1}{2^{\frac{5}{2}}} (|10\rangle [|f(00)\rangle + |-f(00)\rangle - |f(00)\rangle + |-f(00)\rangle]) \\
&\quad + \frac{1}{2^{\frac{5}{2}}} (|11\rangle [|f(00)\rangle - |-f(00)\rangle - |f(00)\rangle + |-f(00)\rangle]) \\
&\quad - \frac{1}{2^{\frac{5}{2}}} (|11\rangle [|f(00)\rangle + |-f(00)\rangle + |f(00)\rangle - |-f(00)\rangle]) \\
&= \frac{1}{2^{\frac{5}{2}}} 4 (|00\rangle [|f(00)\rangle - |-f(00)\rangle] + |01\rangle .0 + |10\rangle .0 + |11\rangle .0) \\
&= \frac{1}{\sqrt{2}} (|00\rangle [|f(00)\rangle - |-f(00)\rangle])
\end{aligned} \tag{83}$$

untuk fungsi konstan $f_1(00) = 0$, kita dapatkan

$$\begin{aligned}
|\psi_3\rangle &= \frac{1}{\sqrt{2}} |00\rangle [|0\rangle - |1\rangle] \\
&= \frac{1}{\sqrt{2}} [|000\rangle - |001\rangle]
\end{aligned} \tag{84}$$

sedangkan untuk fungsi setimbang $f_2(00) = 1$ maka didapatkan:

$$\begin{aligned} |\psi_3\rangle &= \frac{1}{\sqrt{2}} |00\rangle [|1\rangle - |0\rangle] \\ &= -\frac{1}{\sqrt{2}} [|000\rangle - |001\rangle] \end{aligned} \quad (85)$$

maka dengan demikian didapatkan bahwa $|\psi_{out2}\rangle = -|\psi_{out1}\rangle$.

untuk fungsi setimbang, diambil contoh evaluasi untuk f_8 dan f_{11} , yaitu sebagai berikut:

untuk f_8 diketahui dari tabel bahwa $f_8(00) = f_8(10) = 1$ dan $f_8(01) = f_8(11)$, maka dengan demikian didapatkan:

$$\begin{aligned} |\psi_3\rangle &= \frac{1}{2^{\frac{5}{2}}} (|00\rangle [|1\rangle - |0\rangle + |0\rangle - |1\rangle + |1\rangle - |0\rangle + |0\rangle - |1\rangle]) \\ &+ \frac{1}{2^{\frac{5}{2}}} (|01\rangle [|1\rangle - |0\rangle - |0\rangle + |1\rangle + |1\rangle - |0\rangle - |0\rangle + |1\rangle]) \\ &+ \frac{1}{2^{\frac{5}{2}}} (|10\rangle [|1\rangle - |0\rangle + |0\rangle - |1\rangle - |1\rangle + |0\rangle - |0\rangle + |1\rangle]) \\ &+ \frac{1}{2^{\frac{5}{2}}} (|11\rangle [|1\rangle - |0\rangle - |0\rangle + |1\rangle - |1\rangle + |0\rangle + |0\rangle - |1\rangle]) \quad (86) \\ &= \frac{1}{2^{\frac{5}{2}}} 4 |01\rangle (|1\rangle - |0\rangle) \\ &= \frac{1}{\sqrt{2}} (|011\rangle - |010\rangle) \end{aligned}$$

sedangkan untuk fungsi setimbang f_{11} diketahui bahwa $f_{11}(00) = f_{11}(10) = 0$ dan $f_{11}(01) = f_{11}(11) = 1$, sehingga didapatkan

sebagai berikut:

$$\begin{aligned}
|\psi_3\rangle &= \frac{1}{2^{\frac{5}{2}}} (|00\rangle [|0\rangle - |1\rangle + |1\rangle - |0\rangle + |0\rangle - |1\rangle + |1\rangle - |0\rangle]) \\
&+ \frac{1}{2^{\frac{5}{2}}} (|01\rangle [|0\rangle - |1\rangle - |1\rangle + |0\rangle + |0\rangle - |1\rangle - |1\rangle + |0\rangle]) \\
&+ \frac{1}{2^{\frac{5}{2}}} (|10\rangle [|0\rangle - |1\rangle + |1\rangle - |0\rangle - |0\rangle + |1\rangle - |1\rangle + |0\rangle]) \\
&+ \frac{1}{2^{\frac{5}{2}}} (|11\rangle [|0\rangle - |1\rangle - |1\rangle + |0\rangle - |0\rangle + |1\rangle + |1\rangle - |0\rangle]) \quad (87) \\
&= \frac{1}{2^{\frac{5}{2}}} 4 |01\rangle (|0\rangle - |1\rangle) \\
&= -\frac{1}{\sqrt{2}} (|011\rangle - |010\rangle)
\end{aligned}$$

maka dengan demikian diperoleh bahwa $|\psi_{out11}\rangle = -|\psi_{out8}\rangle$.
 untuk selanjutnya juga dilakukan evaluasi terhadap fungsi-
 fungsi setimbang yang lain dan di dapatkan bahwa

$$|\psi_{out}\rangle = \left\{ \begin{array}{ll} |00\rangle \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}} \right] & \text{untuk } f_{k0} \quad \text{konstan0} \\ -|00\rangle \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}} \right] & \text{untuk } f_{k1} \quad \text{konstan1} \\ -|10\rangle \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}} \right] & \text{untuk } f_{s1} \quad \text{setimbang} \\ -|01\rangle \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}} \right] & \text{untuk } f_{s2} \quad \text{setimbang} \\ -|11\rangle \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}} \right] & \text{untuk } f_{s3} \quad \text{setimbang} \\ |11\rangle \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}} \right] & \text{untuk } f_{s4} \quad \text{setimbang} \\ |01\rangle \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}} \right] & \text{untuk } f_{s5} \quad \text{setimbang} \\ |10\rangle \left[\frac{|0\rangle - |1\rangle}{\sqrt{2}} \right] & \text{untuk } f_{s6} \quad \text{setimbang} \end{array} \right. \quad (88)$$

3.4 Algoritma Deutsch-Jozsa pada sistem 3 qubit

inti daripada algoritma Deutsch-jozsa adalah memetakan qubit banyak ke qubit tunggal. Dalam sistem 3 qubit kotak hitam memiliki masukan keadaan:

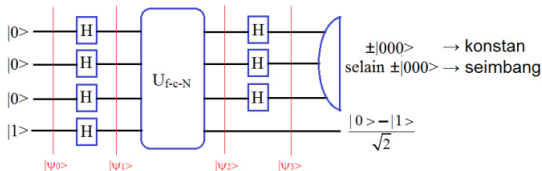
$$|\psi\rangle = |x_2x_1x_0\rangle = |000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle + |101\rangle + |110\rangle + |111\rangle \quad (89)$$

berdasarkan masukan tersebut maka dapat dibuat tabel kemungkinan-kemungkinan fungsi konstan dan setimbang dalam sistem 3 qubit yaitu sebagai berikut:

Tabel 3.2 Kemungkinan Fungsi-Fungsi Kotak Hitam Pada sistem 3 qubit

$ x_2x_1x_0\rangle$	f_{k0}	f_{k0}	f_{k1}	f_{s2}	f_{s3}	f_{s4}	f_{s5}	f_{s6}	f_{s7}	f_{s8}	f_{s9}
$ 000\rangle$	0	1	0	0	0	0	0	0	0	0	0
$ 001\rangle$	0	1	0	1	1	1	1	0	0	0	0
$ 010\rangle$	0	1	1	1	0	0	1	0	1	0	1
$ 011\rangle$	0	1	1	0	1	1	0	1	0	1	0
$ 100\rangle$	0	1	1	1	0	1	1	1	1	0	1
$ 101\rangle$	0	1	1	0	1	0	0	1	1	1	0
$ 110\rangle$	0	1	0	0	1	0	1	0	1	1	1
$ 111\rangle$	0	1	0	1	0	1	0	1	0	1	1

berikut adalah diagram algoritma Deutsch-Josza 3 qubit :



Gambar 3.4 Algoritma Deutsch-Josza 3 qubit

misalkan qubit masukan memiliki keadaan

$$|\psi_{in}\rangle = |0\rangle \otimes |0\rangle \otimes |0\rangle \otimes |1\rangle = |0001\rangle \quad (90)$$

dari qubit masukan tersebut diaplikasikan transformasi Walsh-Hadamard dan didapatkan :

$$\begin{aligned} |\psi_1\rangle &= (H \otimes H \otimes H \otimes H) |\psi_{in}\rangle \\ &= H |0\rangle \otimes H |0\rangle \otimes H |0\rangle \otimes H |1\rangle \\ &= \frac{1}{4} [(|0\rangle + |1\rangle) (|0\rangle + |1\rangle) (|0\rangle + |1\rangle) (|0\rangle - |1\rangle)] \\ &= \frac{1}{4} [|0000\rangle - |0001\rangle + |0010\rangle - |0011\rangle] \\ &\quad + \frac{1}{4} [|0100\rangle - |0101\rangle + |0110\rangle - |0111\rangle] \\ &\quad + \frac{1}{4} [|1000\rangle - |1001\rangle + |1010\rangle - |1011\rangle] \\ &\quad + \frac{1}{4} [|1100\rangle - |1101\rangle + |1110\rangle - |1111\rangle] \end{aligned} \quad (91)$$

kemudian diaplikasikan operator u_f pada $|\psi_1\rangle$ sehingga didapatkan:

$$\begin{aligned} |\psi_2\rangle &= \frac{1}{4} [U |0000\rangle - U |0001\rangle + U |0010\rangle - U |0011\rangle] \\ &\quad + \frac{1}{4} [U |0100\rangle - U |0101\rangle + U |0110\rangle - U |0111\rangle] \\ &\quad + \frac{1}{4} [U |1000\rangle - U |1001\rangle + U |1010\rangle - U |1011\rangle] \\ &\quad + \frac{1}{4} [U |1100\rangle - U |1101\rangle + U |1110\rangle - U |1111\rangle] \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{4} [|0000 \oplus f(000)\rangle - |0001 \oplus f(000)\rangle] \\
&+ \frac{1}{4} [|0010 \oplus f(001)\rangle - |0011 \oplus f(001)\rangle] \\
&+ \frac{1}{4} [|0100 \oplus f(010)\rangle - |0101 \oplus f(010)\rangle] \\
&+ \frac{1}{4} [|0110 \oplus f(011)\rangle - |0111 \oplus f(011)\rangle] \\
&+ \frac{1}{4} [|1000 \oplus f(100)\rangle - |1001 \oplus f(100)\rangle] \\
&+ \frac{1}{4} [|1010 \oplus f(101)\rangle - |1011 \oplus f(101)\rangle] \\
&+ \frac{1}{4} [|1100 \oplus f(110)\rangle - |1101 \oplus f(110)\rangle] \\
&+ \frac{1}{4} [|1110 \oplus f(111)\rangle - |1111 \oplus f(111)\rangle] \\
&= \frac{1}{4} [|000\rangle (|0 \oplus f(000)\rangle - |1 \oplus f(000)\rangle)] \\
&+ \frac{1}{4} [|001\rangle (|0 \oplus f(001)\rangle - |1 \oplus f(001)\rangle)] \\
&+ \frac{1}{4} [|010\rangle (|0 \oplus f(010)\rangle - |1 \oplus f(010)\rangle)] \\
&+ \frac{1}{4} [|011\rangle (|0 \oplus f(011)\rangle - |1 \oplus f(011)\rangle)] \\
&+ \frac{1}{4} [|100\rangle (|0 \oplus f(100)\rangle - |1 \oplus f(100)\rangle)] \\
&+ \frac{1}{4} [|101\rangle (|0 \oplus f(101)\rangle - |1 \oplus f(101)\rangle)] \\
&+ \frac{1}{4} [|110\rangle (|0 \oplus f(110)\rangle - |1 \oplus f(110)\rangle)] \\
&+ \frac{1}{4} [|111\rangle (|0 \oplus f(111)\rangle - |1 \oplus f(111)\rangle)]
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{4} [|000\rangle (|f(000)\rangle - |-f(000)\rangle)] \\
&+ \frac{1}{4} [|001\rangle (|f(001)\rangle - |-f(001)\rangle)] \\
&+ \frac{1}{4} [|010\rangle (|f(010)\rangle - |-f(010)\rangle)] \\
&+ \frac{1}{4} [|011\rangle (|f(011)\rangle - |-f(011)\rangle)] \\
&+ \frac{1}{4} [|100\rangle (|f(100)\rangle - |-f(100)\rangle)] \\
&+ \frac{1}{4} [|101\rangle (|f(101)\rangle - |-f(101)\rangle)] \\
&+ \frac{1}{4} [|110\rangle (|f(110)\rangle - |-f(110)\rangle)] \\
&+ \frac{1}{4} [|111\rangle (|f(111)\rangle - |-f(111)\rangle)]
\end{aligned} \tag{92}$$

kemudian diaplikasin transformasi walsh-hadamard lagi sehingga didapatkan $|\psi_3\rangle$, yaitu sebagai berikut:

$$\begin{aligned}
|\psi_3\rangle &= (H \otimes H \otimes H \otimes I) |\psi_2\rangle \\
&= \frac{1}{4} [H |0\rangle \otimes H |0\rangle \otimes H |0\rangle \otimes I (|f(000)\rangle - |-f(000)\rangle)] \\
&+ \frac{1}{4} [H |0\rangle \otimes H |0\rangle \otimes H |1\rangle \otimes I (|f(001)\rangle - |-f(001)\rangle)] \\
&+ \frac{1}{4} [H |0\rangle \otimes H |1\rangle \otimes H |0\rangle \otimes I (|f(010)\rangle - |-f(010)\rangle)] \\
&+ \frac{1}{4} [H |0\rangle \otimes H |1\rangle \otimes H |1\rangle \otimes I (|f(011)\rangle - |-f(011)\rangle)] \\
&+ \frac{1}{4} [H |1\rangle \otimes H |0\rangle \otimes H |0\rangle \otimes I (|f(100)\rangle - |-f(100)\rangle)] \\
&+ \frac{1}{4} [H |1\rangle \otimes H |0\rangle \otimes H |1\rangle \otimes I (|f(101)\rangle - |-f(101)\rangle)] \\
&+ \frac{1}{4} [H |1\rangle \otimes H |1\rangle \otimes H |0\rangle \otimes I (|f(110)\rangle - |-f(110)\rangle)] \\
&+ \frac{1}{4} [H |1\rangle \otimes H |1\rangle \otimes H |1\rangle \otimes I (|f(111)\rangle - |-f(111)\rangle)]
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{2^{\frac{7}{2}}} [(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)(|f(000)\rangle - |-f(000)\rangle)] \\
&+ \frac{1}{2^{\frac{7}{2}}} [(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)(|0\rangle - |1\rangle)(|f(001)\rangle - |-f(001)\rangle)] \\
&+ \frac{1}{2^{\frac{7}{2}}} [(|0\rangle + |1\rangle)(|0\rangle - |1\rangle)(|0\rangle + |1\rangle)(|f(010)\rangle - |-f(010)\rangle)] \\
&+ \frac{1}{2^{\frac{7}{2}}} [(|0\rangle + |1\rangle)(|0\rangle - |1\rangle)(|0\rangle - |1\rangle)(|f(011)\rangle - |-f(011)\rangle)] \\
&+ \frac{1}{2^{\frac{7}{2}}} [(|0\rangle - |1\rangle)(|0\rangle + |1\rangle)(|0\rangle + |1\rangle)(|f(100)\rangle - |-f(100)\rangle)] \\
&+ \frac{1}{2^{\frac{7}{2}}} [(|0\rangle - |1\rangle)(|0\rangle + |1\rangle)(|0\rangle - |1\rangle)(|f(101)\rangle - |-f(101)\rangle)] \\
&+ \frac{1}{2^{\frac{7}{2}}} [(|0\rangle - |1\rangle)(|0\rangle - |1\rangle)(|0\rangle + |1\rangle)(|f(110)\rangle - |-f(110)\rangle)] \\
&- \frac{1}{2^{\frac{7}{2}}} [(|0\rangle - |1\rangle)(|0\rangle - |1\rangle)(|0\rangle - |1\rangle)(|f(111)\rangle - |-f(111)\rangle)]
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{2^{\frac{7}{2}}} [(|000\rangle + |001\rangle + |010\rangle + |011\rangle + |100\rangle + |101\rangle + |110\rangle + |111\rangle) (|f(000)\rangle - |-f(000)\rangle)] \\
&+ \frac{1}{2^{\frac{7}{2}}} [(|000\rangle - |001\rangle + |010\rangle - |011\rangle + |100\rangle - |101\rangle + |110\rangle - |111\rangle) (|f(001)\rangle - |-f(001)\rangle)] \\
&+ \frac{1}{2^{\frac{7}{2}}} [(|000\rangle + |001\rangle - |010\rangle - |011\rangle + |100\rangle + |101\rangle - |110\rangle - |111\rangle) (|f(010)\rangle - |-f(010)\rangle)] \\
&+ \frac{1}{2^{\frac{7}{2}}} [(|000\rangle - |001\rangle - |010\rangle + |011\rangle + |100\rangle - |101\rangle - |110\rangle + |111\rangle) (|f(011)\rangle - |-f(011)\rangle)] \\
&+ \frac{1}{2^{\frac{7}{2}}} [(|000\rangle + |001\rangle + |010\rangle + |011\rangle - |100\rangle - |101\rangle - |110\rangle - |111\rangle) (|f(100)\rangle - |-f(100)\rangle)] \\
&+ \frac{1}{2^{\frac{7}{2}}} [(|000\rangle - |001\rangle + |010\rangle - |011\rangle - |100\rangle + |101\rangle - |110\rangle + |111\rangle) (|f(101)\rangle - |-f(101)\rangle)] \\
&+ \frac{1}{2^{\frac{7}{2}}} [(|000\rangle + |001\rangle - |010\rangle - |011\rangle - |100\rangle - |101\rangle + |110\rangle + |111\rangle) (|f(110)\rangle - |-f(110)\rangle)] \\
&+ \frac{1}{2^{\frac{7}{2}}} [(|000\rangle - |001\rangle - |010\rangle - |011\rangle - |100\rangle - |101\rangle - |110\rangle - |111\rangle) (|f(111)\rangle - |-f(111)\rangle)]
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{2^{\frac{7}{2}}} [|000\rangle (|f(000)\rangle - |-f(000)\rangle + |f(001)\rangle - |-f(001)\rangle)] \\
&+ \frac{1}{2^{\frac{7}{2}}} [|000\rangle (|f(010)\rangle - |-f(010)\rangle + |f(011)\rangle - |-f(011)\rangle)] \\
&+ \frac{1}{2^{\frac{7}{2}}} [|000\rangle (|f(100)\rangle - |-f(100)\rangle + |f(101)\rangle - |-f(101)\rangle)] \\
&+ \frac{1}{2^{\frac{7}{2}}} [|000\rangle (|f(110)\rangle - |-f(110)\rangle + |f(111)\rangle - |-f(111)\rangle)] \\
&+ \frac{1}{2^{\frac{7}{2}}} [|001\rangle (|f(000)\rangle - |-f(000)\rangle - |f(001)\rangle + |-f(001)\rangle)] \\
&+ \frac{1}{2^{\frac{7}{2}}} [|001\rangle (|f(010)\rangle - |-f(010)\rangle - |f(011)\rangle + |-f(011)\rangle)] \\
&+ \frac{1}{2^{\frac{7}{2}}} [|001\rangle (|f(100)\rangle - |-f(100)\rangle - |f(101)\rangle + |-f(101)\rangle)] \\
&+ \frac{1}{2^{\frac{7}{2}}} [|001\rangle (|f(110)\rangle - |-f(110)\rangle - |f(111)\rangle + |-f(111)\rangle)] \\
&+ \frac{1}{2^{\frac{7}{2}}} [|010\rangle (|f(000)\rangle - |-f(000)\rangle + |f(001)\rangle - |-f(001)\rangle)] \\
&+ \frac{1}{2^{\frac{7}{2}}} [|010\rangle (-|f(010)\rangle + |-f(010)\rangle - |f(011)\rangle + |-f(011)\rangle)] \\
&+ \frac{1}{2^{\frac{7}{2}}} [|010\rangle (|f(100)\rangle - |-f(100)\rangle + |f(101)\rangle - |-f(101)\rangle)] \\
&+ \frac{1}{2^{\frac{7}{2}}} [|010\rangle (-|f(110)\rangle + |-f(110)\rangle - |f(111)\rangle + |-f(111)\rangle)] \\
&+ \frac{1}{2^{\frac{7}{2}}} [|011\rangle (|f(000)\rangle - |-f(000)\rangle - |f(001)\rangle + |-f(001)\rangle)]
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{2^{\frac{7}{2}}} [|011\rangle (-|f(010)\rangle + |-f(010)\rangle + |f(011)\rangle - |-f(011)\rangle)] \\
& + \frac{1}{2^{\frac{7}{2}}} [|011\rangle (|f(100)\rangle - |-f(100)\rangle - |f(101)\rangle + |-f(101)\rangle)] \\
& + \frac{1}{2^{\frac{7}{2}}} [|011\rangle (-|f(110)\rangle + |-f(110)\rangle + |f(111)\rangle - |-f(111)\rangle)] \\
& + \frac{1}{2^{\frac{7}{2}}} [|100\rangle (|f(000)\rangle - |-f(000)\rangle + |f(001)\rangle - |-f(001)\rangle)] \\
& + \frac{1}{2^{\frac{7}{2}}} [|100\rangle (|f(010)\rangle - |-f(010)\rangle + |f(011)\rangle - |-f(011)\rangle)] \\
& + \frac{1}{2^{\frac{7}{2}}} [|100\rangle (-|f(100)\rangle + |-f(100)\rangle - |f(101)\rangle + |-f(101)\rangle)] \\
& + \frac{1}{2^{\frac{7}{2}}} [|100\rangle (-|f(110)\rangle + |-f(110)\rangle - |f(111)\rangle + |-f(111)\rangle)] \\
& + \frac{1}{2^{\frac{7}{2}}} [|101\rangle (|f(000)\rangle - |-f(000)\rangle - |f(001)\rangle + |-f(001)\rangle)] \\
& + \frac{1}{2^{\frac{7}{2}}} [|101\rangle (|f(010)\rangle - |-f(010)\rangle - |f(011)\rangle + |-f(011)\rangle)] \\
& + \frac{1}{2^{\frac{7}{2}}} [|101\rangle (-|f(100)\rangle + |-f(100)\rangle + |f(101)\rangle - |-f(101)\rangle)] \\
& + \frac{1}{2^{\frac{7}{2}}} [|101\rangle (-|f(110)\rangle + |-f(110)\rangle + |f(111)\rangle - |-f(111)\rangle)] \\
& + \frac{1}{2^{\frac{7}{2}}} [|110\rangle (|f(000)\rangle - |-f(000)\rangle + |f(001)\rangle - |-f(001)\rangle)] \\
& + \frac{1}{2^{\frac{7}{2}}} [|110\rangle (-|f(010)\rangle + |-f(010)\rangle - |f(011)\rangle + |-f(011)\rangle)] \\
& + \frac{1}{2^{\frac{7}{2}}} [|110\rangle (-|f(100)\rangle + |-f(100)\rangle - |f(101)\rangle + |-f(101)\rangle)] \\
& + \frac{1}{2^{\frac{7}{2}}} [|110\rangle (|f(110)\rangle - |-f(110)\rangle + |f(111)\rangle - |-f(111)\rangle)]
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{2^{\frac{7}{2}}} [|111\rangle (|f(000)\rangle - |-f(000)\rangle - |f(001)\rangle + |-f(001)\rangle)] \\
& + \frac{1}{2^{\frac{7}{2}}} [|111\rangle (-|f(010)\rangle + |-f(010)\rangle + |f(011)\rangle - |-f(011)\rangle)] \\
& + \frac{1}{2^{\frac{7}{2}}} [|111\rangle (-|f(100)\rangle + |-f(100)\rangle + |f(101)\rangle - |-f(101)\rangle)] \\
& + \frac{1}{2^{\frac{7}{2}}} [|111\rangle (|f(110)\rangle - |-f(110)\rangle - |f(111)\rangle + |-f(111)\rangle)]
\end{aligned}$$

apabila f merupakan fungsi konstan maka $f(000) = f(001) = f(010) = f(011) = f(100) = f(101) = f(110) = f(111)$ sehingga didapatkan:

$$\begin{aligned}
|\psi_{out}\rangle &= \frac{1}{2^{\frac{7}{2}}} [|000\rangle .8 (|f(000)\rangle - |-f(000)\rangle) + |001\rangle .0 + |010\rangle .0 + |011\rangle \\
&= \frac{1}{2^{\frac{7}{2}}} .2^3 |000\rangle [|f(000)\rangle - |-f(000)\rangle] \\
&= \frac{1}{\sqrt{2}} |000\rangle [|f(000)\rangle - |-f(000)\rangle]
\end{aligned}$$

apabila f merupakan fungsi konstan $f_k 0 = f(000) = 0$ maka didapatkan:

$$|\psi_{out}\rangle = \frac{1}{\sqrt{2}} |000\rangle [|0\rangle - |1\rangle] \quad (94)$$

dan apabila fungsi f merupakan fungsi konstan $f_{k1} = f(0000) = 1$ maka didapatkan :

$$|\psi_{out}\rangle = -\frac{1}{\sqrt{2}} |000\rangle [|0\rangle - |1\rangle] \quad (95)$$

maka dengan demikian didapatkan keluaran $|\psi_{out1}\rangle = -|\psi_{out0}\rangle$ kemudian apabila fungsi f merupakan fungsi yang setimbang kita misalkan untuk f_3 dimana $f_3(000) = f_3(001) = f_3(010) = f_3(011) = 0$ dan $f_3(100) = f_3(101) = f_3(110) = f_3(111) = 1$ maka :

$$\begin{aligned}
|\psi_{out}\rangle = & \frac{1}{2^{\frac{7}{2}}} [|000\rangle (|0\rangle - |1\rangle + |0\rangle - |1\rangle + |0\rangle - |1\rangle + |0\rangle - |1\rangle + |0\rangle - |1\rangle + |0\rangle - |1\rangle + |0\rangle - |1\rangle + |0\rangle - |1\rangle)] \\
& + \frac{1}{2^{\frac{7}{2}}} [|001\rangle (|0\rangle - |1\rangle - |0\rangle + |1\rangle + |0\rangle - |1\rangle - |0\rangle + |1\rangle + |0\rangle - |1\rangle - |0\rangle + |1\rangle + |0\rangle - |1\rangle - |0\rangle + |1\rangle)] \\
& + \frac{1}{2^{\frac{7}{2}}} [|010\rangle (|0\rangle - |1\rangle + |0\rangle - |1\rangle - |0\rangle + |1\rangle - |0\rangle + |1\rangle + |0\rangle - |1\rangle + |0\rangle - |1\rangle - |0\rangle + |1\rangle - |0\rangle + |1\rangle)] \\
& + \frac{1}{2^{\frac{7}{2}}} [|011\rangle (|0\rangle - |1\rangle - |0\rangle + |1\rangle - |0\rangle + |1\rangle + |0\rangle - |1\rangle + |0\rangle - |1\rangle - |0\rangle + |1\rangle - |0\rangle + |1\rangle + |0\rangle - |1\rangle)] \\
& + \frac{1}{2^{\frac{7}{2}}} [|100\rangle (|0\rangle - |1\rangle + |0\rangle - |1\rangle + |0\rangle - |1\rangle + |0\rangle - |1\rangle - |0\rangle + |1\rangle - |0\rangle + |1\rangle - |0\rangle + |1\rangle - |0\rangle + |1\rangle)] \\
& + \frac{1}{2^{\frac{7}{2}}} [|101\rangle (|0\rangle - |1\rangle - |0\rangle + |1\rangle + |0\rangle - |1\rangle - |0\rangle + |1\rangle - |0\rangle + |1\rangle + |0\rangle - |1\rangle - |0\rangle + |1\rangle + |0\rangle - |1\rangle)] \\
& + \frac{1}{2^{\frac{7}{2}}} [|110\rangle (|0\rangle - |1\rangle + |0\rangle - |1\rangle - |0\rangle + |1\rangle - |0\rangle + |1\rangle - |0\rangle + |1\rangle - |0\rangle + |1\rangle + |0\rangle - |1\rangle + |0\rangle - |1\rangle)] \\
& + \frac{1}{2^{\frac{7}{2}}} [|111\rangle (|0\rangle - |1\rangle - |0\rangle + |1\rangle - |0\rangle + |1\rangle + |0\rangle - |1\rangle - |0\rangle + |1\rangle + |0\rangle - |1\rangle + |0\rangle - |1\rangle - |0\rangle + |1\rangle)]
\end{aligned} \tag{96}$$

sehingga didapatkan

$$|\psi_{out3}\rangle = \frac{1}{2^{\frac{7}{2}}} . 2^3 |100\rangle [|0\rangle - |1\rangle] = \frac{1}{\sqrt{2}} |100\rangle [|0\rangle - |1\rangle] \tag{97}$$

3.5 Algoritma Deutsch-Josza Pada sistem 4 qubit

Dalam aplikasi algoritma 4 qubit maka langkah yang harus kita cari pertama kali adalah kemungkinan -kemungkinan fungsi konstan dan fungsi setimbang, yaitu sebagai berikut: sebelumnya kita definisikan bahwa qubit masukan

$$|\psi\rangle = |x_3x_2x_1x_0\rangle \quad (98)$$

maka dapat kita ambil beberapa kemungkinan untuk fungsi konstan dan fungsi setimbang yaitu sebagai berikut:

$$\begin{aligned} f_{k0} &= 0 && (\textit{konstan0}) \\ f_{k1} &= 1 && (\textit{konstan1}) \\ f_{s0} &= x_3 && (\textit{setimbang0}) \\ f_{s1} &= x_1 \oplus x_0 && (\textit{setimbang1}) \\ f_{s2} &= x_2 \oplus x_1 \oplus x_1 && (\textit{setimbang2}) \\ f_{s3} &= x_3 \oplus x_2 \oplus x_1 \oplus x_1 && (\textit{setimbang3}) \\ f_{s4} &= x_2x_1 \oplus x_0 && (\textit{setimbang4}) \end{aligned} \quad (99)$$

maka dapat kita buat tabel untuk contoh fungsi-fungsi kotak hitam di atas, yaitu sebagai berikut:

$ x_3x_2x_1x_0\rangle$	f_{k0}	f_{k1}	f_{s0}	f_{s1}	f_{s2}	f_{s3}	f_{s4}
$ 0000\rangle$	0	1	0	0	0	0	0
$ 0001\rangle$	0	1	0	1	1	1	1
$ 0010\rangle$	0	1	0	1	1	1	0
$ 0011\rangle$	0	1	0	0	0	0	1
$ 0100\rangle$	0	1	0	0	1	1	0
$ 0101\rangle$	0	1	0	1	0	0	1
$ 0110\rangle$	0	1	0	1	0	0	1
$ 0111\rangle$	0	1	0	0	1	1	0
$ 1000\rangle$	0	1	1	0	0	1	0
$ 1001\rangle$	0	1	1	1	1	0	1
$ 1010\rangle$	0	1	1	1	1	0	0
$ 1011\rangle$	0	1	1	0	0	1	1
$ 1100\rangle$	0	1	1	0	1	0	0
$ 1101\rangle$	0	1	1	1	0	1	1
$ 1110\rangle$	0	1	1	1	0	1	1
$ 1111\rangle$	0	1	1	0	1	0	0
total kemungkinan	1	1	8	12	8	2	24

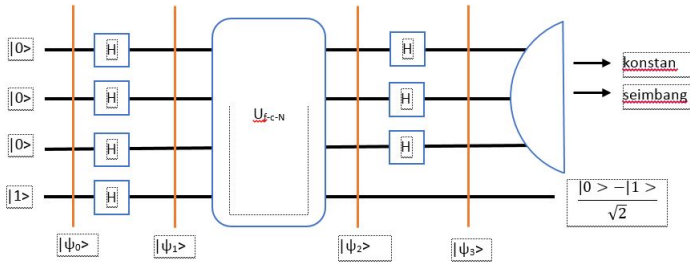
kemudian dikarenakan operator unitary memenuhi:

$$\begin{aligned}
 U_k &= U_f \otimes I \\
 U_f &= (-1)^f(x) |\psi\rangle
 \end{aligned} \tag{100}$$

maka dengan demikian operator unitary untuk fungsi kotak hitam dapat di tuliskan sebagai berikut:

$$\begin{aligned}
 U_{k0} &= I \otimes I \otimes I \otimes I \\
 U_{k1} &= -I \otimes I \otimes I \otimes I \\
 U_{s0} &= I \otimes I \otimes I \otimes \sigma_z \\
 U_{s1} &= I \otimes I \otimes \sigma_z \otimes \sigma_z \\
 U_{s2} &= I \otimes \sigma_z \otimes \sigma_z \otimes \sigma_z \\
 U_{s3} &= \sigma_z \otimes \sigma_z \otimes \sigma_z \otimes \sigma_z \\
 U_{s4} &= I \otimes ([I \otimes \sigma_z] \oplus [\sigma_z \otimes \sigma_z])
 \end{aligned} \tag{101}$$

berikut adalah diagram algoritma Deutsch-Josza 4 qubit :



Gambar 3.5 algoritma Deutsch-Josza 4 qubit

berdasarkan diagram diatas maka dapat diketahui bahwa fungsi keadaan input yaitu $|\psi_{in}\rangle = |00001\rangle$ kemudian diaplikasikan transformasi walsh-Hadamard pada fungsi keadaan

input tersebut dihasilkan sebagai berikut:

$$\begin{aligned}
|\psi_1\rangle &= H \otimes H \otimes H \otimes H \otimes H |00001\rangle \\
&= \left(\frac{1}{\sqrt{2}}\right)^5 (|0\rangle + |1\rangle)(|0\rangle + |1\rangle) \\
&\quad \times (|0\rangle + |1\rangle)(|0\rangle + |1\rangle)(|0\rangle - |1\rangle) \\
&= \left(\frac{1}{\sqrt{2}}\right)^5 (|00000\rangle - |00001\rangle + |00010\rangle - |00011\rangle) \\
&\quad + \left(\frac{1}{\sqrt{2}}\right)^5 (|00100\rangle - |00101\rangle + |00110\rangle - |00111\rangle) \\
&\quad + \left(\frac{1}{\sqrt{2}}\right)^5 (|01000\rangle - |01001\rangle + |01010\rangle - |01011\rangle) \\
&\quad + \left(\frac{1}{\sqrt{2}}\right)^5 (|01100\rangle - |01101\rangle + |01110\rangle - |01111\rangle) \\
&\quad + \left(\frac{1}{\sqrt{2}}\right)^5 (|10000\rangle - |10001\rangle + |10010\rangle - |10011\rangle) \\
&\quad + \left(\frac{1}{\sqrt{2}}\right)^5 (|10100\rangle - |10101\rangle + |10110\rangle - |10111\rangle) \\
&\quad + \left(\frac{1}{\sqrt{2}}\right)^5 (|11000\rangle - |11001\rangle + |11010\rangle - |11011\rangle) \\
&\quad + \left(\frac{1}{\sqrt{2}}\right)^5 (|11100\rangle - |11101\rangle + |11110\rangle - |11111\rangle)
\end{aligned} \tag{102}$$

kemudian diaplikasikan u_f pada $|\psi_1\rangle$ menurut persamaan $U_f : |xy\rangle \rightarrow |xy \oplus f(x)\rangle$ didapatkan $|\psi_2\rangle$ sebagai berikut:

$$\begin{aligned}
|\psi_2\rangle = & \left(\frac{1}{\sqrt{2}}\right)^5 (|00000 \oplus f(0000)\rangle - |00001 \oplus f(0000)\rangle) \\
& + \left(\frac{1}{\sqrt{2}}\right)^5 (|00010 \oplus f(0001)\rangle - |00011 \oplus f(0001)\rangle) \\
& + \left(\frac{1}{\sqrt{2}}\right)^5 (|00100 \oplus f(0010)\rangle - |00101 \oplus f(0010)\rangle) \\
& + \left(\frac{1}{\sqrt{2}}\right)^5 (|00110 \oplus f(0011)\rangle - |00111 \oplus f(0011)\rangle) \\
& + \left(\frac{1}{\sqrt{2}}\right)^5 (|01000 \oplus f(0100)\rangle - |01001 \oplus f(0100)\rangle) \\
& + \left(\frac{1}{\sqrt{2}}\right)^5 (|01010 \oplus f(0101)\rangle - |01011 \oplus f(0101)\rangle) \\
& + \left(\frac{1}{\sqrt{2}}\right)^5 (|01100 \oplus f(0110)\rangle - |01101 \oplus f(0110)\rangle) \\
& + \left(\frac{1}{\sqrt{2}}\right)^5 (|01110 \oplus f(0111)\rangle - |01111 \oplus f(0111)\rangle) \\
& + \left(\frac{1}{\sqrt{2}}\right)^5 (|10000 \oplus f(1000)\rangle - |10001 \oplus f(1000)\rangle) \\
& + \left(\frac{1}{\sqrt{2}}\right)^5 (|10010 \oplus f(1001)\rangle - |10011 \oplus f(1001)\rangle) \\
& + \left(\frac{1}{\sqrt{2}}\right)^5 (|10100 \oplus f(1010)\rangle - |10101 \oplus f(1010)\rangle) \\
& + \left(\frac{1}{\sqrt{2}}\right)^5 (|10110 \oplus f(1011)\rangle - |10111 \oplus f(1011)\rangle) \\
& + \left(\frac{1}{\sqrt{2}}\right)^5 (|11000 \oplus f(1100)\rangle - |11001 \oplus f(1100)\rangle) \\
& + \left(\frac{1}{\sqrt{2}}\right)^5 (|11010 \oplus f(1101)\rangle - |11011 \oplus f(1101)\rangle) \\
& + \left(\frac{1}{\sqrt{2}}\right)^5 (|11100 \oplus f(1110)\rangle - |11101 \oplus f(1110)\rangle) \\
& + \left(\frac{1}{\sqrt{2}}\right)^5 (|11110 \oplus f(1111)\rangle - |11111 \oplus f(1111)\rangle)
\end{aligned} \tag{103}$$

$$\begin{aligned}
&= \left(\frac{1}{\sqrt{2}}\right)^5 |0000\rangle (|f(0000)\rangle - |-f(0000)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^5 |0001\rangle (|f(0001)\rangle - |-f(0001)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^5 |0010\rangle (|f(0010)\rangle - |-f(0010)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^5 |0011\rangle (|f(0011)\rangle - |-f(0011)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^5 |0100\rangle (|f(0100)\rangle - |-f(0100)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^5 |0101\rangle (|f(0101)\rangle - |-f(0101)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^5 |0110\rangle (|f(0110)\rangle - |-f(0110)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^5 |0111\rangle (|f(0111)\rangle - |-f(0111)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^5 |1000\rangle (|f(1000)\rangle - |-f(1000)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^5 |1001\rangle (|f(1001)\rangle - |-f(1001)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^5 |1010\rangle (|f(1010)\rangle - |-f(1010)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^5 |1011\rangle (|f(1011)\rangle - |-f(1011)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^5 |1100\rangle (|f(1100)\rangle - |-f(1100)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^5 |1101\rangle (|f(1101)\rangle - |-f(1101)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^5 |1110\rangle (|f(1110)\rangle - |-f(1110)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^5 |1111\rangle (|f(1111)\rangle - |-f(1111)\rangle)
\end{aligned} \tag{104}$$

berdasarkan diagram diatas setelah mendapatkan $|\psi_2\rangle$ dioperasikan dengan transformasi Walsh-Hadamard lagi, diper-

oleh $|\psi_3\rangle$ yaitu sebagai berikut:

$$\begin{aligned}
 |\psi_3\rangle &= \left(\frac{1}{\sqrt{2}} \right)^4 H \otimes H \otimes H \otimes H \otimes I |\psi_2\rangle \\
 &= A + B + C + D + E + F + G + H + I + J \quad (105) \\
 &\quad + K + L + M + N + O + P
 \end{aligned}$$

dimana A=suku $|0000\rangle$, B=suku $|0001\rangle$, C=suku $|0010\rangle$, D=suku $|0011\rangle$, E=suku $|0100\rangle$, F=suku $|0101\rangle$, G=suku $|0110\rangle$, H=suku $|0111\rangle$, I=suku $|1000\rangle$, J=suku $|1001\rangle$, K=suku $|1010\rangle$, L=suku $|1011\rangle$, M=suku $|1100\rangle$, N=suku $|1101\rangle$, O=suku $|1110\rangle$, P=suku $|1111\rangle$ maka dapat diuraikan hasil operasi dari masing-masing suku yaitu sebagai berikut:

untuk $A = \text{suku } |0000\rangle$

$$\begin{aligned}
&= \left(\frac{1}{\sqrt{2}}\right)^9 |0000\rangle (|f(0000)\rangle - |-f(0000)\rangle + |f(0001)\rangle - |-f(0001)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0000\rangle (|f(0010)\rangle - |-f(0010)\rangle + |f(0011)\rangle - |-f(0011)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0000\rangle (|f(0100)\rangle - |-f(0100)\rangle + |f(0101)\rangle - |-f(0101)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0000\rangle (|f(0110)\rangle - |-f(0110)\rangle + |f(0111)\rangle - |-f(0111)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0000\rangle (|f(1000)\rangle - |-f(1000)\rangle + |f(1001)\rangle - |-f(1001)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0000\rangle (|f(1010)\rangle - |-f(1010)\rangle + |f(1011)\rangle - |-f(1011)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0000\rangle (|f(1100)\rangle - |-f(1100)\rangle + |f(1101)\rangle - |-f(1101)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0000\rangle (|f(1110)\rangle - |-f(1110)\rangle + |f(1111)\rangle - |-f(1111)\rangle)
\end{aligned} \tag{106}$$

untuk B= suku $|0001\rangle$

$$\begin{aligned}
&= \left(\frac{1}{\sqrt{2}}\right)^9 |0000\rangle (|f(0001)\rangle - |-f(0000)\rangle - |f(0001)\rangle + |-f(0001)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0001\rangle (|f(0010)\rangle - |-f(0010)\rangle - |f(0011)\rangle + |-f(0011)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0001\rangle (|f(0100)\rangle - |-f(0100)\rangle - |f(0101)\rangle + |-f(0101)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0001\rangle (|f(0110)\rangle - |-f(0110)\rangle - |f(0111)\rangle + |-f(0111)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0001\rangle (|f(1000)\rangle - |-f(1000)\rangle - |f(1001)\rangle + |-f(1001)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0001\rangle (|f(1010)\rangle - |-f(1010)\rangle - |f(1011)\rangle + |-f(1011)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0001\rangle (|f(1100)\rangle - |-f(1100)\rangle - |f(1101)\rangle + |-f(1101)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0001\rangle (|f(1110)\rangle - |-f(1110)\rangle - |f(1111)\rangle + |-f(1111)\rangle)
\end{aligned} \tag{107}$$

untuk C= suku $|0010\rangle$

$$\begin{aligned}
&= \left(\frac{1}{\sqrt{2}}\right)^9 |0000\rangle (|f(0000)\rangle - |-f(0000)\rangle + |f(0001)\rangle - |-f(0001)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0001\rangle (-|f(0010)\rangle + |-f(0010)\rangle - |f(0011)\rangle + |-f(0011)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0001\rangle (|f(0100)\rangle - |-f(0100)\rangle + |f(0101)\rangle - |-f(0101)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0001\rangle (-|f(0110)\rangle + |-f(0110)\rangle - |f(0111)\rangle + |-f(0111)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0001\rangle (|f(1000)\rangle - |-f(1000)\rangle + |f(1001)\rangle - |-f(1001)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0001\rangle (-|f(1010)\rangle + |-f(1010)\rangle - |f(1011)\rangle + |-f(1011)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0001\rangle (|f(1100)\rangle - |-f(1100)\rangle + |f(1101)\rangle - |-f(1101)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0001\rangle (-|f(1110)\rangle + |-f(1110)\rangle - |f(1111)\rangle + |-f(1111)\rangle)
\end{aligned} \tag{108}$$

untuk D=suku $|0011\rangle$

$$\begin{aligned}
&= \left(\frac{1}{\sqrt{2}}\right)^9 |0011\rangle (|f(0000)\rangle - |-f(0000)\rangle - |f(0001)\rangle + |-f(0001)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0011\rangle (-|f(0010)\rangle + |-f(0010)\rangle + |f(0011)\rangle - |-f(0011)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0011\rangle (|f(0100)\rangle - |-f(0100)\rangle - |f(0101)\rangle + |-f(0101)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0011\rangle (-|f(0110)\rangle + |-f(0110)\rangle + |f(0111)\rangle - |-f(0111)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0011\rangle (|f(1000)\rangle - |-f(1000)\rangle - |f(1001)\rangle + |-f(1001)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0011\rangle (-|f(1010)\rangle + |-f(1010)\rangle + |f(1011)\rangle - |-f(1011)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0011\rangle (|f(1100)\rangle - |-f(1100)\rangle - |f(1101)\rangle + |-f(1101)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0001\rangle (-|f(1110)\rangle + |-f(1110)\rangle + |f(1111)\rangle - |-f(1111)\rangle)
\end{aligned} \tag{109}$$

untuk E=suku $|0100\rangle$

$$\begin{aligned}
&= \left(\frac{1}{\sqrt{2}}\right)^9 |0100\rangle (|f(0000)\rangle - |-f(0000)\rangle + |f(0001)\rangle - |-f(0001)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0100\rangle (|f(0010)\rangle - |-f(0010)\rangle + |f(0011)\rangle - |-f(0011)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0100\rangle (-|f(0100)\rangle + |-f(0100)\rangle - |f(0101)\rangle + |-f(0101)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0100\rangle (-|f(0110)\rangle + |-f(0110)\rangle - |f(0111)\rangle + |-f(0111)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0100\rangle (|f(1000)\rangle - |-f(1000)\rangle + |f(1001)\rangle - |-f(1001)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0100\rangle (|f(1010)\rangle - |-f(1010)\rangle + |f(1011)\rangle - |-f(1011)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0100\rangle (-|f(1100)\rangle + |-f(1100)\rangle - |f(1101)\rangle + |-f(1101)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0100\rangle (-|f(1110)\rangle + |-f(1110)\rangle - |f(1111)\rangle + |-f(1111)\rangle)
\end{aligned} \tag{110}$$

untuk $F = \text{suku } |0101\rangle$

$$\begin{aligned}
&= \left(\frac{1}{\sqrt{2}} \right)^9 |0101\rangle (|f(0000)\rangle - |-f(0000)\rangle - |f(0001)\rangle + |-f(0001)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}} \right)^9 |0101\rangle (|f(0010)\rangle - |-f(0010)\rangle - |f(0011)\rangle + |-f(0011)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}} \right)^9 |0101\rangle (-|f(0100)\rangle + |-f(0100)\rangle + |f(0101)\rangle - |-f(0101)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}} \right)^9 |0101\rangle (-|f(0110)\rangle + |-f(0110)\rangle + |f(0111)\rangle - |-f(0111)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}} \right)^9 |0101\rangle (|f(1000)\rangle - |-f(1000)\rangle - |f(1001)\rangle + |-f(1001)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}} \right)^9 |0101\rangle (|f(1010)\rangle - |-f(1010)\rangle - |f(1011)\rangle + |-f(1011)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}} \right)^9 |0101\rangle (-|f(1100)\rangle + |-f(1100)\rangle + |f(1101)\rangle - |-f(1101)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}} \right)^9 |0101\rangle (-|f(1110)\rangle + |-f(1110)\rangle + |f(1111)\rangle - |-f(1111)\rangle)
\end{aligned} \tag{111}$$

untuk $G=suku$ $|0110\rangle$

$$\begin{aligned}
&= \left(\frac{1}{\sqrt{2}}\right)^9 |0110\rangle (|f(0000)\rangle - |-f(0000)\rangle + |f(0001)\rangle - |-f(0001)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0110\rangle (-|f(0010)\rangle + |-f(0010)\rangle - |f(0011)\rangle + |-f(0011)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0110\rangle (-|f(0100)\rangle + |-f(0100)\rangle - |f(0101)\rangle + |-f(0101)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0110\rangle (|f(0110)\rangle - |-f(0110)\rangle + |f(0111)\rangle - |-f(0111)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0110\rangle (|f(1000)\rangle - |-f(1000)\rangle + |f(1001)\rangle - |-f(1001)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0110\rangle (-|f(1010)\rangle + |-f(1010)\rangle - |f(1011)\rangle + |-f(1011)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0110\rangle (-|f(1100)\rangle + |-f(1100)\rangle - |f(1101)\rangle + |-f(1101)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0110\rangle (|f(1110)\rangle - |-f(1110)\rangle + |f(1111)\rangle - |-f(1111)\rangle)
\end{aligned} \tag{112}$$

untuk H=suku $|0111\rangle$

$$\begin{aligned}
&= \left(\frac{1}{\sqrt{2}}\right)^9 |0111\rangle (|f(0000)\rangle - |-f(0000)\rangle - |f(0001)\rangle + |-f(0001)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0111\rangle (-|f(0010)\rangle + |-f(0010)\rangle + |f(0011)\rangle - |-f(0011)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0111\rangle (-|f(0100)\rangle + |-f(0100)\rangle + |f(0101)\rangle - |-f(0101)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0111\rangle (|f(0110)\rangle - |-f(0110)\rangle - |f(0111)\rangle + |-f(0111)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0111\rangle (|f(1000)\rangle - |-f(1000)\rangle - |f(1001)\rangle + |-f(1001)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0111\rangle (-|f(1010)\rangle + |-f(1010)\rangle + |f(1011)\rangle - |-f(1011)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0111\rangle (-|f(1100)\rangle + |-f(1100)\rangle + |f(1101)\rangle - |-f(1101)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |0111\rangle (|f(1110)\rangle - |-f(1110)\rangle - |f(1111)\rangle + |-f(1111)\rangle)
\end{aligned} \tag{113}$$

untuk $I=suku$ $|1000\rangle$

$$\begin{aligned}
&= \left(\frac{1}{\sqrt{2}}\right)^9 |1000\rangle (|f(0000)\rangle - |-f(0000)\rangle + |f(0001)\rangle - |-f(0001)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |1000\rangle (|f(0010)\rangle - |-f(0010)\rangle + |f(0011)\rangle - |-f(0011)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |1000\rangle (|f(0100)\rangle - |-f(0100)\rangle + |f(0101)\rangle - |-f(0101)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |1000\rangle (|f(0110)\rangle - |-f(0110)\rangle + |f(0111)\rangle - |-f(0111)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |1000\rangle (-|f(1000)\rangle + |-f(1000)\rangle - |f(1001)\rangle + |-f(1001)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |1000\rangle (-|f(1010)\rangle + |-f(1010)\rangle - |f(1011)\rangle + |-f(1011)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |1000\rangle (-|f(1100)\rangle + |-f(1100)\rangle - |f(1101)\rangle + |-f(1101)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |1000\rangle (-|f(1110)\rangle + |-f(1110)\rangle - |f(1111)\rangle + |-f(1111)\rangle)
\end{aligned} \tag{114}$$

untuk $J = \text{suku } |1001\rangle$

$$\begin{aligned}
&= \left(\frac{1}{\sqrt{2}} \right)^9 |1001\rangle (|f(0000)\rangle - |-f(0000)\rangle - |f(0001)\rangle + |-f(0001)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}} \right)^9 |1001\rangle (|f(0010)\rangle - |-f(0010)\rangle - |f(0011)\rangle + |-f(0011)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}} \right)^9 |1001\rangle (|f(0100)\rangle - |-f(0100)\rangle - |f(0101)\rangle + |-f(0101)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}} \right)^9 |1001\rangle (|f(0110)\rangle - |-f(0110)\rangle - |f(0111)\rangle + |-f(0111)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}} \right)^9 |1001\rangle (-|f(1000)\rangle + |-f(1000)\rangle + |f(1001)\rangle - |-f(1001)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}} \right)^9 |1001\rangle (-|f(1010)\rangle + |-f(1010)\rangle + |f(1011)\rangle - |-f(1011)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}} \right)^9 |1001\rangle (-|f(1100)\rangle + |-f(1100)\rangle + |f(1101)\rangle - |-f(1101)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}} \right)^9 |1001\rangle (-|f(1110)\rangle + |-f(1110)\rangle + |f(1111)\rangle + |-f(1111)\rangle)
\end{aligned} \tag{115}$$

untuk $K = \text{suku } |1010\rangle$

$$\begin{aligned}
&= \left(\frac{1}{\sqrt{2}}\right)^9 |1010\rangle (|f(0000)\rangle - |-f(0000)\rangle + |f(0001)\rangle - |-f(0001)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |1010\rangle (-|f(0010)\rangle + |-f(0010)\rangle - |f(0011)\rangle + |-f(0011)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |1010\rangle (|f(0100)\rangle - |-f(0100)\rangle + |f(0101)\rangle - |-f(0101)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |1010\rangle (-|f(0110)\rangle + |-f(0110)\rangle - |f(0111)\rangle + |-f(0111)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |1010\rangle (-|f(1000)\rangle + |-f(1000)\rangle - |f(1001)\rangle + |-f(1001)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |1010\rangle (|f(1010)\rangle - |-f(1010)\rangle + |f(1011)\rangle - |-f(1011)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |1010\rangle (-|f(1100)\rangle + |-f(1100)\rangle - |f(1101)\rangle + |-f(1101)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |1010\rangle (|f(1110)\rangle - |-f(1110)\rangle + |f(1111)\rangle - |-f(1111)\rangle)
\end{aligned} \tag{116}$$

untuk $L =$ suku $|1011\rangle$

$$\begin{aligned}
&= \left(\frac{1}{\sqrt{2}}\right)^9 |1011\rangle (|f(0000)\rangle - |-f(0000)\rangle - |f(0001)\rangle + |-f(0001)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |1011\rangle (-|f(0010)\rangle + |-f(0010)\rangle + |f(0011)\rangle - |-f(0011)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |1011\rangle (|f(0100)\rangle - |-f(0100)\rangle - |f(0101)\rangle + |-f(0101)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |1011\rangle (-|f(0110)\rangle + |-f(0110)\rangle + |f(0111)\rangle - |-f(0111)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |1011\rangle (-|f(1000)\rangle + |-f(1000)\rangle + |f(1001)\rangle - |-f(1001)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |1011\rangle (|f(1010)\rangle - |-f(1010)\rangle - |f(1011)\rangle + |-f(1011)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |1011\rangle (-|f(1100)\rangle + |-f(1100)\rangle - |f(1101)\rangle + |-f(1101)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |1011\rangle (|f(1110)\rangle - |-f(1110)\rangle + |f(1111)\rangle - |-f(1111)\rangle)
\end{aligned} \tag{117}$$

untuk $M = \text{suku } |1100\rangle$

$$\begin{aligned}
&= \left(\frac{1}{\sqrt{2}} \right)^9 |1100\rangle (|f(0000)\rangle - |-f(0000)\rangle + |f(0001)\rangle - |-f(0001)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}} \right)^9 |1100\rangle (|f(0010)\rangle - |-f(0010)\rangle + |f(0011)\rangle - |-f(0011)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}} \right)^9 |1100\rangle (-|f(0100)\rangle + |-f(0100)\rangle - |f(0101)\rangle + |-f(0101)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}} \right)^9 |1100\rangle (-|f(0110)\rangle + |-f(0110)\rangle - |f(0111)\rangle + |-f(0111)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}} \right)^9 |1100\rangle (-|f(1000)\rangle + |-f(1000)\rangle - |f(1001)\rangle + |-f(1001)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}} \right)^9 |1100\rangle (-|f(1010)\rangle + |-f(1010)\rangle - |f(1011)\rangle + |-f(1011)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}} \right)^9 |1100\rangle (|f(1100)\rangle - |-f(1100)\rangle + |f(1101)\rangle - |-f(1101)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}} \right)^9 |1100\rangle (|f(1110)\rangle - |-f(1110)\rangle + |f(1111)\rangle - |-f(1111)\rangle)
\end{aligned} \tag{118}$$

untuk N= suku $|1101\rangle$

$$\begin{aligned}
&= \left(\frac{1}{\sqrt{2}}\right)^9 |1101\rangle (|f(0000)\rangle - |-f(0000)\rangle - |f(0001)\rangle + |-f(0001)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |1101\rangle (|f(0010)\rangle - |-f(0010)\rangle - |f(0011)\rangle + |-f(0011)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |1101\rangle (-|f(0100)\rangle + |-f(0100)\rangle + |f(0101)\rangle - |-f(0101)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |1101\rangle (-|f(0110)\rangle + |-f(0110)\rangle + |f(0111)\rangle - |-f(0111)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |1101\rangle (-|f(1000)\rangle + |-f(1000)\rangle + |f(1001)\rangle - |-f(1001)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |1101\rangle (-|f(1010)\rangle + |-f(1010)\rangle + |f(1011)\rangle - |-f(1011)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |1101\rangle (|f(1100)\rangle - |-f(1100)\rangle + |f(1101)\rangle - |-f(1101)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |1101\rangle (|f(1110)\rangle - |-f(1110)\rangle + |f(1111)\rangle - |-f(1111)\rangle)
\end{aligned} \tag{119}$$

untuk O=suku $|1110\rangle$

$$\begin{aligned}
&= \left(\frac{1}{\sqrt{2}}\right)^9 |1110\rangle (|f(0000)\rangle - |-f(0000)\rangle + |f(0001)\rangle - |-f(0001)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |1110\rangle (-|f(0010)\rangle + |-f(0010)\rangle - |f(0011)\rangle + |-f(0011)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |1110\rangle (-|f(0100)\rangle + |-f(0100)\rangle - |f(0101)\rangle + |-f(0101)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |1110\rangle (|f(0110)\rangle - |-f(0110)\rangle + |f(0111)\rangle - |-f(0111)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |1110\rangle (-|f(1000)\rangle + |-f(1000)\rangle - |f(1001)\rangle + |-f(1001)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |1110\rangle (|f(1010)\rangle - |-f(1010)\rangle + |f(1011)\rangle - |-f(1011)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |1110\rangle (|f(1100)\rangle - |-f(1100)\rangle + |f(1101)\rangle - |-f(1101)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}}\right)^9 |1110\rangle (-|f(1110)\rangle + |-f(1110)\rangle - |f(1111)\rangle + |-f(1111)\rangle)
\end{aligned} \tag{120}$$

untuk $P = \text{suku } |1111\rangle$

$$\begin{aligned}
&= \left(\frac{1}{\sqrt{2}} \right)^9 |1111\rangle (|f(0000)\rangle - |-f(0000)\rangle - |f(0001)\rangle + |-f(0001)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}} \right)^9 |1111\rangle (-|f(0010)\rangle + |-f(0010)\rangle + |f(0011)\rangle - |-f(0011)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}} \right)^9 |1111\rangle (-|f(0100)\rangle + |-f(0100)\rangle + |f(0101)\rangle - |-f(0101)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}} \right)^9 |1111\rangle (|f(0110)\rangle - |-f(0110)\rangle - |f(0111)\rangle + |-f(0111)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}} \right)^9 |1111\rangle (-|f(1000)\rangle + |-f(1000)\rangle + |f(1001)\rangle - |-f(1001)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}} \right)^9 |1111\rangle (|f(1010)\rangle - |-f(1010)\rangle - |f(1011)\rangle + |-f(1011)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}} \right)^9 |1111\rangle (|f(1100)\rangle - |-f(1100)\rangle - |f(1101)\rangle + |-f(1101)\rangle) \\
&+ \left(\frac{1}{\sqrt{2}} \right)^9 |1111\rangle (-|f(1110)\rangle + |-f(1110)\rangle + |f(1111)\rangle - |-f(1111)\rangle)
\end{aligned} \tag{121}$$

untuk fungsi konstan $f(0000) = f(0001) = f(0010) = f(0011) = f(0100) = f(0101) = f(0110) = f(0111) = f(1000) = f(1001) = f(1010) = f(1011) = f(1100) = f(1101) = f(1110) = f(1111) = 0$ didapatkan :

$$|\psi_{k0}\rangle = \frac{1}{\sqrt{2}} |0000\rangle [|0\rangle - |1\rangle] \quad (122)$$

untuk fungsi konstan $f(0000) = f(0001) = f(0010) = f(0011) = f(0100) = f(0101) = f(0110) = f(0111) = f(1000) = f(1001) = f(1010) = f(1011) = f(1100) = f(1101) = f(1110) = f(1111) = 1$ didapatkan :

$$|\psi_{k1}\rangle = -\frac{1}{\sqrt{2}} |0000\rangle [|0\rangle - |1\rangle] \quad (123)$$

untuk fungsi setimbang $f(0000) = f(0001) = f(0010) = f(0011) = f(0100) = f(0101) = f(0110) = 0$ dan $f(0111) = f(1000) = f(1001) = f(1010) = f(1011) = f(1100) = f(1101) = f(1110) = f(1111) = 1$ didapatkan

$$|\psi_{s0}\rangle = \frac{1}{\sqrt{2}} |1000\rangle [|0\rangle - |1\rangle] \quad (124)$$

untuk fungsi setimbang s_1 $f(0000) = 0, f(0001) = f(0010) = 1, f(0011) = f(0100) = 0, f(0101) = f(0110) = 1, f(0111) = f(1000) = 0, f(1001) = f(1010) = 1, f(1011) = f(1100) = 0, f(1101) = f(1110) = 1, f(1111) = 0$

$$|\psi_{s1}\rangle = \frac{1}{\sqrt{2}} |0011\rangle [|0\rangle - |1\rangle] \quad (125)$$

untuk fungsi setimbang s_2 $f(0000) = 0, f(0001) = f(0010) = 1, f(0011) = 0, f(0100) = 1, f(0101) = f(0110) = 0, f(0111) = 1, f(1000) = 0, f(1001) = f(1010) = 1, f(1011) = 0, f(1100) = 1, f(1101) = f(1110) = 1, f(1111) = 0$ didapatkan :

$$|\psi_{s2}\rangle = \frac{1}{\sqrt{2}} |0111\rangle [|0\rangle - |1\rangle] \quad (126)$$

untuk fungsi setimbang s_3 $f(0000) = f(0001) = f(0010) = f(0011) = f(0100) = f(0101) = f(0110) = f(0111) = f(1000) = f(1001) = f(1010) = f(1011) = f(1100) = f(1101) = f(1110) = f(1111) = 0$ didapatkan :

$$|\psi_{s3}\rangle = \frac{1}{\sqrt{2}} |1111\rangle [|0\rangle - |1\rangle] \quad (127)$$

”Halaman ini sengaja dikosongkan”

BAB IV

SISTEM N-QUBIT

Sebelum membahas permasalahan dalam sistem N-qubit maka harus dibahas terlebih dahulu untuk sistem 1-qubit, 2 qubit, 3 qubit dan 4 qubit untuk mendapatkan pola dari pada masing-masing sistem, sehingga dapat diketahui apabila sistem memiliki pola atau hanya perluasan saja tanpa pola umum

4.1 Sistem 1, 2 dan 3 qubit

Pada sistem 1 qubit. Level energi dihasilkan oleh 1 partikel, dimana hamiltonian untuk presesi bebas qubit tunggal adalah:

$$\begin{aligned} H_0 &= -\hbar\gamma B_0 I_z \\ &= \hbar\omega_0 I_z \end{aligned} \tag{128}$$

dimana $\omega_0 = -\gamma B_0$ dan ω_0 merupakan frekuensi sudut larmor dan γ merupakan rasio giromagnetik. kemudian diambil $\hbar = 1$ sehingga didapatkan:

$$H_0 = \omega_0 I_z \tag{129}$$

untuk spin tunggal masukan keadaannya adalah:

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle \tag{130}$$

maka dengan demikian kita dapat mengoperasikan hamiltonian pada keadaan tersebut dan diperoleh sebagai berikut:

$$\begin{aligned}
H_0 |\psi\rangle &= \omega_0 I_z (|0\rangle + |1\rangle) \\
&= \omega_0 (I_z |0\rangle + I_z |1\rangle) \\
&= \omega_0 \left(\frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right) \\
&= \omega_0 \left[\frac{1}{2} \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \frac{1}{2} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right] \\
&= \omega_0 \left(\frac{1}{2} |0\rangle - \frac{1}{2} |1\rangle \right) \\
&= \frac{1}{2} \omega_0 (|0\rangle - |1\rangle)
\end{aligned} \tag{131}$$

maka dengan demikian diperoleh dua nilai level energi yaitu sebagai berikut:

$$E_a = m\omega_0 = \begin{cases} E_0 = \frac{1}{2}\omega_0 & \text{untuk } a = |0\rangle \text{ dan } m = \frac{1}{2} \\ E_1 = -\frac{1}{2}\omega_0 & \text{untuk } a = |1\rangle \text{ dan } m = -\frac{1}{2} \end{cases} \tag{132}$$

Pada sistem NMR 2 qubit interaksi yang terjadi diasumsikan sebagai interaksi heisenberg yaitu interaksi yang diperanguri oleh atom tetangga yang terdekat sehingga hamiltonian sistem adalah sebagai berikut:

$$H = \omega_0 I \otimes I_z + \omega_1 I_z \otimes I + 2\pi J_{10} I_z \otimes I_z \tag{133}$$

dimana ω_0 dan ω_1 merupakan frekuensi sudut larmor untuk qubit ke 0 dan qubit ke 1, sedangkan J_{10} merupakan kopling skalar yang melibatkan qubit-0 dan qubit-1. kemudian vektor keadaan untuk sistem 2 qubit adalah sebagai berikut:

$$|\psi\rangle = c_0 |00\rangle + c_1 |01\rangle + c_2 |10\rangle + c_3 |11\rangle \tag{134}$$

dengan c_0, c_1, c_2, c_3 merupakan koefisien kompleks. apabila H bekerja pada vektor keadaan maka diperoleh:

$$\begin{aligned}
H|\psi\rangle &= (\omega_0 I \otimes I_z + \omega_1 I_z \otimes I + 2\pi J_{10} I_z \otimes I_z) |\psi\rangle \\
&= \omega_0 I \otimes I_z (c_0 |00\rangle + c_1 |01\rangle + c_2 |10\rangle + c_3 |11\rangle) \\
&\quad + \omega_1 I_z \otimes I (c_0 |00\rangle + c_1 |01\rangle + c_2 |10\rangle + c_3 |11\rangle) \\
&\quad + 2\pi J_{10} I_z \otimes I_z (c_0 |00\rangle + c_1 |01\rangle + c_2 |10\rangle + c_3 |11\rangle) \\
&= \omega_0 (c_0 I |0\rangle I_z |0\rangle + c_1 I |0\rangle I_z |1\rangle + c_2 I |1\rangle I_z |0\rangle + c_3 I |1\rangle I_z |1\rangle) \\
&\quad + \omega_1 (c_0 I_z |0\rangle I |0\rangle + c_1 I_z |0\rangle I |1\rangle + c_2 I_z |1\rangle I |0\rangle + c_3 I_z |1\rangle I |1\rangle) \\
&\quad + 2\pi J_{10} (c_0 I_z |0\rangle I_z |0\rangle + c_1 I_z |0\rangle I_z |1\rangle + c_2 I_z |1\rangle I_z |0\rangle) \\
&\quad + 2\pi J_{10} (c_3 I_z |1\rangle I_z |1\rangle)
\end{aligned}$$

$$\begin{aligned}
&= \omega_0 \left(c_0 |0\rangle \frac{1}{2} |0\rangle + c_1 |0\rangle \left(-\frac{1}{2} \right) |1\rangle + c_2 |1\rangle \frac{1}{2} |0\rangle \right) \\
&+ \omega_0 \left(c_3 |1\rangle \left(-\frac{1}{2} \right) |1\rangle \right) \\
&+ \omega_1 \left(c_0 \frac{1}{2} |0\rangle |0\rangle + c_1 \frac{1}{2} |0\rangle |1\rangle + c_2 \left(-\frac{1}{2} \right) |1\rangle |0\rangle \right) \\
&+ \omega_1 \left(c_3 \left(-\frac{1}{2} \right) |1\rangle |1\rangle \right) \\
&+ 2\pi J_{10} \left(c_0 \frac{1}{2} |0\rangle \frac{1}{2} |0\rangle + c_1 \frac{1}{2} |0\rangle \left(-\frac{1}{2} \right) |1\rangle + c_2 \left(-\frac{1}{2} \right) |1\rangle \frac{1}{2} |0\rangle \right) \\
&+ 2\pi J_{10} \left(c_3 \left(-\frac{1}{2} \right) |1\rangle \left(-\frac{1}{2} \right) |1\rangle \right) \tag{135} \\
&= \omega_0 \left[\frac{1}{2} C_0 |00\rangle - \frac{1}{2} C_1 |01\rangle + \frac{1}{2} C_2 |10\rangle - \frac{1}{2} C_3 |11\rangle \right] \\
&+ \omega_1 \left[\frac{1}{2} C_0 |00\rangle + \frac{1}{2} C_1 |01\rangle + -da C_2 |10\rangle - \frac{1}{2} C_3 |11\rangle \right] \\
&+ 2\pi J_{10} \left[\frac{1}{4} C_0 |00\rangle - \frac{1}{4} C_1 |01\rangle - \frac{1}{4} C_2 |10\rangle + \frac{1}{4} C_3 |11\rangle \right] \\
&= \frac{1}{2} (\omega_0 + \omega_1 + \pi J_{10}) c_0 |00\rangle + \frac{1}{2} (-\omega_0 - \omega_1 + \pi J_{10}) c_1 |01\rangle \\
&+ \frac{1}{2} (\omega_0 - \omega_1 - \pi J_{10}) c_2 |10\rangle + \frac{1}{2} (-\omega_0 - \omega_1 + \pi J_{10}) c_3 |11\rangle
\end{aligned}$$

maka dengan demikian diperoleh bentuk

$$H |\psi\rangle = E_{2,0} c_0 |00\rangle + E_{2,1} c_1 |01\rangle + E_{2,2} c_2 |10\rangle + E_{2,3} c_3 |11\rangle \tag{136}$$

sehingga dapat dibuat tabel level-level energi untuk sistem 2 qubit yaitu sebagai berikut:

n	$E_{2,n}$
0	$\frac{1}{2}(\omega_0 + \omega_1 + \pi J_{10})$
1	$\frac{1}{2}(-\omega_0 - \omega_1 + \pi J_{10})$
2	$\frac{1}{2}(\omega_0 - \omega_1 - \pi J_{10})$
3	$\frac{1}{2}(-\omega_0 - \omega_1 + \pi J_{10})$

Level Energi pada sistem 3 qubit ditentukan oleh energi 3 partikel dan interaksinya. untuk cataan bahwa interaksi yang terjadi adalah interaksi heisenberg yakni interaksi yang hanya dipengaruhi oleh atom tetangga terdekat. Bentuk Hamiltonian untuk sistem 3 qubit adalah sebagai berikut:

$$H = \omega_0 I \otimes I \otimes I_z + \omega_1 I \otimes I_z \otimes I + \omega_2 I_z \otimes I \otimes I + 2\pi J_{10} I \otimes I_z \otimes I_z + 2\pi J_{20} I_z \otimes I \otimes I_z + 2\pi J_{21} I_z \otimes I_z \otimes I \quad (137)$$

sedangkan untuk vektor keadaan 3 qubit adalah sebagai berikut:

$$|\psi\rangle = c_0 |000\rangle + c_1 |001\rangle + c_2 |010\rangle + c_3 |011\rangle + c_4 |100\rangle + c_5 |101\rangle + c_6 |110\rangle + c_7 |111\rangle \quad (138)$$

dengan c_1, c_2, \dots, c_7 merupakan koefisien kompleks. untuk mendapatkan nilai level energi dari keadaan 3 qubit maka dioperasikan hamiltonian pada vektor keadaannya yaitu se-

bagai berikut:

$$\begin{aligned}
H|\psi\rangle &= (\omega_0 I \otimes I \otimes I_z + \omega_1 I \otimes I_z \otimes I + \omega_2 I_z \otimes I \otimes I) |\psi\rangle \\
&+ (2\pi J_{10} I \otimes I_z \otimes I_z + 2\pi j_{20} I_z \otimes I \otimes I_z + 2\pi j_{21} I_z \otimes I_z \otimes I) |\psi\rangle \\
&= \omega_0 I \otimes I \otimes I_z (c_0 |000\rangle + c_1 |001\rangle + c_2 |010\rangle + c_3 |011\rangle) \\
&+ \omega_0 I \otimes I \otimes I_z (c_4 |100\rangle + c_5 |101\rangle + c_6 |110\rangle + c_7 |111\rangle) \\
&+ \omega_1 I \otimes I_z \otimes I (c_0 |000\rangle + c_1 |001\rangle + c_2 |010\rangle + c_3 |011\rangle) \\
&+ \omega_1 I \otimes I_z \otimes I (c_4 |100\rangle + c_5 |101\rangle + c_6 |110\rangle + c_7 |111\rangle) \\
&+ \omega_2 I_z \otimes I \otimes I (c_0 |000\rangle + c_1 |001\rangle + c_2 |010\rangle + c_3 |011\rangle) \\
&+ \omega_2 I_z \otimes I \otimes I (c_4 |100\rangle + c_5 |101\rangle + c_6 |110\rangle + c_7 |111\rangle) \\
&+ 2\pi J_{10} I \otimes I_z \otimes I_z (c_0 |000\rangle + c_1 |001\rangle + c_2 |010\rangle + c_3 |011\rangle) \\
&+ 2\pi J_{10} I \otimes I_z \otimes I_z (c_4 |100\rangle + c_5 |101\rangle + c_6 |110\rangle + c_7 |111\rangle) \\
&+ 2\pi j_{20} I_z \otimes I \otimes I_z (c_0 |000\rangle + c_1 |001\rangle + c_2 |010\rangle + c_3 |011\rangle) \\
&+ 2\pi j_{20} I_z \otimes I \otimes I_z (c_4 |100\rangle + c_5 |101\rangle + c_6 |110\rangle + c_7 |111\rangle) \\
&+ 2\pi j_{21} I_z \otimes I_z \otimes I (c_0 |000\rangle + c_1 |001\rangle + c_2 |010\rangle + c_3 |011\rangle) \\
&+ 2\pi j_{21} I_z \otimes I_z \otimes I (c_4 |100\rangle + c_5 |101\rangle + c_6 |110\rangle + c_7 |111\rangle) \\
&= \omega_0 (c_0 I |0\rangle I |0\rangle I_z |0\rangle + c_1 I |0\rangle I |0\rangle I_z |1\rangle + c_2 I |0\rangle I |1\rangle I_z |0\rangle) \\
&+ \omega_0 (c_3 I |0\rangle I |1\rangle I_z |1\rangle + c_4 I |1\rangle I |0\rangle I_z |0\rangle + c_5 I |1\rangle I |0\rangle I_z |1\rangle) \\
&+ \omega_0 (c_6 I |1\rangle I |1\rangle I_z |0\rangle + c_7 I |1\rangle I |1\rangle I_z |1\rangle) \\
&+ \omega_1 (c_0 I |0\rangle I_z |0\rangle I |0\rangle + c_1 I |0\rangle I_z |0\rangle I |1\rangle + c_2 I |0\rangle I_z |1\rangle I |0\rangle) \\
&+ \omega_1 (c_3 I |0\rangle I_z |1\rangle I |1\rangle + c_4 I |1\rangle I_z |0\rangle I |0\rangle + c_5 I |1\rangle I_z |0\rangle I |1\rangle) \\
&+ \omega_1 (c_6 I |1\rangle I_z |1\rangle I |0\rangle + c_7 I |1\rangle I_z |1\rangle I |1\rangle) \\
&+ \omega_2 (c_0 I_z |0\rangle I |0\rangle I |0\rangle + c_1 I_z |0\rangle I |0\rangle I |1\rangle + c_2 I_z |0\rangle I |1\rangle I |0\rangle) \\
&+ \omega_1 (c_3 I_z |0\rangle I |1\rangle I |1\rangle + c_4 I_z |1\rangle I |0\rangle I |0\rangle + c_5 I_z |1\rangle I |0\rangle I |1\rangle) \\
&+ \omega_1 (c_6 I_z |1\rangle I |1\rangle I |0\rangle + c_7 I_z |1\rangle I |1\rangle I |1\rangle) \\
&+ 2\pi J_{10} (c_0 I |0\rangle I_z |0\rangle I_z |0\rangle + c_1 I |0\rangle I_z |0\rangle I_z |1\rangle) \\
&+ 2\pi J_{10} (c_2 I |0\rangle I_z |1\rangle I_z |0\rangle + c_3 I |0\rangle I_z |1\rangle I_z |1\rangle) \\
&+ 2\pi J_{10} (c_4 I |1\rangle I_z |0\rangle I_z |0\rangle + c_5 I |1\rangle I_z |0\rangle I_z |1\rangle) \\
&+ 2\pi J_{10} (c_6 I |1\rangle I_z |1\rangle I_z |0\rangle + c_7 I |1\rangle I_z |1\rangle I_z |1\rangle)
\end{aligned}$$

$$\begin{aligned}
& + 2\pi J_{20} (c_0 I_z |0\rangle I |0\rangle I_z |0\rangle + c_1 I_z |0\rangle I |0\rangle I_z |1\rangle + c_2 I_z |0\rangle I |1\rangle I_z |0\rangle) \\
& + 2\pi J_{20} (c_3 I_z |0\rangle I |1\rangle I_z |1\rangle + c_4 I_z |1\rangle I |0\rangle I_z |0\rangle + c_5 I_z |1\rangle I |0\rangle I_z |1\rangle) \\
& + 2\pi J_{20} (c_6 I_z |1\rangle I |1\rangle I_z |0\rangle + c_7 I_z |1\rangle I |1\rangle I_z |1\rangle) \\
& + 2\pi J_{21} (c_0 I_z |0\rangle I_z |0\rangle I |0\rangle + c_1 I_z |0\rangle I_z |0\rangle I |1\rangle + c_2 I_z |0\rangle I_z |1\rangle I |0\rangle) \\
& + 2\pi J_{21} (c_3 I_z |0\rangle I_z |1\rangle I_z |1\rangle + c_4 I_z |1\rangle I_z |0\rangle I |0\rangle + c_5 I_z |1\rangle I_z |0\rangle I |1\rangle) \\
& + 2\pi J_{21} (c_6 I_z |1\rangle I_z |1\rangle I |0\rangle + c_7 I_z |1\rangle I_z |1\rangle I |1\rangle) \\
& = \omega_0 \left(c_0 |0\rangle |0\rangle \frac{1}{2} |0\rangle + c_1 |0\rangle |0\rangle \left(-\frac{1}{2}\right) |1\rangle + c_2 |0\rangle |1\rangle \frac{1}{2} |0\rangle \right) \\
& + \omega_0 \left(c_3 |0\rangle |1\rangle \left(-\frac{1}{2}\right) |1\rangle + c_4 |1\rangle |0\rangle \frac{1}{2} |0\rangle + c_5 |1\rangle |0\rangle \left(-\frac{1}{2}\right) |1\rangle \right) \\
& + \omega_0 \left(c_6 |1\rangle |1\rangle \frac{1}{2} |0\rangle + c_7 |1\rangle |1\rangle \left(-\frac{1}{2}\right) |1\rangle \right) \\
& + \omega_1 \left(c_0 |0\rangle \frac{1}{2} |0\rangle |0\rangle + c_1 |0\rangle \frac{1}{2} |0\rangle |1\rangle + c_2 |0\rangle \left(-\frac{1}{2}\right) |1\rangle |0\rangle \right) \\
& + \omega_1 \left(c_3 |0\rangle \left(-\frac{1}{2}\right) |1\rangle |1\rangle + c_4 |1\rangle \frac{1}{2} |0\rangle |0\rangle + c_5 |1\rangle \frac{1}{2} |0\rangle |1\rangle \right) \\
& + \omega_1 \left(c_6 |1\rangle \left(-\frac{1}{2}\right) |1\rangle |0\rangle + c_7 |1\rangle \left(-\frac{1}{2}\right) |1\rangle |1\rangle \right) \\
& + \omega_2 \left(c_0 \frac{1}{2} |0\rangle |0\rangle |0\rangle + c_1 \frac{1}{2} |0\rangle |0\rangle |1\rangle + c_2 \frac{1}{2} |0\rangle |1\rangle |0\rangle \right) \\
& + \omega_2 \left(c_3 \frac{1}{2} |0\rangle |1\rangle |1\rangle + c_4 \left(-\frac{1}{2}\right) |1\rangle |0\rangle |0\rangle + c_5 \left(-\frac{1}{2}\right) |1\rangle |0\rangle |1\rangle \right) \\
& + \omega_2 \left(c_6 \left(-\frac{1}{2}\right) |1\rangle |1\rangle |0\rangle + c_7 \left(-\frac{1}{2}\right) |1\rangle |1\rangle |1\rangle \right) \\
& + 2\pi J_{10} \left(c_0 |0\rangle \frac{1}{2} |0\rangle \frac{1}{2} |0\rangle + c_1 |0\rangle \frac{1}{2} |0\rangle \left(-\frac{1}{2}\right) |1\rangle + c_2 |0\rangle \left(-\frac{1}{2}\right) |1\rangle \frac{1}{2} |0\rangle \right) \\
& + 2\pi J_{10} \left(c_3 |0\rangle \left(-\frac{1}{2}\right) |1\rangle \left(-\frac{1}{2}\right) |1\rangle + c_4 |1\rangle \frac{1}{2} |0\rangle \frac{1}{2} |0\rangle + c_5 |1\rangle \frac{1}{2} |0\rangle \left(-\frac{1}{2}\right) |1\rangle \right) \\
& + 2\pi J_{10} \left(c_6 |1\rangle \left(-\frac{1}{2}\right) |1\rangle \frac{1}{2} |0\rangle + c_7 |1\rangle \left(-\frac{1}{2}\right) |1\rangle \left(-\frac{1}{2}\right) |1\rangle \right) \\
& + 2\pi J_{20} \left(c_0 \frac{1}{2} |0\rangle |0\rangle \frac{1}{2} |0\rangle + c_1 \frac{1}{2} |0\rangle |0\rangle \left(-\frac{1}{2}\right) |1\rangle + c_2 \frac{1}{2} |0\rangle |1\rangle \frac{1}{2} |0\rangle \right) \\
& + 2\pi J_{20} \left(c_3 \frac{1}{2} |0\rangle |1\rangle \left(-\frac{1}{2}\right) |1\rangle + c_4 \left(-\frac{1}{2}\right) |1\rangle |0\rangle \frac{1}{2} |0\rangle + c_5 \left(-\frac{1}{2}\right) |1\rangle |0\rangle \left(-\frac{1}{2}\right) |1\rangle \right) \\
& + 2\pi J_{20} \left(c_6 \left(-\frac{1}{2}\right) |1\rangle |1\rangle \frac{1}{2} |0\rangle + c_7 \left(-\frac{1}{2}\right) |1\rangle |1\rangle \left(-\frac{1}{2}\right) |1\rangle \right) \\
& + 2\pi J_{21} \left(c_0 \frac{1}{2} |0\rangle \frac{1}{2} |0\rangle |0\rangle + c_1 \frac{1}{2} |0\rangle \frac{1}{2} |0\rangle |1\rangle + c_2 \frac{1}{2} |0\rangle \left(-\frac{1}{2}\right) |1\rangle |0\rangle \right) \\
& + 2\pi J_{21} \left(c_3 \frac{1}{2} |0\rangle \left(-\frac{1}{2}\right) |1\rangle |1\rangle + c_4 \left(-\frac{1}{2}\right) |1\rangle \frac{1}{2} |0\rangle |0\rangle + c_5 \left(-\frac{1}{2}\right) |1\rangle \frac{1}{2} |0\rangle |1\rangle \right) \\
& + 2\pi J_{21} \left(c_6 \left(-\frac{1}{2}\right) |1\rangle \left(-\frac{1}{2}\right) |1\rangle |0\rangle + c_7 \left(-\frac{1}{2}\right) |1\rangle \left(-\frac{1}{2}\right) |1\rangle |1\rangle \right)
\end{aligned}$$

$$\begin{aligned}
&= \frac{\omega_0}{2} (c_0 |000\rangle - c_1 |001\rangle + c_2 |010\rangle - c_3 |011\rangle) \\
&+ \frac{\omega_0}{2} (c_4 |100\rangle - c_5 |101\rangle + c_6 |110\rangle - c_7 |111\rangle) \\
&+ \frac{\omega_1}{2} (c_0 |000\rangle + c_1 |001\rangle - c_2 |010\rangle - c_3 |011\rangle) \\
&+ \frac{\omega_1}{2} (c_4 |100\rangle + c_5 |101\rangle - c_6 |110\rangle - c_7 |111\rangle) \\
&+ \frac{\omega_2}{2} (c_0 |000\rangle + c_1 |001\rangle + c_2 |010\rangle + c_3 |011\rangle) \\
&+ \frac{\omega_2}{2} (-c_4 |100\rangle - c_5 |101\rangle - c_6 |110\rangle - c_7 |111\rangle) \\
&+ \frac{\pi J_{10}}{2} (c_0 |000\rangle - c_1 |001\rangle - c_2 |010\rangle + c_3 |011\rangle) \\
&+ \frac{\pi J_{10}}{2} (c_4 |100\rangle - c_5 |101\rangle - c_6 |110\rangle + c_7 |111\rangle) \\
&+ \frac{\pi J_{20}}{2} (c_0 |000\rangle - c_1 |001\rangle + c_2 |010\rangle - c_3 |011\rangle) \\
&+ \frac{\pi J_{20}}{2} (-c_4 |100\rangle + c_5 |101\rangle - c_6 |110\rangle + c_7 |111\rangle) \\
&+ \frac{\pi J_{21}}{2} (c_0 |000\rangle + c_1 |001\rangle - c_2 |010\rangle - c_3 |011\rangle) \\
&+ \frac{\pi J_{21}}{2} (-c_4 |100\rangle - c_5 |101\rangle + c_6 |110\rangle + c_7 |111\rangle) \\
&= \frac{\omega_0 + \omega_1 + \omega_2 + \pi J_{10} + \pi J_{20} + \pi J_{21}}{2} c_0 |000\rangle \\
&+ \frac{-\omega_0 + \omega_1 + \omega_2 - \pi J_{10} - \pi J_{20} + \pi J_{21}}{2} c_1 |001\rangle \\
&+ \frac{\omega_0 - \omega_1 + \omega_2 - \pi J_{10} + \pi J_{20} - \pi J_{21}}{2} c_2 |010\rangle \\
&+ \frac{-\omega_0 - \omega_1 + \omega_2 + \pi J_{10} - \pi J_{20} - \pi J_{21}}{2} c_3 |011\rangle \\
&+ \frac{\omega_0 + \omega_1 - \omega_2 + \pi J_{10} - \pi J_{20} - \pi J_{21}}{2} c_4 |100\rangle \\
&+ \frac{-\omega_0 + \omega_1 - \omega_2 - \pi J_{10} + \pi J_{20} - \pi J_{21}}{2} c_5 |101\rangle \\
&+ \frac{\omega_0 - \omega_1 - \omega_2 - \pi J_{10} - \pi J_{20} + \pi J_{21}}{2} c_6 |110\rangle \\
&+ \frac{-\omega_0 - \omega_1 - \omega_2 + \pi J_{10} + \pi J_{20} + \pi J_{21}}{2} c_7 |111\rangle
\end{aligned} \tag{139}$$

maka dengan demikian bentuk diatas sama dengan bentuk

$$\begin{aligned}
H |\psi\rangle &= E_{3,0}c_0 |000\rangle + E_{3,1}c_1 |001\rangle + E_{3,2}c_2 |010\rangle + E_{3,3}c_3 |011\rangle \\
&+ E_{3,4}c_4 |100\rangle + E_{3,5}c_5 |101\rangle + E_{3,6}c_6 |110\rangle + E_{3,7}c_7 |111\rangle \tag{140}
\end{aligned}$$

sehingga dapat dibuat tabel level energi sebagai berikut:

n	$E_{3,n}$
0	$\frac{\omega_0 + \omega_1 + \omega_2 + \pi J_{10} + \pi J_{20} + \pi J_{21}}{2}$
1	$\frac{-\omega_0 + \omega_1 + \omega_2 - \pi J_{10} - \pi J_{20} + \pi J_{21}}{2}$
2	$\frac{\omega_0 - \omega_1 + \omega_2 - \pi J_{10} + \pi J_{20} - \pi J_{21}}{2}$
3	$\frac{-\omega_0 - \omega_1 + \omega_2 + \pi J_{10} - \pi J_{20} - \pi J_{21}}{2}$
4	$\frac{\omega_0 + \omega_1 - \omega_2 + \pi J_{10} - \pi J_{20} - \pi J_{21}}{2}$
5	$\frac{-\omega_0 + \omega_1 - \omega_2 - \pi J_{10} + \pi J_{20} - \pi J_{21}}{2}$
6	$\frac{\omega_0 - \omega_1 - \omega_2 - \pi J_{10} - \pi J_{20} + \pi J_{21}}{2}$
7	$\frac{-\omega_0 - \omega_1 - \omega_2 + \pi J_{10} + \pi J_{20} + \pi J_{21}}{2}$

4.2 Sistem 4 qubit

pada sistem 4 qubit, level energi dihasilkan dari interaksi 4 partikel, dimana bentuk hamiltonian dari sistem 4 qubit adalah sebagai berikut:

$$\begin{aligned}
H = & \omega_0 I \otimes I \otimes I \otimes I_z + \omega_1 I \otimes I \otimes I_z \otimes I + \omega_2 I \otimes I_z \otimes I \otimes I \\
& + \omega_3 I_z \otimes I \otimes I \otimes I + 2\pi J_{10} I \otimes I \otimes I_z \otimes I_z \\
& + 2\pi J_{20} I \otimes I_z \otimes I \otimes I_z + 2\pi J_{30} I_z \otimes I \otimes I_z \\
& + 2\pi J_{21} I \otimes I_z \otimes I_z \otimes I + 2\pi J_{31} I_z \otimes I \otimes I_z \otimes I \\
& + 2\pi J_{32} I_z \otimes I_z \otimes I \otimes I
\end{aligned} \tag{141}$$

sedangkan vektor keadaan untuk 4 qubit adalah sebagai berikut:

$$\begin{aligned}
|\psi\rangle = & c_0 |0000\rangle + c_1 |0001\rangle + c_2 |0010\rangle \\
& + c_3 |0011\rangle + c_4 |0100\rangle + c_5 |0101\rangle \\
& + c_6 |0110\rangle + c_7 |0111\rangle + c_8 |1000\rangle + \\
& + c_9 |1001\rangle + c_{10} |1010\rangle + c_{11} |1011\rangle \\
& + c_{12} |1100\rangle + c_{13} |1101\rangle + c_{14} |1110\rangle \\
& + c_{15} |1111\rangle
\end{aligned} \tag{142}$$

kemudian dilakukan operasi berikut:

$$\begin{aligned}
H |\psi\rangle &= \omega_0 I \otimes I \otimes I \otimes I_z |\psi\rangle \\
&+ \omega_1 I \otimes I \otimes I_z \otimes I |\psi\rangle \\
&+ \omega_2 I \otimes I_z \otimes I \otimes I |\psi\rangle \\
&+ \omega_3 I_z \otimes I \otimes I \otimes I |\psi\rangle \\
&+ 2\pi J_{10} I \otimes I \otimes I_z \otimes I_z |\psi\rangle \\
&+ 2\pi J_{20} I \otimes I_z \otimes I \otimes I_z |\psi\rangle \\
&+ 2\pi J_{30} I_z \otimes I \otimes I_z |\psi\rangle \\
&+ 2\pi J_{21} I \otimes I_z \otimes I_z \otimes I |\psi\rangle \\
&+ 2\pi J_{31} I_z \otimes I \otimes I_z \otimes I |\psi\rangle \\
&+ 2\pi J_{32} I_z \otimes I_z \otimes I \otimes I |\psi\rangle \\
&= \omega_0 (c_0 I |0\rangle I |0\rangle I |0\rangle I_z |0\rangle + c_1 I |0\rangle I |0\rangle I |0\rangle I_z |1\rangle) \\
&+ \omega_0 (c_2 I |0\rangle I |0\rangle I |1\rangle I_z |0\rangle + c_3 I |0\rangle I |0\rangle I |1\rangle I_z |1\rangle) \\
&+ \omega_0 (c_4 I |0\rangle I |1\rangle I |0\rangle I_z |0\rangle + c_5 I |0\rangle I |1\rangle I |1\rangle I_z |1\rangle) \\
&+ \omega_0 (c_6 I |0\rangle I |1\rangle I |1\rangle I_z |1\rangle + c_7 I |1\rangle I |0\rangle I |0\rangle I_z |0\rangle) \\
&+ \omega_0 (c_8 I |1\rangle I |0\rangle I |0\rangle I_z |1\rangle + c_9 I |1\rangle I |0\rangle I |1\rangle I_z |0\rangle) \\
&+ \omega_0 (c_{10} I |1\rangle I |0\rangle I |1\rangle I_z |1\rangle + c_{11} I |1\rangle I |1\rangle I |0\rangle I_z |0\rangle) \\
&+ \omega_0 (c_{12} I |1\rangle I |1\rangle I |0\rangle I_z |1\rangle + c_{13} I |1\rangle I |1\rangle I |0\rangle I_z |1\rangle) \\
&+ \omega_0 (c_{14} I |1\rangle I |1\rangle I |1\rangle I_z |0\rangle + c_{15} I |1\rangle I |1\rangle I |1\rangle I_z |1\rangle) \\
&+ \omega_1 (c_0 I |0\rangle I |0\rangle I_z |0\rangle I |0\rangle + c_1 I |0\rangle I |0\rangle I_z |0\rangle I |1\rangle) \\
&+ \omega_1 (c_2 I |0\rangle I |0\rangle I_z |1\rangle I |0\rangle + c_3 I |0\rangle I |0\rangle I_z |1\rangle I |1\rangle) \\
&+ \omega_1 (c_4 I |0\rangle I |1\rangle I_z |0\rangle I |0\rangle + c_5 I |0\rangle I |1\rangle I_z |1\rangle I |1\rangle) \\
&+ \omega_1 (c_6 I |0\rangle I |1\rangle I_z |1\rangle I |1\rangle + c_7 I |1\rangle I |0\rangle I_z |0\rangle I |0\rangle) \\
&+ \omega_1 (c_8 I |1\rangle I |0\rangle I_z |0\rangle I |1\rangle + c_9 I |1\rangle I |0\rangle I_z |1\rangle I |0\rangle) \\
&+ \omega_1 (c_{10} I |1\rangle I |0\rangle I_z |1\rangle I |1\rangle + c_{11} I |1\rangle I |1\rangle I_z |0\rangle I |0\rangle) \\
&+ \omega_1 (c_{12} I |1\rangle I |1\rangle I_z |0\rangle I |1\rangle + c_{13} I |1\rangle I |1\rangle I_z |0\rangle I |1\rangle) \\
&+ \omega_1 (c_{14} I |1\rangle I |1\rangle I |1\rangle I |0\rangle + c_{15} I |1\rangle I |1\rangle I_z |1\rangle I |1\rangle)
\end{aligned}$$

$$\begin{aligned}
& + \omega_2 (c_0 I |0\rangle I_z |0\rangle I |0\rangle I |0\rangle + c_1 I |0\rangle I_z |0\rangle I |0\rangle I |1\rangle) \\
& + \omega_2 (c_2 I |0\rangle I_z |0\rangle I |1\rangle I |0\rangle + c_3 I |0\rangle I_z |0\rangle I |1\rangle I |1\rangle) \\
& + \omega_2 (c_4 I |0\rangle I_z |1\rangle I |0\rangle I |0\rangle + c_5 I |0\rangle I_z |1\rangle I |1\rangle I |1\rangle) \\
& + \omega_2 (c_6 I |0\rangle I_z |1\rangle I |1\rangle I |1\rangle + c_7 I |1\rangle I_z |0\rangle I |0\rangle I |0\rangle) \\
& + \omega_2 (c_8 I |1\rangle I_z |0\rangle I |0\rangle I |1\rangle + c_9 I |1\rangle I_z |0\rangle I |1\rangle I |0\rangle) \\
& + \omega_2 (c_{10} I |1\rangle I_z |0\rangle I |1\rangle I |1\rangle + c_{11} I |1\rangle I_z |1\rangle I |0\rangle I |0\rangle) \\
& + \omega_2 (c_{12} I |1\rangle I_z |1\rangle I |0\rangle I |1\rangle + c_{13} I |1\rangle I_z |1\rangle I |0\rangle I |1\rangle) \\
& + \omega_2 (c_{14} I |1\rangle I_z |1\rangle I |1\rangle I |0\rangle + c_{15} I |1\rangle I_z |1\rangle I |1\rangle I |1\rangle) \\
& + \omega_3 (c_0 I_z |0\rangle I |0\rangle I |0\rangle I |0\rangle + c_1 I_z |0\rangle I |0\rangle I |0\rangle I |1\rangle) \\
& + \omega_3 (c_2 I_z |0\rangle I |0\rangle I |1\rangle I |0\rangle + c_3 I_z |0\rangle I |0\rangle I |1\rangle I |1\rangle) \\
& + \omega_3 (c_4 I_z |0\rangle I |1\rangle I |0\rangle I |0\rangle + c_5 I_z |0\rangle I |1\rangle I |1\rangle I |1\rangle) \\
& + \omega_3 (c_6 I_z |0\rangle I |1\rangle I |1\rangle I |1\rangle + c_7 I_z |1\rangle I |0\rangle I |0\rangle I |0\rangle) \\
& + \omega_3 (c_8 I_z |1\rangle I |0\rangle I |0\rangle I |1\rangle + c_9 I_z |1\rangle I |0\rangle I |1\rangle I |0\rangle) \\
& + \omega_3 (c_{10} I_z |1\rangle I |0\rangle I |1\rangle I |1\rangle + c_{11} I_z |1\rangle I |1\rangle I |0\rangle I |0\rangle) \\
& + \omega_1 (c_{12} I_z |1\rangle I |1\rangle I |0\rangle I |1\rangle + c_{13} I_z |1\rangle I |1\rangle I |0\rangle I_z |1\rangle) \\
& + \omega_3 (c_{14} I_z |1\rangle I |1\rangle I |1\rangle I |0\rangle + c_{15} I_z |1\rangle I |1\rangle I |1\rangle I |1\rangle) \\
& + 2\pi J_{10} (c_0 I |0\rangle I |0\rangle I_z |0\rangle I_z |0\rangle + c_1 I |0\rangle I |0\rangle I_z |0\rangle I_z |1\rangle) \\
& + 2\pi J_{10} (c_2 I |0\rangle I |0\rangle I_z |1\rangle I_z |0\rangle + c_3 I |0\rangle I |0\rangle I_z |1\rangle I_z |1\rangle) \\
& + 2\pi J_{10} (c_4 I |0\rangle I |1\rangle I_z |0\rangle I_z |0\rangle + c_5 I |0\rangle I |1\rangle I_z |1\rangle I_z |1\rangle) \\
& + 2\pi J_{10} (c_6 I |0\rangle I |1\rangle I_z |1\rangle I_z |1\rangle + c_7 I |1\rangle I |0\rangle I_z |0\rangle I_z |0\rangle) \\
& + 2\pi J_{10} (c_8 I |1\rangle I |0\rangle I_z |0\rangle I_z |1\rangle + c_9 I |1\rangle I |0\rangle I_z |1\rangle I_z |0\rangle) \\
& + 2\pi J_{10} (c_{10} I |1\rangle I |0\rangle I_z |1\rangle I_z |1\rangle + c_{11} I |1\rangle I |1\rangle I_z |0\rangle I_z |0\rangle) \\
& + 2\pi J_{10} (c_{12} I |1\rangle I |1\rangle I_z |0\rangle I_z |1\rangle + c_{13} I |1\rangle I |1\rangle I_z |0\rangle I_z |1\rangle) \\
& + 2\pi J_{10} (c_{14} I |1\rangle I |1\rangle I_z |1\rangle I_z |0\rangle + c_{15} I |1\rangle I |1\rangle I |1\rangle I |1\rangle)
\end{aligned}$$

$$\begin{aligned}
& + 2\pi J_{20} (c_0 I |0\rangle I_z |0\rangle I |0\rangle I_z |0\rangle + c_1 I |0\rangle I_z |0\rangle I |0\rangle I_z |1\rangle) \\
& + 2\pi J_{20} (c_2 I |0\rangle I_z |0\rangle I |1\rangle I_z |0\rangle + c_3 I |0\rangle I_z |0\rangle I |1\rangle I_z |1\rangle) \\
& + 2\pi J_{20} (c_4 I |0\rangle I_z |1\rangle I |0\rangle I_z |0\rangle + c_5 I |0\rangle I_z |1\rangle I |1\rangle I_z |1\rangle) \\
& + 2\pi J_{20} (c_6 I |0\rangle I_z |1\rangle I |1\rangle I_z |1\rangle + c_7 I |1\rangle I_z |0\rangle I |0\rangle I_z |0\rangle) \\
& + 2\pi J_{20} (c_8 I |1\rangle I_z |0\rangle I |0\rangle I_z |1\rangle + c_9 I |1\rangle I_z |0\rangle I |1\rangle I_z |0\rangle) \\
& + 2\pi J_{20} (c_{10} I |1\rangle I_z |0\rangle I |1\rangle I_z |1\rangle + c_{11} I |1\rangle I_z |1\rangle I |0\rangle I_z |0\rangle) \\
& + 2\pi J_{20} (c_{12} I |1\rangle I_z |1\rangle I |0\rangle I_z |1\rangle + c_{13} I |1\rangle I_z |1\rangle I |0\rangle I_z |1\rangle) \\
& + 2\pi J_{20} (c_{14} I |1\rangle I_z |1\rangle I |1\rangle I_z |0\rangle + c_{15} I |1\rangle I_z |1\rangle I |1\rangle I_z |1\rangle) \\
& + 2\pi J_{30} (c_0 I_z |0\rangle I |0\rangle I |0\rangle I_z |0\rangle + c_1 I_z |0\rangle I |0\rangle I |0\rangle I_z |1\rangle) \\
& + 2\pi J_{30} (c_2 I_z |0\rangle I |0\rangle I |1\rangle I_z |0\rangle + c_3 I_z |0\rangle I |0\rangle I |1\rangle I_z |1\rangle) \\
& + 2\pi J_{30} (c_4 I_z |0\rangle I |1\rangle I |0\rangle I_z |0\rangle + c_5 I_z |0\rangle I |1\rangle I |1\rangle I_z |1\rangle) \\
& + 2\pi J_{30} (c_6 I_z |0\rangle I |1\rangle I |1\rangle I_z |1\rangle + c_7 I_z |1\rangle I |0\rangle I |0\rangle I_z |0\rangle) \\
& + 2\pi J_{30} (c_8 I_z |1\rangle I |0\rangle I |0\rangle I_z |1\rangle + c_9 I_z |1\rangle I |0\rangle I |1\rangle I_z |0\rangle) \\
& + 2\pi J_{30} (c_{10} I_z |1\rangle I |0\rangle I |1\rangle I_z |1\rangle + c_{11} I_z |1\rangle I |1\rangle I |0\rangle I_z |0\rangle) \\
& + 2\pi J_{30} (c_{12} I_z |1\rangle I |1\rangle I |0\rangle I_z |1\rangle + c_{13} I_z |1\rangle I |1\rangle I |0\rangle I_z |1\rangle) \\
& + 2\pi J_{30} (c_{14} I_z |1\rangle I |1\rangle I |1\rangle I_z |0\rangle + c_{15} I_z |1\rangle I |1\rangle I |1\rangle I_z |1\rangle) \\
& + 2\pi J_{21} (c_0 I |0\rangle I_z |0\rangle I_z |0\rangle I |0\rangle + c_1 I |0\rangle I_z |0\rangle I_z |0\rangle I |1\rangle) \\
& + 2\pi J_{21} (c_2 I |0\rangle I_z |0\rangle I_z |1\rangle I |0\rangle + c_3 I |0\rangle I_z |0\rangle I_z |1\rangle I |1\rangle) \\
& + 2\pi J_{21} (c_4 I |0\rangle I_z |1\rangle I_z |0\rangle I |0\rangle + c_5 I |0\rangle I |1\rangle I |1\rangle I |1\rangle) \\
& + 2\pi J_{21} (c_6 I |0\rangle I_z |1\rangle I_z |1\rangle I |1\rangle + c_7 I |1\rangle I_z |0\rangle I_z |0\rangle I |0\rangle) \\
& + 2\pi J_{21} (c_8 I |1\rangle I_z |0\rangle I_z |0\rangle I |1\rangle + c_9 I |1\rangle I_z |0\rangle I_z |1\rangle I |0\rangle) \\
& + 2\pi J_{21} (c_{10} I |1\rangle I |0\rangle I_z |1\rangle I_z |1\rangle + c_{11} I |1\rangle I_z |1\rangle I_z |0\rangle I |0\rangle) \\
& + 2\pi J_{21} (c_{12} I |1\rangle I_z |1\rangle I_z |0\rangle I |1\rangle + c_{13} I |1\rangle I_z |1\rangle I |0\rangle I_z |1\rangle) \\
& + 2\pi J_{21} (c_{14} I |1\rangle I |1\rangle I |1\rangle I |0\rangle + c_{15} I |1\rangle I_z |1\rangle I_z |1\rangle I |1\rangle)
\end{aligned}$$

$$\begin{aligned}
& + 2\pi J_{31} (c_0 I_z |0\rangle I |0\rangle I_z |0\rangle I |0\rangle + c_1 I_z |0\rangle I |0\rangle I_z |0\rangle I |1\rangle) \\
& + 2\pi J_{31} (c_2 I_z |0\rangle I |0\rangle I_z |1\rangle I |0\rangle + c_3 I_z |0\rangle I |0\rangle I_z |1\rangle I |1\rangle) \\
& + 2\pi J_{31} (c_4 I_z |0\rangle I |1\rangle I_z |0\rangle I |0\rangle + c_5 I_z |0\rangle I |1\rangle I_z |1\rangle I |1\rangle) \\
& + 2\pi J_{31} (c_6 I_z |0\rangle I |1\rangle I_z |1\rangle I |1\rangle + c_7 I_z |1\rangle I |0\rangle I_z |0\rangle I |0\rangle) \\
& + 2\pi J_{31} (c_8 I_z |1\rangle I |0\rangle I_z |0\rangle I |1\rangle + c_9 I_z |1\rangle I |0\rangle I_z |1\rangle I |0\rangle) \\
& + 2\pi J_{31} (c_{10} I_z |1\rangle I |0\rangle I_z |1\rangle I |1\rangle + c_{11} I_z |1\rangle I |1\rangle I_z |0\rangle I |0\rangle) \\
& + 2\pi J_{31} (c_{12} I_z |1\rangle I |1\rangle I_z |0\rangle I |1\rangle + c_{13} I_z |1\rangle I |1\rangle I_z |0\rangle I |1\rangle) \\
& + 2\pi J_{31} (c_{14} I_z |1\rangle I |1\rangle I_z |1\rangle I |0\rangle + c_{15} I_z |1\rangle I |1\rangle I_z |1\rangle I |1\rangle) \\
& + 2\pi J_{32} (c_0 I_z |0\rangle I_z |0\rangle I |0\rangle I |0\rangle + c_1 I_z |0\rangle I_z |0\rangle I |0\rangle I |1\rangle) \\
& + 2\pi J_{32} (c_2 I_z |0\rangle I_z |0\rangle I |1\rangle I |0\rangle + c_3 I_z |0\rangle I_z |0\rangle I |1\rangle I |1\rangle) \\
& + 2\pi J_{32} (c_4 I_z |0\rangle I_z |1\rangle I |0\rangle I |0\rangle + c_5 I_z |0\rangle I_z |1\rangle I |1\rangle I |1\rangle) \\
& + 2\pi J_{32} (c_6 I_z |0\rangle I_z |1\rangle I |1\rangle I |1\rangle + c_7 I_z |1\rangle I_z |0\rangle I |0\rangle I |0\rangle) \\
& + 2\pi J_{32} (c_8 I_z |1\rangle I_z |0\rangle I |0\rangle I |1\rangle + c_9 I_z |1\rangle I_z |0\rangle I |1\rangle I |0\rangle) \\
& + 2\pi J_{32} (c_{10} I_z |1\rangle I_z |0\rangle I |1\rangle I |1\rangle + c_{11} I_z |1\rangle I_z |1\rangle I |0\rangle I |0\rangle) \\
& + 2\pi J_{32} (c_{12} I_z |1\rangle I_z |1\rangle I |0\rangle I |1\rangle + c_{13} I_z |1\rangle I_z |1\rangle I |0\rangle I |1\rangle) \\
& + 2\pi J_{32} (c_{14} I_z |1\rangle I |1\rangle I_z |1\rangle I |0\rangle + c_{15} I_z |1\rangle I_z |1\rangle I |1\rangle I |1\rangle)
\end{aligned}$$

$$\begin{aligned}
&= \omega_0 \left(c_0 |0\rangle |0\rangle |0\rangle \frac{1}{2} |0\rangle + c_1 |0\rangle |0\rangle |0\rangle \left(-\frac{1}{2}\right) |1\rangle + c_2 |0\rangle |0\rangle |1\rangle \frac{1}{2} |0\rangle \right) \\
&+ \omega_0 \left(c_3 |0\rangle |0\rangle |1\rangle \left(-\frac{1}{2}\right) |1\rangle + c_4 |0\rangle |1\rangle |0\rangle \frac{1}{2} |0\rangle + c_5 |0\rangle |1\rangle |0\rangle \left(-\frac{1}{2}\right) |1\rangle \right) \\
&+ \omega_0 \left(c_6 |0\rangle |1\rangle |1\rangle \frac{1}{2} |0\rangle + c_7 |0\rangle |1\rangle |1\rangle \left(-\frac{1}{2}\right) |1\rangle + c_8 |1\rangle |0\rangle |0\rangle \frac{1}{2} |0\rangle \right) \\
&+ \omega_0 \left(c_9 |1\rangle |0\rangle |0\rangle \left(-\frac{1}{2}\right) |1\rangle + c_{10} |1\rangle |0\rangle |1\rangle \frac{1}{2} |0\rangle + c_{11} |1\rangle |0\rangle |1\rangle \left(-\frac{1}{2}\right) |1\rangle \right) \\
&+ \omega_0 \left(c_{12} |1\rangle |1\rangle |0\rangle \frac{1}{2} |0\rangle + c_{13} |1\rangle |1\rangle |0\rangle \left(-\frac{1}{2}\right) |1\rangle + c_{14} |1\rangle |1\rangle |1\rangle \frac{1}{2} |0\rangle \right) \\
&+ \omega_0 \left(c_{15} |1\rangle |1\rangle |1\rangle \left(-\frac{1}{2}\right) |1\rangle \right) \\
&+ \omega_1 \left(c_0 |0\rangle |0\rangle \frac{1}{2} |0\rangle |0\rangle + c_1 |0\rangle |0\rangle \frac{1}{2} |0\rangle |1\rangle + c_2 |0\rangle |0\rangle \left(-\frac{1}{2}\right) |1\rangle |0\rangle \right) \\
&+ \omega_1 \left(c_3 |0\rangle |0\rangle \left(-\frac{1}{2}\right) |1\rangle |1\rangle + c_4 |0\rangle |1\rangle \frac{1}{2} |0\rangle |0\rangle + c_5 |0\rangle |1\rangle \frac{1}{2} |0\rangle |1\rangle \right) \\
&+ \omega_1 \left(c_6 |0\rangle |1\rangle \left(-\frac{1}{2}\right) |1\rangle |0\rangle + c_7 |0\rangle |1\rangle \left(-\frac{1}{2}\right) |1\rangle |1\rangle + c_8 |1\rangle |0\rangle \frac{1}{2} |0\rangle |0\rangle \right) \\
&+ \omega_1 \left(c_9 |1\rangle |0\rangle \frac{1}{2} |0\rangle |1\rangle + c_{10} |1\rangle |0\rangle \left(-\frac{1}{2}\right) |1\rangle |0\rangle + c_{11} |1\rangle |0\rangle \left(-\frac{1}{2}\right) |1\rangle |1\rangle \right) \\
&+ \omega_1 \left(c_{12} |1\rangle |1\rangle \frac{1}{2} |0\rangle |0\rangle + c_{13} |1\rangle |1\rangle \frac{1}{2} |0\rangle |1\rangle + c_{14} |1\rangle |1\rangle \left(-\frac{1}{2}\right) |1\rangle |0\rangle \right) \\
&+ \omega_1 \left(c_{15} |1\rangle |1\rangle \left(-\frac{1}{2}\right) |1\rangle |1\rangle \right) \\
&+ \omega_2 \left(c_0 |0\rangle \frac{1}{2} |0\rangle |0\rangle |0\rangle + c_1 |0\rangle \frac{1}{2} |0\rangle |0\rangle |1\rangle + c_2 |0\rangle \frac{1}{2} |0\rangle |1\rangle |0\rangle \right) \\
&+ \omega_2 \left(c_3 |0\rangle \frac{1}{2} |0\rangle |1\rangle |1\rangle + c_4 |0\rangle \frac{1}{2} |0\rangle |1\rangle |0\rangle |0\rangle + c_5 |0\rangle \frac{1}{2} |1\rangle |0\rangle |1\rangle \right) \\
&+ \omega_2 \left(c_6 |0\rangle \left(-\frac{1}{2}\right) |1\rangle |1\rangle |0\rangle + c_7 |0\rangle \left(-\frac{1}{2}\right) |1\rangle |1\rangle |1\rangle + c_8 |1\rangle \frac{1}{2} |0\rangle |0\rangle |0\rangle \right) \\
&+ \omega_2 \left(c_9 |1\rangle \frac{1}{2} |0\rangle |0\rangle |1\rangle + c_{10} |1\rangle \frac{1}{2} |0\rangle |1\rangle |0\rangle + c_{11} |1\rangle \frac{1}{2} |0\rangle |1\rangle |1\rangle \right) \\
&+ \omega_2 \left(c_{12} |1\rangle \left(-\frac{1}{2}\right) |1\rangle |0\rangle |0\rangle + c_{13} |1\rangle \left(-\frac{1}{2}\right) |1\rangle |0\rangle |1\rangle + c_{14} |1\rangle \left(-\frac{1}{2}\right) |1\rangle |1\rangle |0\rangle \right) \\
&+ \omega_2 \left(c_{15} |1\rangle \left(-\frac{1}{2}\right) |1\rangle |1\rangle |1\rangle \right) \\
&+ \omega_3 \left(c_0 \frac{1}{2} |0\rangle |0\rangle |0\rangle |0\rangle + c_1 \frac{1}{2} |0\rangle |0\rangle |0\rangle |1\rangle + c_2 \frac{1}{2} |0\rangle |0\rangle |1\rangle |0\rangle \right) \\
&+ \omega_3 \left(c_3 \frac{1}{2} |0\rangle |0\rangle |1\rangle |1\rangle + c_4 \frac{1}{2} |0\rangle |1\rangle |0\rangle |0\rangle + c_5 \frac{1}{2} |0\rangle |1\rangle |0\rangle |1\rangle \right) \\
&+ \omega_3 \left(c_6 \frac{1}{2} |0\rangle |1\rangle |1\rangle |0\rangle + c_7 \frac{1}{2} |0\rangle |1\rangle |1\rangle |1\rangle + c_8 \left(-\frac{1}{2}\right) |1\rangle |0\rangle |0\rangle |0\rangle \right) \\
&+ \omega_3 \left(c_9 \left(-\frac{1}{2}\right) |1\rangle |0\rangle |0\rangle |1\rangle + c_{10} \left(-\frac{1}{2}\right) |1\rangle |0\rangle |1\rangle |0\rangle + c_{11} \left(-\frac{1}{2}\right) |1\rangle |0\rangle |1\rangle |1\rangle \right)
\end{aligned}$$

$$\begin{aligned}
& + 2\pi J_{21} \left(c_6 |0\rangle \left(-\frac{1}{2}\right) |1\rangle \left(-\frac{1}{2}\right) |1\rangle |0\rangle + c_7 |0\rangle \left(-\frac{1}{2}\right) |1\rangle \left(-\frac{1}{2}\right) |1\rangle |1\rangle \right) \\
& + 2\pi J_{21} \left(c_8 |1\rangle \frac{1}{2} |0\rangle \frac{1}{2} |0\rangle |0\rangle + c_9 |1\rangle \frac{1}{2} |0\rangle \frac{1}{2} |0\rangle |1\rangle \right) \\
& + 2\pi J_{21} \left(c_{10} |1\rangle \frac{1}{2} |0\rangle \left(-\frac{1}{2}\right) |1\rangle |0\rangle + c_{11} |1\rangle \frac{1}{2} |0\rangle \left(-\frac{1}{2}\right) |1\rangle |1\rangle \right) \\
& + 2\pi J_{31} \left(c_{12} |1\rangle \left(-\frac{1}{2}\right) |1\rangle \frac{1}{2} |0\rangle |0\rangle + c_{13} |1\rangle \left(-\frac{1}{2}\right) |1\rangle \frac{1}{2} |0\rangle |1\rangle \right) \\
& + 2\pi J_{31} \left(c_{14} |1\rangle |1\rangle \left(-\frac{1}{2}\right) |1\rangle \frac{1}{2} |0\rangle + c_{15} |1\rangle \left(-\frac{1}{2}\right) |1\rangle \left(-\frac{1}{2}\right) |1\rangle |1\rangle \right) \\
& + 2\pi J_{31} \left(c_0 \frac{1}{2} |0\rangle |0\rangle \frac{1}{2} |0\rangle |0\rangle + c_1 \frac{1}{2} |0\rangle |0\rangle \frac{1}{2} |0\rangle |1\rangle \right) \\
& + 2\pi J_{31} \left(c_2 \frac{1}{2} |0\rangle |0\rangle \left(-\frac{1}{2}\right) |1\rangle |0\rangle + c_3 \frac{1}{2} |0\rangle |0\rangle \left(-\frac{1}{2}\right) |1\rangle |1\rangle \right) \\
& + 2\pi J_{31} \left(c_4 \frac{1}{2} |0\rangle |1\rangle \frac{1}{2} |0\rangle |0\rangle + c_5 \frac{1}{2} |0\rangle |1\rangle \frac{1}{2} |0\rangle |1\rangle \right) \\
& + 2\pi J_{31} \left(c_6 \frac{1}{2} |0\rangle |1\rangle \left(-\frac{1}{2}\right) |1\rangle |0\rangle + c_7 \frac{1}{2} |0\rangle |1\rangle \left(-\frac{1}{2}\right) |1\rangle |1\rangle \right) \\
& + 2\pi J_{31} \left(c_8 \left(-\frac{1}{2}\right) |1\rangle |0\rangle \frac{1}{2} |0\rangle |0\rangle + c_9 \left(-\frac{1}{2}\right) |1\rangle |0\rangle \frac{1}{2} |0\rangle |1\rangle \right) \\
& + 2\pi J_{31} \left(c_{10} \left(-\frac{1}{2}\right) |1\rangle |0\rangle \left(-\frac{1}{2}\right) |1\rangle |0\rangle + c_{11} \left(-\frac{1}{2}\right) |1\rangle |0\rangle \left(-\frac{1}{2}\right) |1\rangle |1\rangle \right) \\
& + 2\pi J_{31} \left(c_{12} \left(-\frac{1}{2}\right) |1\rangle |1\rangle \frac{1}{2} |0\rangle |0\rangle + c_{13} \left(-\frac{1}{2}\right) |1\rangle |1\rangle \frac{1}{2} |0\rangle |1\rangle \right) \\
& + 2\pi J_{31} \left(c_{14} \left(-\frac{1}{2}\right) |1\rangle |1\rangle \left(-\frac{1}{2}\right) |1\rangle |0\rangle + c_{15} \left(-\frac{1}{2}\right) |1\rangle |1\rangle \left(-\frac{1}{2}\right) |1\rangle |1\rangle \right) \\
& + 2\pi J_{32} \left(c_0 \frac{1}{2} |0\rangle \frac{1}{2} |0\rangle |0\rangle |0\rangle + c_1 \frac{1}{2} |0\rangle \frac{1}{2} |0\rangle |0\rangle |1\rangle + c_2 \frac{1}{2} |0\rangle \frac{1}{2} |0\rangle |1\rangle |0\rangle \right) \\
& + 2\pi J_{32} \left(c_3 \frac{1}{2} |0\rangle \frac{1}{2} |0\rangle |1\rangle |1\rangle + c_4 \frac{1}{2} |0\rangle \left(-\frac{1}{2}\right) |1\rangle |0\rangle |0\rangle + c_5 \frac{1}{2} |0\rangle \left(-\frac{1}{2}\right) |1\rangle |0\rangle |1\rangle \right) \\
& + 2\pi J_{32} \left(c_6 \frac{1}{2} |0\rangle \left(-\frac{1}{2}\right) |1\rangle |1\rangle |0\rangle + c_7 \frac{1}{2} |0\rangle \left(-\frac{1}{2}\right) |1\rangle |1\rangle |1\rangle + c_8 \left(-\frac{1}{2}\right) |1\rangle \frac{1}{2} |0\rangle |0\rangle |0\rangle \right) \\
& + 2\pi J_{32} \left(c_9 \left(-\frac{1}{2}\right) |1\rangle \frac{1}{2} |0\rangle |0\rangle |1\rangle + c_{10} \left(-\frac{1}{2}\right) |1\rangle \frac{1}{2} |0\rangle |1\rangle |0\rangle + c_{11} \left(-\frac{1}{2}\right) |1\rangle \frac{1}{2} |0\rangle |1\rangle |1\rangle \right) \\
& + 2\pi J_{32} \left(c_{12} \left(-\frac{1}{2}\right) |1\rangle \left(-\frac{1}{2}\right) |1\rangle |0\rangle |0\rangle + c_{13} \left(-\frac{1}{2}\right) |1\rangle \left(-\frac{1}{2}\right) |1\rangle |0\rangle |1\rangle + c_{14} \left(-\frac{1}{2}\right) |1\rangle \left(-\frac{1}{2}\right) |1\rangle |1\rangle |0\rangle \right) \\
& + 2\pi J_{32} \left(c_{15} \left(-\frac{1}{2}\right) |1\rangle \left(-\frac{1}{2}\right) |1\rangle |1\rangle |1\rangle \right) \\
& = \frac{\omega_0}{2} (c_0 |0000\rangle - c_1 |0001\rangle + c_2 |0010\rangle)
\end{aligned}$$

$$\begin{aligned}
& + \frac{\omega_0}{2} (-c_3 |0011\rangle + c_4 |0100\rangle - c_5 |0101\rangle) \\
& + \frac{\omega_0}{2} (c_6 |0110\rangle - c_7 |0111\rangle + c_8 |1000\rangle) \\
& + \frac{\omega_0}{2} (-c_9 |1001\rangle + c_{10} |1010\rangle - c_{11} |1011\rangle) \\
& + \frac{\omega_0}{2} (c_{12} |1100\rangle - c_{13} |1101\rangle + c_{14} |1110\rangle) \\
& - \frac{\omega_0}{2} c_{15} |1111\rangle \\
& + \frac{\omega_1}{2} (c_0 |0000\rangle + c_1 |0001\rangle - c_2 |0010\rangle) \\
& + \frac{\omega_1}{2} (-c_3 |0011\rangle + c_4 |0100\rangle + c_5 |0101\rangle) \\
& + \frac{\omega_1}{2} (-c_6 |0110\rangle - c_7 |0111\rangle + c_8 |1000\rangle) \\
& + \frac{\omega_1}{2} (+c_9 |1001\rangle - c_{10} |1010\rangle - c_{11} |1011\rangle) \\
& + \frac{\omega_1}{2} (c_{12} |1100\rangle + c_{13} |1101\rangle - c_{14} |1110\rangle) \\
& - \frac{\omega_1}{2} c_{15} |1111\rangle \\
& + \frac{\omega_2}{2} (c_0 |0000\rangle + c_1 |0001\rangle + c_2 |0010\rangle) \\
& + \frac{\omega_2}{2} (+c_3 |0011\rangle - c_4 |0100\rangle - c_5 |0101\rangle) \\
& + \frac{\omega_2}{2} (-c_6 |0110\rangle - c_7 |0111\rangle + c_8 |1000\rangle) \\
& + \frac{\omega_2}{2} (c_9 |1001\rangle + c_{10} |1010\rangle + c_{11} |1011\rangle) \\
& + \frac{\omega_2}{2} (-c_{12} |1100\rangle - c_{13} |1101\rangle - c_{14} |1110\rangle)
\end{aligned}$$

$$\begin{aligned}
& -\frac{\omega_2}{2}c_{15}|1111\rangle \\
& +\frac{\omega_3}{2}(c_0|0000\rangle+c_1|0001\rangle+c_2|0010\rangle) \\
& +\frac{\omega_3}{2}(c_3|0011\rangle+c_4|0100\rangle+c_5|0101\rangle) \\
& +\frac{\omega_3}{2}(+c_6|0110\rangle+c_7|0111\rangle-c_8|1000\rangle) \\
& +\frac{\omega_3}{2}(-c_9|1001\rangle-c_{10}|1010\rangle-c_{11}|1011\rangle) \\
& +\frac{\omega_3}{2}(-c_{12}|1100\rangle-c_{13}|1101\rangle-c_{14}|1110\rangle) \\
& -\frac{\omega_3}{2}c_{15}|1111\rangle \\
& +\frac{\pi J_{10}}{2}(c_0|0000\rangle-c_1|0001\rangle-c_2|0010\rangle) \\
& +\frac{\pi J_{10}}{2}(c_3|0011\rangle+c_4|0100\rangle-c_5|0101\rangle) \\
& +\frac{\pi J_{10}}{2}(-c_6|0110\rangle+c_7|0111\rangle+c_8|1000\rangle) \\
& +\frac{\pi J_{10}}{2}(-c_9|1001\rangle-c_{10}|1010\rangle+c_{11}|1011\rangle) \\
& +\frac{\pi J_{10}}{2}(c_{12}|1100\rangle-c_{13}|1101\rangle-c_{14}|1110\rangle) \\
& +\frac{\pi J_{10}}{2}c_{15}|1111\rangle \\
& +\frac{\pi J_{20}}{2}(c_0|0000\rangle-c_1|0001\rangle+c_2|0010\rangle) \\
& +\frac{\pi J_{20}}{2}(-c_3|0011\rangle-c_4|0100\rangle+c_5|0101\rangle) \\
& +\frac{\pi J_{20}}{2}(-c_6|0110\rangle+c_7|0111\rangle+c_8|1000\rangle)
\end{aligned}$$

$$\begin{aligned}
& + \frac{\pi J_{20}}{2} (-c_9 |1001\rangle + c_{10} |1010\rangle - c_{11} |1011\rangle) \\
& + \frac{\pi J_{20}}{2} (-c_{12} |1100\rangle + c_{13} |1101\rangle - c_{14} |1110\rangle) \\
& + \frac{\pi J_{20}}{2} c_{15} |1111\rangle \\
& + \frac{\pi J_{30}}{2} (c_0 |0000\rangle - c_1 |0001\rangle + c_2 |0010\rangle) \\
& + \frac{\pi J_{30}}{2} (-c_3 |0011\rangle + c_4 |0100\rangle - c_5 |0101\rangle) \\
& + \frac{\pi J_{30}}{2} (+c_6 |0110\rangle - c_7 |0111\rangle - c_8 |1000\rangle) \\
& + \frac{\pi J_{30}}{2} (c_9 |1001\rangle - c_{10} |1010\rangle + c_{11} |1011\rangle) \\
& + \frac{\pi J_{30}}{2} (-c_{12} |1100\rangle + c_{13} |1101\rangle - c_{14} |1110\rangle) \\
& + \frac{\pi J_{30}}{2} c_{15} |1111\rangle \\
& + \frac{\pi J_{21}}{2} (c_0 |0000\rangle + c_1 |0001\rangle - c_2 |0010\rangle) \\
& + \frac{\pi J_{21}}{2} (-c_3 |0011\rangle - c_4 |0100\rangle - c_5 |0101\rangle) \\
& + \frac{\pi J_{21}}{2} (+c_6 |0110\rangle + c_7 |0111\rangle + c_8 |1000\rangle) \\
& + \frac{\pi J_{21}}{2} (c_9 |1001\rangle - c_{10} |1010\rangle - c_{11} |1011\rangle) \\
& + \frac{\pi J_{21}}{2} (-c_{12} |1100\rangle - c_{13} |1101\rangle + c_{14} |1110\rangle) \\
& + \frac{\pi J_{21}}{2} c_{15} |1111\rangle
\end{aligned}$$

$$\begin{aligned}
& + \frac{\pi J_{31}}{2} (c_0 |0000\rangle + c_1 |0001\rangle - c_2 |0010\rangle) \\
& + \frac{\pi J_{31}}{2} (-c_3 |0011\rangle + c_4 |0100\rangle + c_5 |0101\rangle) \\
& + \frac{\pi J_{31}}{2} (-c_6 |0110\rangle - c_7 |0111\rangle - c_8 |1000\rangle) \\
& + \frac{\pi J_{31}}{2} (-c_9 |1001\rangle + c_{10} |1010\rangle + c_{11} |1011\rangle) \\
& + \frac{\pi J_{31}}{2} (-c_{12} |1100\rangle - c_{13} |1101\rangle + c_{14} |1110\rangle) \\
& + \frac{\pi J_{31}}{2} c_{15} |1111\rangle \\
& + \frac{\pi J_{32}}{2} (c_0 |0000\rangle + c_1 |0001\rangle + c_2 |0010\rangle) \\
& + \frac{\pi J_{32}}{2} (c_3 |0011\rangle - c_4 |0100\rangle - c_5 |0101\rangle) \\
& + \frac{\pi J_{32}}{2} (-c_6 |0110\rangle - c_7 |0111\rangle - c_8 |1000\rangle) \\
& + \frac{\pi J_{32}}{2} (-c_9 |1001\rangle - c_{10} |1010\rangle - c_{11} |1011\rangle) \\
& + \frac{\pi J_{32}}{2} (c_{12} |1100\rangle + c_{13} |1101\rangle + c_{14} |1110\rangle) \\
& + \frac{\pi J_{32}}{2} c_{15} |1111\rangle
\end{aligned}$$

$$\begin{aligned}
&= \frac{\omega_0 + \omega_1 + \omega_2 + \omega_3 + \pi J_{10} + \pi J_{20} + \pi J_{30} + \pi J_{21} + \pi J_{31} + \pi J_{32}}{2} c_0 |0000\rangle) \\
&+ \frac{-\omega_0 + \omega_1 + \omega_2 + \omega_3 - \pi J_{10} - \pi J_{20} - \pi J_{30} + \pi J_{21} + \pi J_{31} + \pi J_{32}}{2} c_1 |0001\rangle) \\
&+ \frac{\omega_0 - \omega_1 + \omega_2 + \omega_3 - \pi J_{10} + \pi J_{20} + \pi J_{30} - \pi J_{21} - \pi J_{31} + \pi J_{32}}{2} c_2 |0010\rangle) \\
&+ \frac{-\omega_0 - \omega_1 + \omega_2 + \omega_3 + \pi J_{10} - \pi J_{20} - \pi J_{30} - \pi J_{21} - \pi J_{31} + \pi J_{32}}{2} c_3 |0011\rangle) \\
&+ \frac{\omega_0 + \omega_1 - \omega_2 + \omega_3 + \pi J_{10} - \pi J_{20} + \pi J_{30} - \pi J_{21} + \pi J_{31} - \pi J_{32}}{2} c_4 |0100\rangle) \\
&+ \frac{-\omega_0 + \omega_1 - \omega_2 + \omega_3 - \pi J_{10} + \pi J_{20} - \pi J_{30} - \pi J_{21} + \pi J_{31} - \pi J_{32}}{2} c_5 |0101\rangle) \\
&+ \frac{\omega_0 - \omega_1 - \omega_2 + \omega_3 - \pi J_{10} - \pi J_{20} + \pi J_{30} + \pi J_{21} - \pi J_{31} - \pi J_{32}}{2} c_6 |0110\rangle) \\
&+ \frac{-\omega_0 - \omega_1 - \omega_2 + \omega_3 + \pi J_{10} + \pi J_{20} - \pi J_{30} + \pi J_{21} - \pi J_{31} - \pi J_{32}}{2} c_7 |0111\rangle) \\
&+ \frac{\omega_0 + \omega_1 + \omega_2 - \omega_3 + \pi J_{10} + \pi J_{20} - \pi J_{30} + \pi J_{21} - \pi J_{31} - \pi J_{32}}{2} c_8 |1000\rangle) \\
&+ \frac{-\omega_0 + \omega_1 + \omega_2 - \omega_3 - \pi J_{10} - \pi J_{20} + \pi J_{30} + \pi J_{21} - \pi J_{31} - \pi J_{32}}{2} c_9 |1001\rangle) \\
&+ \frac{\omega_0 - \omega_1 + \omega_2 - \omega_3 - \pi J_{10} + \pi J_{20} - \pi J_{30} - \pi J_{21} + \pi J_{31} - \pi J_{32}}{2} c_{10} |1010\rangle) \\
&+ \frac{-\omega_0 - \omega_1 + \omega_2 - \omega_3 + \pi J_{10} - \pi J_{20} + \pi J_{30} - \pi J_{21} + \pi J_{31} - \pi J_{32}}{2} c_{11} |1011\rangle) \\
&+ \frac{\omega_0 + \omega_1 - \omega_2 - \omega_3 + \pi J_{10} - \pi J_{20} - \pi J_{30} - \pi J_{21} - \pi J_{31} + \pi J_{32}}{2} c_{12} |1100\rangle) \\
&+ \frac{-\omega_0 + \omega_1 - \omega_2 - \omega_3 - \pi J_{10} + \pi J_{20} + \pi J_{30} - \pi J_{21} - \pi J_{31} + \pi J_{32}}{2} c_{13} |1101\rangle) \\
&+ \frac{\omega_0 - \omega_1 - \omega_2 - \omega_3 - \pi J_{10} - \pi J_{20} - \pi J_{30} + \pi J_{21} + \pi J_{31} + \pi J_{32}}{2} c_{14} |1110\rangle) \\
&+ \frac{-\omega_0 - \omega_1 - \omega_2 - \omega_3 + \pi J_{10} + \pi J_{20} + \pi J_{30} + \pi J_{21} + \pi J_{31} + \pi J_{32}}{2} c_{15} |1111\rangle)
\end{aligned} \tag{144}$$

maka dengan demikian dapat leel energi dalam 4 qubit dapat di buat dalam bentuk tabel berrikut:

n	$E_{4,n}$
0	$\frac{\omega_0+\omega_1+\omega_2+\omega_3+\pi J_{10}+\pi J_{20}+\pi J_{30}+\pi J_{21}+\pi J_{31}+\pi J_{32}}{2}$
1	$\frac{-\omega_0+\omega_1+\omega_2+\omega_3-\pi J_{10}-\pi J_{20}-\pi J_{30}+\pi J_{21}+\pi J_{31}+\pi J_{32}}{2}$
2	$\frac{\omega_0-\omega_1+\omega_2+\omega_3-\pi J_{10}+\pi J_{20}+\pi J_{30}-\pi J_{21}-\pi J_{31}+\pi J_{32}}{2}$
3	$\frac{-\omega_0-\omega_1+\omega_2+\omega_3+\pi J_{10}-\pi J_{20}-\pi J_{30}-\pi J_{21}-\pi J_{31}+\pi J_{32}}{2}$
4	$\frac{\omega_0+\omega_1-\omega_2+\omega_3+\pi J_{10}-\pi J_{20}+\pi J_{30}-\pi J_{21}+\pi J_{31}-\pi J_{32}}{2}$
5	$\frac{-\omega_0+\omega_1-\omega_2+\omega_3-\pi J_{10}+\pi J_{20}-\pi J_{30}-\pi J_{21}+\pi J_{31}-\pi J_{32}}{2}$
6	$\frac{\omega_0-\omega_1-\omega_2+\omega_3-\pi J_{10}-\pi J_{20}+\pi J_{30}+\pi J_{21}-\pi J_{31}-\pi J_{32}}{2}$
7	$\frac{-\omega_0-\omega_1-\omega_2+\omega_3+\pi J_{10}+\pi J_{20}-\pi J_{30}+\pi J_{21}-\pi J_{31}-\pi J_{32}}{2}$
8	$\frac{\omega_0+\omega_1+\omega_2-\omega_3+\pi J_{10}+\pi J_{20}-\pi J_{30}+\pi J_{21}-\pi J_{31}-\pi J_{32}}{2} C_8$
9	$\frac{-\omega_0+\omega_1+\omega_2-\omega_3-\pi J_{10}-\pi J_{20}+\pi J_{30}+\pi J_{21}-\pi J_{31}-\pi J_{32}}{2}$
10	$\frac{\omega_0-\omega_1+\omega_2-\omega_3-\pi J_{10}+\pi J_{20}-\pi J_{30}-\pi J_{21}+\pi J_{31}-\pi J_{32}}{2}$
11	$\frac{-\omega_0-\omega_1+\omega_2-\omega_3+\pi J_{10}-\pi J_{20}+\pi J_{30}-\pi J_{21}+\pi J_{31}-\pi J_{32}}{2}$
12	$\frac{\omega_0+\omega_1-\omega_2-\omega_3+\pi J_{10}-\pi J_{20}-\pi J_{30}-\pi J_{21}-\pi J_{31}+\pi J_{32}}{2}$
13	$\frac{-\omega_0+\omega_1-\omega_2-\omega_3-\pi J_{10}+\pi J_{20}+\pi J_{30}-\pi J_{21}-\pi J_{31}+\pi J_{32}}{2}$
14	$\frac{\omega_0-\omega_1-\omega_2-\omega_3-\pi J_{10}-\pi J_{20}-\pi J_{30}+\pi J_{21}+\pi J_{31}+\pi J_{32}}{2}$
15	$\frac{-\omega_0-\omega_1-\omega_2-\omega_3+\pi J_{10}+\pi J_{20}+\pi J_{30}+\pi J_{21}+\pi J_{31}+\pi J_{32}}{2}$

maka dengan demikian berdasarkan tabel energi daripada sistem 1 qubit, 2 qubit, 3 qubit didapatkan pola umum untuk sistem level energi untuk N-qubit, yaitu sebagai berikut:

$$E_{N,n} = \sum_{i=1}^{N-1} \left(-\frac{1}{2}\right)^{x_i} \omega_i + \pi \sum_{k=1}^{N-1} \sum_{i=0}^{N-2} \left(-\frac{1}{2}\right)^{x_i \oplus x_k} J_{kl} \quad (145)$$

dengan $k \neq l$

BAB V

PENERAPAN ALGORITMA DEUTSCH-JOZSA DALAM SISTEM 4 QUBIT

Dalam aplikasi algoritma 4 qubit maka langkah yang harus kita cari pertama kali adalah kemungkinan -kemungkinan fungsi konstan dan fungsi setimbang, yaitu sebagai berikut: sebelumnya kita definisikan bahwa qubit masukan

$$|\psi\rangle = |x_3x_2x_1x_0\rangle \quad (146)$$

maka dapat kita ambil beberapa kemungkinan untuk fungsi konstan dan fungsi setimbang yaitu sebagai berikut:

$$\begin{aligned}
 f_{k0} &= 0 && (\text{konstan0}) \\
 f_{k1} &= 1 && (\text{konstan1}) \\
 f_{s0} &= x_3 && (\text{setimbang0}) \\
 f_{s1} &= x_1 \oplus x_0 && (\text{setimbang1}) \\
 f_{s2} &= x_2 \oplus x_1 \oplus x_1 && (\text{setimbang2}) \\
 f_{s3} &= x_3 \oplus x_2 \oplus x_1 \oplus x_1 && (\text{setimbang3}) \\
 f_{s4} &= x_2x_1 \oplus x_0 && (\text{setimbang4})
 \end{aligned} \quad (147)$$

maka dapat kita buat tabel untuk contoh fungsi-fungsi kotak hitam di atas, yaitu sebagai berikut:

$ x_3x_2x_1x_0\rangle$	f_{k0}	f_{k1}	f_{s0}	f_{s1}	f_{s2}	f_{s3}	f_{s4}
$ 0000\rangle$	0	1	0	0	0	0	0
$ 0001\rangle$	0	1	0	1	1	1	1
$ 0010\rangle$	0	1	0	1	1	1	0
$ 0011\rangle$	0	1	0	0	0	0	1
$ 0100\rangle$	0	1	0	0	1	1	0
$ 0101\rangle$	0	1	0	1	0	0	1
$ 0110\rangle$	0	1	0	1	0	0	1
$ 0111\rangle$	0	1	0	0	1	1	0
$ 1000\rangle$	0	1	1	0	0	1	0
$ 1001\rangle$	0	1	1	1	1	0	1
$ 1010\rangle$	0	1	1	1	1	0	0
$ 1011\rangle$	0	1	1	0	0	1	1
$ 1100\rangle$	0	1	1	0	1	0	0
$ 1101\rangle$	0	1	1	1	0	1	1
$ 1110\rangle$	0	1	1	1	0	1	1
$ 1111\rangle$	0	1	1	0	1	0	0
total kemungkinan	1	1	8	12	8	2	24

kemudian dikarenakan operator unitary memenuhi:

$$\begin{aligned}
 U_k &= U_f \otimes I \\
 U_f &= (-1)^f(x) |\psi\rangle
 \end{aligned} \tag{148}$$

maka dengan demikian operator unitary untuk fungsi kotak hitam dapat di tuliskan sebagai berikut:

$$\begin{aligned}
 U_{k0} &= I \otimes I \otimes I \otimes I \\
 U_{k1} &= -I \otimes I \otimes I \otimes I \\
 U_{s0} &= I \otimes I \otimes I \otimes \sigma_z \\
 U_{s1} &= I \otimes I \otimes \sigma_z \otimes \sigma_z \\
 U_{s2} &= I \otimes \sigma_z \otimes \sigma_z \otimes \sigma_z \\
 U_{s3} &= \sigma_z \otimes \sigma_z \otimes \sigma_z \otimes \sigma_z \\
 U_{s4} &= I \otimes ([I \otimes \sigma_z] \oplus [\sigma_z \otimes \sigma_z])
 \end{aligned} \tag{149}$$

di dalam algoritma Deutsh josza untuk 4 qubit dilakukan beberapa operasi terhadap operator, berikut adalah diagram daripada operator-operator tersebut: berdasarkan diagram

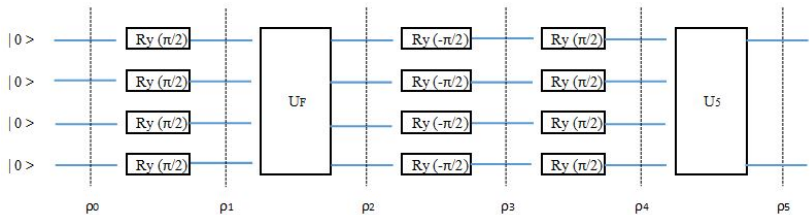


Figure 1: Diagram Blok NMR 4 qubit

diatas maka dapat kita tuliskan bahwa

$$|\psi\rangle = |0000\rangle = |0\rangle \otimes |0\rangle \otimes |0\rangle \otimes |0\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (150)$$

dan untuk operator densitasnya yaitu:

$$\rho_0 = |0000\rangle \langle 0000|$$

$$= \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \otimes \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

maka matrik ρ_0 diatas dapat di pecah kembali dalam direct product matriks berikut:

$$\begin{aligned}
\rho_0 &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \\
&= \left[\frac{1}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \right] \otimes \left[\frac{1}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \right] \\
&\otimes \left[\frac{1}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \right] \otimes \left[\frac{1}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \right] \\
&= \left(\frac{1}{2} I + I_z \right) \otimes \left(\frac{1}{2} I + I_z \right) \otimes \left(\frac{1}{2} I + I_z \right) \otimes \left(\frac{1}{2} I + I_z \right) \\
&= \frac{1}{4} \left[\frac{1}{4} I \otimes I \otimes I \otimes I + \frac{1}{2} (I \otimes I \otimes I \otimes I_z + I \otimes I \otimes I_z \otimes I) \right. \\
&\quad \left. + \frac{1}{4} \left[\frac{1}{2} (I \otimes I_z \otimes I \otimes I + I_z \otimes I \otimes I \otimes I) \right] \right. \\
&\quad \left. + \frac{1}{4} [I \otimes I \otimes I_z \otimes I_z + I \otimes I_z \otimes I \otimes I_z + I \otimes I_z \otimes I_z \otimes I] \right. \\
&\quad \left. + \frac{1}{4} [I_z \otimes I \otimes I \otimes I_z + I_{3z} \otimes I \otimes I_z \otimes I] \right. \\
&\quad \left. + \frac{1}{4} [I_z I_z \otimes I \otimes I + 2(I \otimes I_z \otimes I_z \otimes I_z + I_z \otimes I \otimes I_z \otimes I_z)] \right. \\
&\quad \left. + \frac{1}{4} [2(I_z \otimes I_z \otimes I \otimes I_z + I_z \otimes I_z \otimes I_z \otimes I)] \right. \\
&\quad \left. + I_z \otimes I_z \otimes I_z \otimes I_z \right] \\
&= \frac{1}{4} \left[\frac{1}{4} I + \frac{1}{2} (I_{0z} + I_{1z} + I_{2z} + I_{3z}) + I_{1z} I_{0z} \right] \\
&\quad + \frac{1}{4} [I_{2z} I_{0z} + I_{2z} I_{1z} + I_{3z} I_{0z} + I_{3z} I_{1z}] \\
&\quad + \frac{1}{4} [I_{3z} I_{2z} + 2(I_{2z} I_{1z} I_{0z} + I_{3z} I_{1z} I_{0z} + I_{3z} I_{2z} I_{0z} + I_{3z} I_{2z} I_{1z})] \\
&\quad + \frac{1}{4} [4 I_{3z} I_{2z} I_{1z} I_{0z}]
\end{aligned} \tag{152}$$

maka dalam keadaan kesetimbangan termal ρ_0 dapat didekati dengan :

$$\begin{aligned}\rho_0 &\approx I_{3z} + I_{2z} + I_{1z} + I_{0z} \\ &\approx I_z \otimes I \otimes I \otimes I + I \otimes I \otimes I_z \otimes I + I \otimes I_z \otimes I \otimes I + I_z \otimes I \otimes I \otimes I \\ &\approx \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}\end{aligned}$$

[illegible]

$$= \begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 \end{pmatrix} \quad (153)$$

selain itu dari diagram di atas dapat juga diketahui bahwa operator uniter total dalam algoritma deutsch-josza adalah sebagai berikut:

$$U_{tot} = U_5 U_4 U_3 U_2 U_1 \quad (154)$$

dimana

$$U_1 = R_y\left(\frac{\pi}{2}\right) \otimes R_y\left(\frac{\pi}{2}\right) \otimes R_y\left(\frac{\pi}{2}\right) \otimes R_y\left(\frac{\pi}{2}\right)$$

$$= \frac{1}{4} \begin{pmatrix} 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 \\ 1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & -1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$U_2 = U_f \Rightarrow (\text{fungsi kotak hitam})$$

$$U_3 = R_y \left(\frac{-\pi}{2} \right) \otimes R_y \left(\frac{-\pi}{2} \right) \otimes R_y \left(\frac{-\pi}{2} \right) \otimes R_y \left(\frac{-\pi}{2} \right) \quad (155)$$

$$U_4 = R_y \left(\frac{\pi}{2} \right) \otimes R_y \left(\frac{\pi}{2} \right) \otimes R_y \left(\frac{\pi}{2} \right) \otimes R_y \left(\frac{\pi}{2} \right)$$

$$U_5 = e^{-iHt}$$

maka dari perkalian operator uniter tersebut dapat dilakukan perkalian terlebih dahulu antara $U_4 U_3$, yaitu sebagai berikut:

$$\begin{aligned} U_4 U_3 &= \left[R_y \left(\frac{\pi}{2} \right) \otimes R_y \left(\frac{\pi}{2} \right) \otimes R_y \left(\frac{\pi}{2} \right) \otimes R_y \left(\frac{\pi}{2} \right) \right] \\ &\times \left[R_y \left(\frac{-\pi}{2} \right) \otimes R_y \left(\frac{-\pi}{2} \right) \otimes R_y \left(\frac{-\pi}{2} \right) \otimes R_y \left(\frac{-\pi}{2} \right) \right] \\ &= R_y \left(\frac{\pi}{2} \right) R_y \left(\frac{-\pi}{2} \right) \otimes R_y \left(\frac{\pi}{2} \right) R_y \left(\frac{-\pi}{2} \right) \otimes R_y \left(\frac{\pi}{2} \right) R_y \left(\frac{-\pi}{2} \right) \quad (156) \\ &\otimes R_y \left(\frac{\pi}{2} \right) R_y \left(\frac{-\pi}{2} \right) \\ &= I \otimes I \otimes I \otimes I \end{aligned}$$

sehingga hal ini membuat operator uniter totalnya menjadi

$$\begin{aligned} U_{tot} &= U_5 (I \otimes I \otimes I \otimes I) U_2 U_1 \\ &= U_5 U_2 U_1 \end{aligned} \quad (157)$$

langkah selanjutnya adalah menentukan U_5 , dimana bentuk hamiltonian dalam sistem 4 qubit adalah sebagai berikut:

$$\begin{aligned}
H = & \omega_0 I \otimes I \otimes I \otimes I_z + \omega_1 I \otimes I \otimes I_z \otimes I + \omega_2 I \otimes I_z \otimes I \otimes I \\
& + \omega_3 I_z \otimes I \otimes I \otimes I + 2\pi J_{10} I \otimes I \otimes I_z \otimes I_z \\
& + 2\pi J_{20} I \otimes I_z \otimes I \otimes I_z + 2\pi J_{30} I_z \otimes I \otimes I \otimes I_z \\
& + 2\pi J_{21} I \otimes I_z \otimes I_z \otimes I + 2\pi J_{31} I_z \otimes I \otimes I_z \otimes I \\
& + 2\pi J_{32} I_z \otimes I_z \otimes I \otimes I
\end{aligned} \tag{158}$$

maka didapatkan

$$\begin{aligned}
U_5 = & \exp [-i (\omega_0 I \otimes I \otimes I \otimes I_z + \omega_1 I \otimes I \otimes I_z \otimes I + \omega_2 I \otimes I_z \otimes I \otimes I + \omega_3 I_z \otimes I \otimes I \otimes I) t] \\
& \times \exp [-i (2\pi J_{10} I \otimes I \otimes I_z \otimes I_z + 2\pi J_{20} I \otimes I_z \otimes I \otimes I_z + 2\pi J_{30} I_z \otimes I \otimes I \otimes I_z) t] \\
& \times \exp [-i (2\pi J_{21} I \otimes I_z \otimes I_z \otimes I + 2\pi J_{31} I_z \otimes I \otimes I_z \otimes I + 2\pi J_{32} I_z \otimes I_z \otimes I \otimes I) t] \\
= & R (\omega_0 t) R (\omega_1 t) R (\omega_2 t) R (\omega_3 t) R (\pi J_{10} t) R (\pi J_{20} t) R (\pi J_{30} t) R (\pi J_{21} t) R (\pi J_{31} t) R (\pi J_{32} t)
\end{aligned} \tag{159}$$

kita defnisikan bahwa

$$\begin{aligned}
\theta_0 &= \frac{\omega_0 t}{2} \\
\theta_1 &= \frac{\omega_1 t}{2} \\
\theta_2 &= \frac{\omega_2 t}{2} \\
\theta_3 &= \frac{\omega_3 t}{2} \\
\theta_4 &= \frac{\pi J_{10}}{2} t \\
\theta_5 &= \frac{\pi J_{20}}{2} t \\
\theta_6 &= \frac{\pi J_{30}}{2} t \\
\theta_7 &= \frac{\pi J_{21}}{2} t \\
\theta_8 &= \frac{\pi J_{31}}{2} t \\
\theta_9 &= \frac{\pi J_{32}}{2} t
\end{aligned} \tag{160}$$

maka pers. 14 dapat dituliskan sebagai berikut:

$$U_5 = R(2\theta_0) R(2\theta_1) R(2\theta_2) R(2\theta_3) R(4\theta_4) R(4\theta_5) R(4\theta_6) R(4\theta_7) R(4\theta_8) R(4\theta_9) \quad (161)$$

dimana

$$\begin{aligned}
R(2\theta_0) &= e^{-i\omega_0 I \otimes I \otimes I \otimes I_z t} \\
&= e^{\frac{-i\omega_3 t}{2} I \otimes I \otimes I \otimes \sigma_z} \\
&= \cos \theta_0 I \otimes I \otimes I \otimes I - i \sin \theta_0 I \otimes I \otimes I \otimes \sigma_z \\
&= \cos \theta_0 \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}
\end{aligned}$$

[illegible]

$$\begin{aligned}
R(2\theta_1) &= e^{-i\omega_1 I \otimes I \otimes I_z \otimes I t} \\
&= e^{\frac{-i\omega_1 t}{2} I \otimes I \otimes \sigma_z \otimes I} \\
&= \cos \theta_1 I \otimes I \otimes I \otimes I - i \sin \theta_1 I \otimes I \otimes \sigma_z \otimes I \\
&= \cos \theta_1 \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}
\end{aligned}$$

[illegible]

(163)

$$\begin{aligned}
R(2\theta_2) &= e^{-i\omega_2 I \otimes I \otimes I_z \otimes I t} \\
&= e^{\frac{-i\omega_2 t}{2} I \otimes \sigma_z \otimes I \otimes I} \\
&= \cos \theta_2 I \otimes I \otimes I \otimes I - i \sin \theta_1 I \otimes \sigma_z \otimes I \otimes I \\
&= \cos \theta_2 \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}
\end{aligned}$$

$$-i \sin \theta_2$$

[illegible]

(164)

$$\begin{aligned}
R(2\theta_3) &= e^{-i\omega_3 I_z \otimes I \otimes I \otimes I t} \\
&= e^{\frac{-i\omega_3 t}{2} \sigma_z \otimes I \otimes I \otimes I} \\
&= \cos \theta_3 I \otimes I \otimes I \otimes I - i \sin \theta_3 \sigma_z \otimes I \otimes I \otimes I \\
&= \cos \theta_3 \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}
\end{aligned}$$

[illegible]

[illegible]

(165)

$$\begin{aligned}
R(4\theta_4) &= e^{-i2\pi J_{10} I \otimes I \otimes I_z \otimes I_z t} \\
&= e^{-i\frac{\pi J_{10} t}{2} I \otimes I \otimes \sigma_z \otimes \sigma_z} \\
&= \cos \theta_4 I \otimes I \otimes I \otimes I - i \sin \theta_4 I \otimes I \otimes \sigma_z \otimes \sigma_z \\
&= \cos \theta_4 \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}
\end{aligned}$$

[illegible]

$$\begin{aligned}
R(4\theta_5) &= e^{-i2\pi J_{20} I \otimes I_z \otimes I \otimes I_z t} \\
&= e^{-i\frac{\pi J_{20} t}{2} I \otimes \sigma_z \otimes I \otimes \sigma_z} \\
&= \cos \theta_5 I \otimes I \otimes I \otimes I - i \sin \theta_5 I \otimes \sigma_z \otimes I \otimes \sigma_z \\
&= \cos \theta_5 \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}
\end{aligned}$$

$$-i \sin \theta_5$$

[illegible]

(167)

$$\begin{aligned}
R(4\theta_6) &= e^{-i2\pi J_{30} I_z \otimes I \otimes I \otimes I_z t} \\
&= e^{-i\frac{\pi J_{30} t}{2} \sigma_z \otimes I \otimes I \otimes \sigma_z} \\
&= \cos \theta_6 I \otimes I \otimes I \otimes I - i \sin \theta_6 \sigma_z \otimes I \otimes I \otimes \sigma_z \\
&= \cos \theta_6 \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}
\end{aligned}$$

$$-i \sin \theta_6$$

[illegible]

(168)

$$\begin{aligned}
R(4\theta_7) &= e^{-i2\pi J_{21}I \otimes I_z \otimes I_z \otimes It} \\
&= e^{-i\frac{\pi J_{21}t}{2}I \otimes \sigma_z \otimes \sigma_z \otimes I} \\
&= \cos \theta_7 I \otimes I \otimes I \otimes I - i \sin \theta_7 I \otimes \sigma_z \otimes \sigma_z \otimes I \\
&= \cos \theta_7 \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}
\end{aligned}$$

$$-i \sin \theta_7$$

[illegible]

(169)

$$\begin{aligned}
R(4\theta_8) &= e^{-i2\pi J_{31} I_z \otimes I \otimes I_z \otimes I t} \\
&= e^{-i\frac{\pi J_{31} t}{2} \sigma_z \otimes I \otimes \sigma_z \otimes I} \\
&= \cos \theta_8 I \otimes I \otimes I \otimes I - i \sin \theta_8 \sigma_z \otimes I \otimes \sigma_z \otimes I \\
&= \cos \theta_8 \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}
\end{aligned}$$

[illegible]

[illegible]

(170)

$$\begin{aligned}
R(4\theta_9) &= e^{-i2\pi J_{32} I_z \otimes I_z \otimes I \otimes I t} \\
&= e^{-i\frac{\pi J_{32} t}{2} \sigma_z \otimes \sigma_z \otimes I \otimes I} \\
&= \cos \theta_9 I \otimes I \otimes I \otimes I - i \sin \theta_9 \sigma_z \otimes \sigma_z \otimes I \otimes I \\
&= \cos \theta_9 \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}
\end{aligned}$$

$$-i \sin \theta_9$$

[illegible]

(171)

maka dengan demikian didapatkan untuk operator U_5 adalah

$$U_5 = \begin{pmatrix} e^{\beta_0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & e^{\beta_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{\beta_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{\beta_3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{\beta_4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{\beta_5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_9} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_{10}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_{11}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_{12}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_{13}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_{14}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_{15}} \end{pmatrix} \quad (172)$$

dengan

$$\begin{aligned}
\beta_0 &= -iE_{4,0}t & \beta_1 &= -iE_{4,1}t \\
\beta_2 &= -iE_{4,2}t & \beta_3 &= -iE_{4,3}t \\
\beta_4 &= -iE_{4,4}t & \beta_5 &= -iE_{4,5}t \\
\beta_6 &= -iE_{4,6}t & \beta_7 &= -iE_{4,7}t \\
\beta_8 &= -iE_{4,8}t & \beta_9 &= -iE_{4,9}t \\
\beta_{10} &= -iE_{4,10}t & \beta_{11} &= -iE_{4,11}t \\
\beta_{12} &= -iE_{4,12}t & \beta_{13} &= -iE_{4,13}t \\
\beta_{14} &= -iE_{4,14}t & \beta_{15} &= -iE_{4,15}t
\end{aligned} \tag{173}$$

dan

$$\begin{aligned}
E_0 &= \frac{1}{2} [\omega_0 + \omega_1 + \omega_2 + \omega_3 + \pi J_{10} + \pi J_{20} + \pi J_{30} + \pi J_{21} + \pi J_{31} + \pi J_{32}] \\
E_1 &= \frac{1}{2} [-\omega_0 + \omega_1 + \omega_2 + \omega_3 - \pi J_{10} - \pi J_{20} + \pi J_{30} + \pi J_{21} - \pi J_{31} + \pi J_{32}] \\
E_2 &= \frac{1}{2} [\omega_0 - \omega_1 + \omega_2 + \omega_3 - \pi J_{10} + \pi J_{20} + \pi J_{30} - \pi J_{21} - \pi J_{31} + \pi J_{32}] \\
E_3 &= \frac{1}{2} [-\omega_0 - \omega_1 + \omega_2 + \omega_3 + \pi J_{10} - \pi J_{20} - \pi J_{30} - \pi J_{21} - \pi J_{31} + \pi J_{32}] \\
E_4 &= \frac{1}{2} [\omega_0 + \omega_1 - \omega_2 + \omega_3 + \pi J_{10} + \pi J_{20} + \pi J_{30} - \pi J_{21} + \pi J_{31} - \pi J_{32}] \\
E_5 &= \frac{1}{2} [-\omega_0 + \omega_1 - \omega_2 + \omega_3 - \pi J_{10} + \pi J_{20} - \pi J_{30} - \pi J_{21} + \pi J_{31} - \pi J_{32}] \\
E_6 &= \frac{1}{2} [\omega_0 - \omega_1 - \omega_2 + \omega_3 - \pi J_{10} - \pi J_{20} + \pi J_{30} + \pi J_{21} - \pi J_{31} + \pi J_{32}] \\
E_7 &= \frac{1}{2} [-\omega_0 - \omega_1 - \omega_2 + \omega_3 + \pi J_{10} + \pi J_{20} = \pi J_{30} + \pi J_{21} - \pi J_{31} - \pi J_{32}] \\
E_8 &= \frac{1}{2} [\omega_0 + \omega_1 + \omega_2 - \omega_3 + \pi J_{10} + \pi J_{20} - \pi J_{30} + \pi J_{21} - \pi J_{31} - \pi J_{32}] \\
E_9 &= \frac{1}{2} [-\omega_0 + \omega_1 + \omega_2 - \omega_3 - \pi J_{10} - \pi J_{20} + \pi J_{30} + \pi J_{21} - \pi J_{31} - \pi J_{32}] \\
E_{10} &= \frac{1}{2} [\omega_0 - \omega_1 + \omega_2 - \omega_3 - \pi J_{10} + \pi J_{20} - \pi J_{30} - \pi J_{21} + \pi J_{31} - \pi J_{32}] \\
E_{11} &= \frac{1}{2} [-\omega_0 - \omega_1 + \omega_2 - \omega_3 + \pi J_{10} - \pi J_{20} + \pi J_{30} - \pi J_{21} + \pi J_{31} - \pi J_{32}] \\
E_{12} &= \frac{1}{2} [\omega_0 + \omega_1 - \omega_2 - \omega_3 + \pi J_{10} - \pi J_{20} - \pi J_{30} - \pi J_{21} - \pi J_{31} + \pi J_{32}] \\
E_{13} &= \frac{1}{2} [-\omega_0 + \omega_1 - \omega_2 - \omega_3 - \pi J_{10} + \pi J_{20} + \pi J_{30} - \pi J_{21} - \pi J_{31} + \pi J_{32}] \\
E_{14} &= \frac{1}{2} [\omega_0 - \omega_1 - \omega_2 - \omega_3 - \pi J_{10} - \pi J_{20} - \pi J_{30} + \pi J_{21} + \pi J_{31} + \pi J_{32}] \\
E_{15} &= \frac{1}{2} [-\omega_0 - \omega_1 - \omega_2 - \omega_3 + \pi J_{10} + \pi J_{20} + \pi J_{30} + \pi J_{21} + \pi J_{31} + \pi J_{32}]
\end{aligned} \tag{174}$$

[illegible]

[illegible]

×

[illegible]

$$= \frac{1}{16} \begin{pmatrix} 0 & 8\rho_{0,1} & 8\rho_{0,2} & 0 & 8\rho_{0,4} & 0 & 0 & 0 & 8\rho_{0,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8\rho_{1,0} & 0 & 0 & 8\rho_{1,3} & 0 & 8\rho_{1,5} & 0 & 0 & 0 & 8\rho_{1,9} & 0 & 0 & 0 & 0 & 0 & 0 \\ 8\rho_{2,0} & 0 & 0 & 8\rho_{2,3} & 0 & 0 & 8\rho_{2,6} & 0 & 0 & 0 & 8\rho_{2,10} & 0 & 0 & 0 & 0 & 0 \\ 0 & 8\rho_{3,1} & 8\rho_{3,2} & 0 & 0 & 0 & 0 & 8\rho_{3,7} & 0 & 0 & 8\rho_{3,11} & 0 & 0 & 0 & 0 & 0 \\ 8\rho_{4,0} & 0 & 0 & 0 & 0 & 8\rho_{4,5} & 8\rho_{4,6} & 0 & 0 & 0 & 0 & 0 & 8\rho_{4,12} & 0 & 0 & 0 \\ 0 & 8\rho_{4,1} & 0 & 0 & 8\rho_{5,4} & 0 & 0 & 8\rho_{5,7} & 0 & 0 & 0 & 0 & 0 & 8\rho_{5,13} & 0 & 0 \\ 0 & 0 & 8\rho_{6,2} & 0 & 8\rho_{6,4} & 0 & 0 & 8\rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & 8\rho_{6,14} & 0 \\ 0 & 0 & 0 & 8\rho_{7,3} & 0 & 8\rho_{7,5} & 8\rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8\rho_{7,15} \\ 8\rho_{8,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8\rho_{8,9} & 8\rho_{8,10} & 0 & 8\rho_{8,12} & 0 & 0 & 0 \\ 0 & 8\rho_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & 8\rho_{9,8} & 0 & 0 & 8\rho_{9,11} & 0 & 8\rho_{9,13} & 0 & 0 \\ 0 & 0 & 8\rho_{10,2} & 0 & 0 & 0 & 0 & 0 & 8\rho_{10,8} & 0 & 0 & 8\rho_{10,11} & 0 & 0 & 8\rho_{10,14} & 0 \\ 0 & 0 & 0 & 8\rho_{11,3} & 0 & 0 & 0 & 0 & 0 & 8\rho_{11,9} & 8\rho_{11,10} & 0 & 0 & 0 & 0 & 8\rho_{11,15} \\ 0 & 0 & 0 & 0 & 8\rho_{12,4} & 0 & 0 & 0 & 8\rho_{12,8} & 0 & 0 & 0 & 0 & 8\rho_{12,13} & 8\rho_{12,14} & 0 \\ 0 & 0 & 0 & 0 & 0 & 8\rho_{13,5} & 0 & 0 & 0 & 8\rho_{13,9} & 0 & 0 & 8\rho_{13,12} & 0 & 0 & 8\rho_{13,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 8\rho_{14,6} & 0 & 0 & 0 & 8\rho_{14,10} & 0 & 8\rho_{14,12} & 0 & 0 & 8\rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8\rho_{15,7} & 0 & 0 & 0 & 8\rho_{15,11} & 0 & 8\rho_{15,13} & 8\rho_{15,14} & 0 \end{pmatrix} \quad (177)$$

$$= \frac{1}{2} \begin{pmatrix} 0 & \rho_{0,1} & \rho_{0,2} & 0 & \rho_{0,4} & 0 & 0 & 0 & \rho_{0,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho_{1,0} & 0 & 0 & \rho_{1,3} & 0 & \rho_{1,5} & 0 & 0 & 0 & \rho_{1,9} & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho_{2,0} & 0 & 0 & \rho_{2,3} & 0 & 0 & \rho_{2,6} & 0 & 0 & 0 & \rho_{2,10} & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_{3,1} & \rho_{3,2} & 0 & 0 & 0 & 0 & \rho_{3,7} & 0 & 0 & \rho_{3,11} & 0 & 0 & 0 & 0 & 0 \\ \rho_{4,0} & 0 & 0 & 0 & 0 & \rho_{4,5} & \rho_{4,6} & 0 & 0 & 0 & 0 & 0 & \rho_{4,12} & 0 & 0 & 0 \\ 0 & \rho_{4,1} & 0 & 0 & \rho_{5,4} & 0 & 0 & \rho_{5,7} & 0 & 0 & 0 & 0 & 0 & \rho_{5,13} & 0 & 0 \\ 0 & 0 & \rho_{6,2} & 0 & \rho_{6,4} & 0 & 0 & \rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{6,14} & 0 \\ 0 & 0 & 0 & \rho_{7,3} & 0 & \rho_{7,5} & \rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{7,15} \\ \rho_{8,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{8,9} & \rho_{8,10} & 0 & \rho_{8,12} & 0 & 0 & 0 \\ 0 & \rho_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{9,8} & 0 & 0 & \rho_{9,11} & 0 & \rho_{9,13} & 0 & 0 \\ 0 & 0 & \rho_{10,2} & 0 & 0 & 0 & 0 & 0 & \rho_{10,8} & 0 & 0 & \rho_{10,11} & 0 & 0 & \rho_{10,14} & 0 \\ 0 & 0 & 0 & \rho_{11,3} & 0 & 0 & 0 & 0 & 0 & \rho_{11,9} & \rho_{11,10} & 0 & 0 & 0 & 0 & \rho_{11,15} \\ 0 & 0 & 0 & 0 & \rho_{12,4} & 0 & 0 & 0 & \rho_{12,8} & 0 & 0 & 0 & 0 & \rho_{12,13} & \rho_{12,14} & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_{13,5} & 0 & 0 & 0 & \rho_{13,9} & 0 & 0 & \rho_{13,12} & 0 & 0 & \rho_{13,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho_{14,6} & 0 & 0 & 0 & \rho_{14,10} & 0 & \rho_{14,12} & 0 & 0 & \rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{15,7} & 0 & 0 & 0 & \rho_{15,11} & 0 & \rho_{15,13} & \rho_{15,14} & 0 \end{pmatrix}$$

(178)

dengan

$$\begin{aligned}
\rho_{0,1} &= \rho_{1,0}^* = e^{\beta_0 - \beta_1} & \rho_{6,7} &= \rho_{7,6}^* = e^{\beta_6 - \beta_7} \\
\rho_{0,2} &= \rho_{2,0}^* = e^{\beta_0 - \beta_2} & \rho_{6,14} &= \rho_{14,6}^* = e^{\beta_6 - \beta_{14}} \\
\rho_{0,8} &= \rho_{8,0}^* = e^{\beta_8 - \beta_1} & \rho_{8,9} &= \rho_{9,8}^* = e^{\beta_8 - \beta_9} \\
\rho_{1,3} &= \rho_{3,1}^* = e^{\beta_1 - \beta_3} & \rho_{8,10} &= \rho_{10,8}^* = e^{\beta_8 - \beta_{10}} \\
\rho_{1,5} &= \rho_{5,1}^* = e^{\beta_1 - \beta_5} & \rho_{8,12} &= \rho_{12,8}^* = e^{\beta_8 - \beta_{12}} \\
\rho_{1,9} &= \rho_{9,1}^* = e^{\beta_1 - \beta_9} & \rho_{9,11} &= \rho_{11,9}^* = e^{\beta_9 - \beta_{11}} \\
\rho_{2,3} &= \rho_{3,2}^* = e^{\beta_2 - \beta_3} & \rho_{9,13} &= \rho_{13,9}^* = e^{\beta_9 - \beta_{13}} \\
\rho_{2,6} &= \rho_{6,2}^* = e^{\beta_2 - \beta_6} & \rho_{10,11} &= \rho_{11,10}^* = e^{\beta_{10} - \beta_{11}} \\
\rho_{2,10} &= \rho_{10,2}^* = e^{\beta_2 - \beta_{10}} & \rho_{10,14} &= \rho_{14,10}^* = e^{\beta_{10} - \beta_{14}} \\
\rho_{3,7} &= \rho_{7,3}^* = e^{\beta_3 - \beta_7} & \rho_{12,13} &= \rho_{13,12}^* = e^{\beta_{12} - \beta_{13}} \\
\rho_{3,11} &= \rho_{11,3}^* = e^{\beta_3 - \beta_{11}} & \rho_{12,14} &= \rho_{14,12}^* = e^{\beta_{12} - \beta_{14}} \\
\rho_{4,5} &= \rho_{5,4}^* = e^{\beta_4 - \beta_5} & \rho_{11,15} &= \rho_{15,11}^* = e^{\beta_{11} - \beta_{15}} \\
\rho_{4,6} &= \rho_{6,4}^* = e^{\beta_4 - \beta_6} & \rho_{13,15} &= \rho_{15,13}^* = e^{\beta_{13} - \beta_{15}} \\
\rho_{4,12} &= \rho_{12,4}^* = e^{\beta_4 - \beta_{12}} & \rho_{14,15} &= \rho_{15,14}^* = e^{\beta_{14} - \beta_{15}} \\
\rho_{5,7} &= \rho_{7,5}^* = e^{\beta_5 - \beta_7} & \rho_{15,7} &= \rho_{7,15}^* = e^{\beta_7 - \beta_{15}} \\
\rho_{5,13} &= \rho_{13,5}^* = e^{\beta_5 - \beta_{13}} & \rho_{0,4} &= \rho_{4,0}^* = e^{\beta_0 - \beta_4}
\end{aligned} \tag{179}$$

dan

$$\begin{array}{ll}
\beta_0 - \beta_1 = -i(E_{4,0} - E_{4,1})t & \beta_0 - \beta_2 = -i(E_{4,0} - E_{4,2})t \\
\beta_0 - \beta_4 = -i(E_{4,0} - E_{4,4})t & \beta_0 - \beta_8 = -i(E_{4,0} - E_{4,8})t \\
\beta_1 - \beta_3 = -i(E_{4,1} - E_{4,3})t & \beta_1 - \beta_5 = -i(E_{4,1} - E_{4,5})t \\
\beta_1 - \beta_9 = -i(E_{4,1} - E_{4,9})t & \beta_2 - \beta_3 = -i(E_{4,2} - E_{4,3})t \\
\beta_2 - \beta_6 = -i(E_{4,2} - E_{4,6})t & \beta_2 - \beta_{10} = -i(E_{4,2} - E_{4,10})t \\
\beta_3 - \beta_7 = -i(E_{4,3} - E_{4,7})t & \beta_3 - \beta_{11} = -i(E_{4,3} - E_{4,11})t \\
\beta_4 - \beta_5 = -i(E_{4,4} - E_{4,5})t & \beta_4 - \beta_6 = -i(E_{4,4} - E_{4,6})t \\
\beta_4 - \beta_{12} = -i(E_{4,4} - E_{4,12})t & \beta_5 - \beta_7 = -i(E_{4,5} - E_{4,7})t \\
\beta_5 - \beta_{13} = -i(E_{4,5} - E_{4,13})t & \beta_6 - \beta_7 = -i(E_{4,6} - E_{4,7})t \\
\beta_6 - \beta_{14} = -i(E_{4,6} - E_{4,14})t & \beta_8 - \beta_9 = -i(E_{4,8} - E_{4,9})t \\
\beta_8 - \beta_{10} = -i(E_{4,8} - E_{4,10})t & \beta_8 - \beta_{12} = -i(E_{4,8} - E_{4,12})t \\
\beta_9 - \beta_{11} = -i(E_{4,9} - E_{4,11})t & \beta_9 - \beta_{13} = -i(E_{4,9} - E_{4,13})t \\
\beta_{10} - \beta_{11} = -i(E_{4,10} - E_{4,11})t & \beta_{10} - \beta_{14} = -i(E_{4,10} - E_{4,14})t \\
\beta_{12} - \beta_{13} = -i(E_{4,12} - E_{4,13})t & \beta_{12} - \beta_{14} = -i(E_{4,12} - E_{4,14})t \\
\beta_{11} - \beta_{15} = -i(E_{4,11} - E_{4,15})t & \beta_{13} - \beta_{15} = -i(E_{4,13} - E_{4,15})t \\
\beta_{14} - \beta_{15} = -i(E_{4,14} - E_{4,15})t & \beta_{15} - \beta_7 = -i(E_{4,15} - E_{4,7})t
\end{array} \tag{180}$$

dan

$$\begin{aligned}
E_{4,0} - E_{4,1} &= \omega_0 + \pi J_{10} + \pi J_{20} + \pi J_{30} & E_{4,0} - E_{4,2} &= \omega_1 + \pi J_{10} + \pi J_{21} + \pi J_{31} \\
E_{4,0} - E_{4,4} &= \omega_2 + \pi J_{20} + \pi J_{21} + \pi J_{32} & E_{4,0} - E_{4,8} &= \omega_3 + \pi J_{30} + \pi J_{31} + \pi J_{32} \\
E_{4,1} - E_{4,3} &= \omega_1 - \pi J_{10} + \pi J_{21} + \pi J_{31} & E_{4,1} - E_{4,5} &= \omega_2 - \pi J_{20} + \pi J_{21} + \pi J_{32} \\
E_{4,1} - E_{4,9} &= \omega_3 - \pi J_{30} + \pi J_{310} + \pi J_{32} & E_{4,2} - E_{4,3} &= \omega_0 - \pi J_{10} + \pi J_{20} + \pi J_{30} \\
E_{4,2} - E_{4,6} &= \omega_2 + \pi J_{20} - \pi J_{21} + \pi J_{32} & E_{4,2} - E_{4,10} &= \omega_3 + \pi J_{30} - \pi J_{31} + \pi J_{32} \\
E_{4,3} - E_{4,7} &= \omega_2 - \pi J_{20} - \pi J_{21} + \pi J_{32} & E_{4,3} - E_{4,11} &= \omega_3 - \pi J_{30} - \pi J_{31} + \pi J_{32} \\
E_{4,4} - E_{4,5} &= \omega_0 + \pi J_{10} - \pi J_{20} + \pi J_{30} & E_{4,4} - E_{4,6} &= \omega_1 + \pi J_{10} - \pi J_{21} + \pi J_{31} \\
E_{4,4} - E_{4,12} &= \omega_3 + \pi J_{30} + \pi J_{31} - \pi J_{32} & E_{4,5} - E_{4,7} &= \omega_1 - \pi J_{10} - \pi J_{21} + \pi J_{31} \\
E_{4,5} - E_{4,13} &= \omega_3 - \pi J_{30} + \pi J_{31} - \pi J_{32} & E_{4,6} - E_{4,7} &= \omega_0 - \pi J_{10} - \pi J_{20} + \pi J_{30} \\
E_{4,6} - E_{4,14} &= \omega_3 + \pi J_{30} - \pi J_{31} - \pi J_{32} & E_{4,8} - E_{4,9} &= \omega_0 + \pi J_{10} + \pi J_{20} - \pi J_{30} \\
E_{4,8} - E_{4,10} &= \omega_1 + \pi J_{21} - \pi J_{31} + \pi J_{30} & E_{4,8} - E_{4,12} &= \omega_2 + \pi J_{20} + \pi J_{21} - \pi J_{32} \\
E_{4,9} - E_{4,11} &= \omega_1 - \pi J_{10} + \pi J_{21} - \pi J_{32} & E_{4,10} - E_{4,11} &= \omega_0 - \pi J_{10} + \pi J_{20} - \pi J_{30} \\
E_{4,10} - E_{4,14} &= \omega_2 + \pi J_{20} - \pi J_{21} - \pi J_{32} & E_{4,12} - E_{4,13} &= \omega_0 + \pi J_{10} - \pi J_{20} - \pi J_{30} \\
E_{4,12} - E_{4,14} &= \omega_1 + \pi J_{10} - \pi J_{21} - \pi J_{31} & E_{4,11} - E_{4,15} &= \omega_2 - \pi J_{20} - \pi J_{21} - \pi J_{32} \\
E_{4,13} - E_{4,15} &= \omega_1 - \pi J_{10} - \pi J_{21} - \pi J_{31} & E_{4,14} - E_{4,15} &= \omega_0 - \pi J_{10} - \pi J_{20} - \pi J_{30} \\
E_{4,9} - E_{4,13} &= \omega_2 - \pi J_{20} + \pi J_{21} - \pi J_{32} & E_{4,15} - E_{4,7} &= \omega_3 + \pi J_{30} + \pi J_{31} + \pi J_{32}
\end{aligned} \tag{181}$$

maka dengan demikian suku-suku tidak nol dari matriks ρ_5 menjadi:

$$\begin{aligned}
\rho_{0,1} &= \rho_{1,0}^* = e^{-i(\omega_0 + \pi J_{10} + \pi J_{20} + \pi J_{30})t} \\
\rho_{0,2} &= \rho_{2,0}^* = e^{-i(\omega_1 + \pi J_{10} + \pi J_{21} + \pi J_{31})t} \\
\rho_{0,4} &= \rho_{4,0}^* = e^{-i(\omega_2 + \pi J_{20} + \pi J_{21} + \pi J_{32})t} \\
\rho_{0,8} &= \rho_{8,0}^* = e^{-i(\omega_3 + \pi J_{30} + \pi J_{31} + \pi J_{32})t} \\
\rho_{1,3} &= \rho_{3,1}^* = e^{-i(\omega_1 - \pi J_{10} + \pi J_{21} + \pi J_{31})t} \\
\rho_{1,5} &= \rho_{5,1}^* = e^{-i(\omega_2 - \pi J_{20} + \pi J_{21} + \pi J_{32})t} \\
\rho_{1,9} &= \rho_{9,1}^* = e^{-i(\omega_3 - \pi J_{30} + \pi J_{31} + \pi J_{32})t} \\
\rho_{2,3} &= \rho_{3,2}^* = e^{-i(\omega_0 - \pi J_{10} + \pi J_{20} + \pi J_{30})t} \\
\rho_{2,6} &= \rho_{6,2}^* = e^{-i(\omega_2 + \pi J_{20} - \pi J_{21} + \pi J_{32})t} \\
\rho_{2,10} &= \rho_{10,2}^* = e^{-i(\omega_3 + \pi J_{30} - \pi J_{31} + \pi J_{32})t} \\
\rho_{3,7} &= \rho_{7,3}^* = e^{-i(\omega_2 - \pi J_{20} - \pi J_{21} + \pi J_{32})t} \\
\rho_{3,11} &= \rho_{11,3}^* = e^{-i(\omega_3 - \pi J_{30} - \pi J_{31} + \pi J_{32})t} \\
\rho_{4,5} &= \rho_{5,4}^* = e^{-i(\omega_0 + \pi J_{10} - \pi J_{20} + \pi J_{30})t} \\
\rho_{4,6} &= \rho_{6,4}^* = e^{-i(\omega_1 + \pi J_{10} - \pi J_{21} + \pi J_{31})t} \\
\rho_{4,12} &= \rho_{12,4}^* = e^{-i(\omega_3 + \pi J_{30} + \pi J_{31} - \pi J_{32})t} \\
\rho_{5,7} &= \rho_{7,5}^* = e^{-i(\omega_1 - \pi J_{10} - \pi J_{21} + \pi J_{31})t} \\
\rho_{5,13} &= \rho_{13,5}^* = e^{-i(\omega_3 - \pi J_{30} + \pi J_{31} - \pi J_{32})t} \\
\rho_{6,7} &= \rho_{7,6}^* = e^{-i(\omega_0 - \pi J_{10} - \pi J_{20} + \pi J_{30})t} \\
\rho_{6,14} &= \rho_{14,6}^* = e^{-i(\omega_3 + \pi J_{30} - \pi J_{31} - \pi J_{32})t} \\
\rho_{8,9} &= \rho_{9,8}^* = e^{-i(\omega_0 + \pi J_{10} + \pi J_{20} - \pi J_{30})t} \\
\rho_{8,10} &= \rho_{10,8}^* = e^{-i(\omega_1 + \pi J_{21} - \pi J_{31} + \pi J_{30})t} \\
\rho_{8,12} &= \rho_{12,8}^* = e^{-i(\omega_2 + \pi J_{20} + \pi J_{21} - \pi J_{32})t}
\end{aligned} \tag{182}$$

$$\begin{aligned}
\rho_{9,11} &= \rho_{11,9}^* = e^{-i(\omega_1 - \pi J_{10} + \pi J_{21} - \pi J_{32})t} \\
\rho_{10,11} &= \rho_{11,10}^* = e^{-i(\omega_0 - \pi J_{10} + \pi J_{20} - \pi J_{30})t} \\
\rho_{10,14} &= \rho_{14,10}^* = e^{-i(\omega_2 + \pi J_{20} - \pi J_{21} - \pi J_{32})t} \\
\rho_{12,13} &= \rho_{13,12}^* = e^{-i(\omega_0 + \pi J_{10} - \pi J_{20} - \pi J_{30})t} \\
\rho_{12,14} &= \rho_{14,12}^* = e^{-i(\omega_1 + \pi J_{10} - \pi J_{21} - \pi J_{31})t} \\
\rho_{11,15} &= \rho_{15,11}^* = e^{-i(\omega_2 - \pi J_{20} - \pi J_{21} - \pi J_{32})t} \\
\rho_{7,15} &= \rho_{15,7}^* = e^{-i(\omega_3 - \pi J_{30} - \pi J_{31} - \pi J_{32})t} \\
\rho_{13,15} &= \rho_{15,13}^* = e^{-i(\omega_1 - \pi J_{10} - \pi J_{21} - \pi J_{31})t} \\
\rho_{14,15} &= \rho_{15,14}^* = e^{-i(\omega_0 - \pi J_{10} - \pi J_{20} - \pi J_{30})t} \\
\rho_{9,13} &= \rho_{13,9}^* = e^{-i(\omega_2 - \pi J_{20} + \pi J_{21} - \pi J_{32})t}
\end{aligned} \tag{183}$$

setelah di dapatkan matriks densitas ρ_5 maka dapat diukur nilai magnetisasi dari sistem M_+ , untuk mengukur nilai magnetisasi sebelumnya harus ditentukan terlebih dahulu representasi matriks daripada operator penaik dari masing-masing qubit : yaitu sebagai berikut:

$$\begin{aligned}
 I_{0+} &= I \otimes I \otimes I \otimes (I_x + iI_y) \\
 &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \\
 &= \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (184)
 \end{aligned}$$

$$\begin{aligned}
I_{1+} &= I \otimes I \otimes (I_x + iI_y) \otimes I \\
&= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (185)
\end{aligned}$$

$$\begin{aligned}
I_{2+} &= I \otimes (I_x + iI_y) \otimes I \otimes I \\
&= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (186)
\end{aligned}$$

$$\begin{aligned}
I_{3+} &= (I_x + iI_y) \otimes I \otimes I \otimes I \\
&= \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (187)
\end{aligned}$$

maka dapat ditentukan nilai magnetisasi dari masing-masing qubitnya yaitu sebagai berikut:

[illegible]

$$\times \frac{1}{2} \begin{pmatrix} 0 & \rho_{0,1} & \rho_{0,2} & 0 & \rho_{0,4} & 0 & 0 & 0 & \rho_{0,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho_{1,0} & 0 & 0 & \rho_{1,3} & 0 & \rho_{1,5} & 0 & 0 & 0 & \rho_{1,9} & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho_{2,0} & 0 & 0 & \rho_{2,3} & 0 & 0 & \rho_{2,6} & 0 & 0 & 0 & \rho_{2,10} & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_{3,1} & \rho_{3,2} & 0 & 0 & 0 & 0 & \rho_{3,7} & 0 & 0 & \rho_{3,11} & 0 & 0 & 0 & 0 & 0 \\ \rho_{40} & 0 & 0 & 0 & 0 & \rho_{4,5} & \rho_{4,6} & 0 & 0 & 0 & 0 & 0 & \rho_{4,12} & 0 & 0 & 0 \\ 0 & \rho_{5,1} & 0 & 0 & \rho_{5,4} & 0 & 0 & \rho_{5,7} & 0 & 0 & 0 & 0 & 0 & \rho_{5,13} & 0 & 0 \\ 0 & 0 & \rho_{6,2} & 0 & \rho_{6,4} & 0 & 0 & \rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{6,14} & 0 \\ 0 & 0 & 0 & \rho_{7,3} & 0 & \rho_{7,5} & \rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{7,15} \\ \rho_{8,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{8,9} & \rho_{8,10} & 0 & \rho_{8,12} & 0 & 0 & 0 \\ 0 & \rho_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{9,8} & 0 & 0 & \rho_{9,11} & 0 & \rho_{9,13} & 0 & 0 \\ 0 & 0 & \rho_{10,2} & 0 & 0 & 0 & 0 & 0 & \rho_{10,8} & 0 & 0 & \rho_{10,11} & 0 & 0 & \rho_{10,14} & 0 \\ 0 & 0 & 0 & \rho_{11,3} & 0 & 0 & 0 & 0 & 0 & \rho_{11,9} & \rho_{11,10} & 0 & 0 & 0 & 0 & \rho_{11,15} \\ 0 & 0 & 0 & 0 & \rho_{12,4} & 0 & 0 & 0 & \rho_{12,8} & 0 & 0 & 0 & 0 & \rho_{12,13} & \rho_{12,14} & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_{13,5} & 0 & 0 & 0 & \rho_{13,9} & 0 & 0 & \rho_{13,12} & 0 & 0 & \rho_{13,15} \\ 0 & 0 & & 0 & 0 & 0 & \rho_{14,6} & 0 & 0 & 0 & \rho_{14,10} & 0 & \rho_{14,12} & 0 & 0 & \rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{15,7} & 0 & 0 & 0 & \rho_{15,11} & 0 & \rho_{15,13} & \rho_{15,14} & 0 \end{pmatrix}$$

[illegible]

$$\begin{aligned}
M_{3+} &= \langle I_{3+} \rangle = \text{tr} (I_{3+} \rho_5) \\
&= \rho_{8,0} + \rho_{9,1} + \rho_{10,2} + \rho_{11,3} + \rho_{12,4} + \rho_{13,5} + \rho_{14,6} + \rho_{15,7}
\end{aligned} \tag{188}$$

dan

$$(I_{2+} \rho_5) = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\times \frac{1}{2} \begin{pmatrix} 0 & \rho_{0,1} & \rho_{0,2} & 0 & \rho_{0,4} & 0 & 0 & 0 & \rho_{0,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho_{1,0} & 0 & 0 & \rho_{1,3} & 0 & \rho_{1,5} & 0 & 0 & 0 & \rho_{1,9} & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho_{2,0} & 0 & 0 & \rho_{2,3} & 0 & 0 & \rho_{2,6} & 0 & 0 & 0 & \rho_{2,10} & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_{3,1} & \rho_{3,2} & 0 & 0 & 0 & 0 & \rho_{3,7} & 0 & 0 & \rho_{3,11} & 0 & 0 & 0 & 0 & 0 \\ \rho_{40} & 0 & 0 & 0 & 0 & \rho_{4,5} & \rho_{4,6} & 0 & 0 & 0 & 0 & 0 & \rho_{4,12} & 0 & 0 & 0 \\ 0 & \rho_{5,1} & 0 & 0 & \rho_{5,4} & 0 & 0 & \rho_{5,7} & 0 & 0 & 0 & 0 & 0 & \rho_{5,13} & 0 & 0 \\ 0 & 0 & \rho_{6,2} & 0 & \rho_{6,4} & 0 & 0 & \rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{6,14} & 0 \\ 0 & 0 & 0 & \rho_{7,3} & 0 & \rho_{7,5} & \rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{7,15} \\ \rho_{8,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{8,9} & \rho_{8,10} & 0 & \rho_{8,12} & 0 & 0 & 0 \\ 0 & \rho_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{9,8} & 0 & 0 & \rho_{9,11} & 0 & \rho_{9,13} & 0 & 0 \\ 0 & 0 & \rho_{10,2} & 0 & 0 & 0 & 0 & 0 & \rho_{10,8} & 0 & 0 & \rho_{10,11} & 0 & 0 & \rho_{10,14} & 0 \\ 0 & 0 & 0 & \rho_{11,3} & 0 & 0 & 0 & 0 & 0 & \rho_{11,9} & \rho_{11,10} & 0 & 0 & 0 & 0 & \rho_{11,15} \\ 0 & 0 & 0 & 0 & \rho_{12,4} & 0 & 0 & 0 & \rho_{12,8} & 0 & 0 & 0 & 0 & \rho_{12,13} & \rho_{12,14} & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_{13,5} & 0 & 0 & 0 & \rho_{13,9} & 0 & 0 & \rho_{13,12} & 0 & 0 & \rho_{13,15} \\ 0 & 0 & & 0 & 0 & 0 & \rho_{14,6} & 0 & 0 & 0 & \rho_{14,10} & 0 & \rho_{14,12} & 0 & 0 & \rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{15,7} & 0 & 0 & 0 & \rho_{15,11} & 0 & \rho_{15,13} & \rho_{15,14} & 0 \end{pmatrix}$$

[illegible]

$$\begin{aligned}
M_{2+} &= \langle I_{2+} \rangle = tr(I_{2+}\rho_5) \\
&= \rho_{4,0} + \rho_{5,1} + \rho_{6,2} + \rho_{7,3} + \rho_{12,8} + \rho_{13,9} + \rho_{14,10} + \rho_{15,11}
\end{aligned} \tag{190}$$

dan

$$(I_{1+}\rho_5) = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\times \frac{1}{2} \begin{pmatrix} 0 & \rho_{0,1} & \rho_{0,2} & 0 & \rho_{0,4} & 0 & 0 & 0 & \rho_{0,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho_{1,0} & 0 & 0 & \rho_{1,3} & 0 & \rho_{1,5} & 0 & 0 & 0 & \rho_{1,9} & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho_{2,0} & 0 & 0 & \rho_{2,3} & 0 & 0 & \rho_{2,6} & 0 & 0 & 0 & \rho_{2,10} & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_{3,1} & \rho_{3,2} & 0 & 0 & 0 & 0 & \rho_{3,7} & 0 & 0 & \rho_{3,11} & 0 & 0 & 0 & 0 & 0 \\ \rho_{40} & 0 & 0 & 0 & 0 & \rho_{4,5} & \rho_{4,6} & 0 & 0 & 0 & 0 & 0 & \rho_{4,12} & 0 & 0 & 0 \\ 0 & \rho_{5,1} & 0 & 0 & \rho_{5,4} & 0 & 0 & \rho_{5,7} & 0 & 0 & 0 & 0 & 0 & \rho_{5,13} & 0 & 0 \\ 0 & 0 & \rho_{6,2} & 0 & \rho_{6,4} & 0 & 0 & \rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{6,14} & 0 \\ 0 & 0 & 0 & \rho_{7,3} & 0 & \rho_{7,5} & \rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{7,15} \\ \rho_{8,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{8,9} & \rho_{8,10} & 0 & \rho_{8,12} & 0 & 0 & 0 \\ 0 & \rho_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{9,8} & 0 & 0 & \rho_{9,11} & 0 & \rho_{9,13} & 0 & 0 \\ 0 & 0 & \rho_{10,2} & 0 & 0 & 0 & 0 & 0 & \rho_{10,8} & 0 & 0 & \rho_{10,11} & 0 & 0 & \rho_{10,14} & 0 \\ 0 & 0 & 0 & \rho_{11,3} & 0 & 0 & 0 & 0 & 0 & \rho_{11,9} & \rho_{11,10} & 0 & 0 & 0 & 0 & \rho_{11,15} \\ 0 & 0 & 0 & 0 & \rho_{12,4} & 0 & 0 & 0 & \rho_{12,8} & 0 & 0 & 0 & 0 & \rho_{12,13} & \rho_{12,14} & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_{13,5} & 0 & 0 & 0 & \rho_{13,9} & 0 & 0 & \rho_{13,12} & 0 & 0 & \rho_{13,15} \\ 0 & 0 & & 0 & 0 & 0 & \rho_{14,6} & 0 & 0 & 0 & \rho_{14,10} & 0 & \rho_{14,12} & 0 & 0 & \rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{15,7} & 0 & 0 & 0 & \rho_{15,11} & 0 & \rho_{15,13} & \rho_{15,14} & 0 \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} \rho_{2,0} & 0 & 0 & \rho_{2,3} & 0 & 0 & \rho_{2,6} & 0 & 0 & 0 & \rho_{2,10} & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_{3,1} & \rho_{3,2} & 0 & 0 & 0 & 0 & \rho_{3,7} & 0 & 0 & \rho_{3,11} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_{6,2} & 0 & \rho_{6,4} & 0 & 0 & \rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{6,14} & 0 \\ 0 & 0 & 0 & \rho_{7,3} & 0 & \rho_{7,5} & \rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{7,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_{10,2} & 0 & 0 & 0 & 0 & 0 & \rho_{10,8} & 0 & 0 & \rho_{10,11} & 0 & 0 & \rho_{10,14} & 0 \\ 0 & 0 & 0 & \rho_{11,3} & 0 & 0 & 0 & 0 & 0 & \rho_{11,9} & \rho_{11,10} & 0 & 0 & 0 & 0 & \rho_{11,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho_{14,6} & 0 & 0 & 0 & \rho_{14,10} & 0 & \rho_{14,12} & 0 & 0 & \rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{15,7} & 0 & 0 & 0 & \rho_{15,11} & 0 & \rho_{15,13} & \rho_{15,14} & 0 \end{pmatrix}$$

(191)

$$\begin{aligned}
M_{1+} &= \langle I_{1+} \rangle = tr(I_{1+} \rho_5) \\
&= \rho_{2,0} + \rho_{3,1} + \rho_{6,4} + \rho_{7,5} + \rho_{10,8} + \rho_{11,9} + \rho_{14,12} + \rho_{15,13}
\end{aligned} \tag{192}$$

dan

$$(I_{0+} \rho_5) = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\times \frac{1}{2} \begin{pmatrix} 0 & \rho_{0,1} & \rho_{0,2} & 0 & \rho_{0,4} & 0 & 0 & 0 & \rho_{0,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho_{1,0} & 0 & 0 & \rho_{1,3} & 0 & \rho_{1,5} & 0 & 0 & 0 & \rho_{1,9} & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho_{2,0} & 0 & 0 & \rho_{2,3} & 0 & 0 & \rho_{2,6} & 0 & 0 & 0 & \rho_{2,10} & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_{3,1} & \rho_{3,2} & 0 & 0 & 0 & 0 & \rho_{3,7} & 0 & 0 & \rho_{3,11} & 0 & 0 & 0 & 0 & 0 \\ \rho_{40} & 0 & 0 & 0 & 0 & \rho_{4,5} & \rho_{4,6} & 0 & 0 & 0 & 0 & 0 & \rho_{4,12} & 0 & 0 & 0 \\ 0 & \rho_{5,1} & 0 & 0 & \rho_{5,4} & 0 & 0 & \rho_{5,7} & 0 & 0 & 0 & 0 & 0 & \rho_{5,13} & 0 & 0 \\ 0 & 0 & \rho_{6,2} & 0 & \rho_{6,4} & 0 & 0 & \rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{6,14} & 0 \\ 0 & 0 & 0 & \rho_{7,3} & 0 & \rho_{7,5} & \rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{7,15} \\ \rho_{8,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{8,9} & \rho_{8,10} & 0 & \rho_{8,12} & 0 & 0 & 0 \\ 0 & \rho_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{9,8} & 0 & 0 & \rho_{9,11} & 0 & \rho_{9,13} & 0 & 0 \\ 0 & 0 & \rho_{10,2} & 0 & 0 & 0 & 0 & 0 & \rho_{10,8} & 0 & 0 & \rho_{10,11} & 0 & 0 & \rho_{10,14} & 0 \\ 0 & 0 & 0 & \rho_{11,3} & 0 & 0 & 0 & 0 & 0 & \rho_{11,9} & \rho_{11,10} & 0 & 0 & 0 & 0 & \rho_{11,15} \\ 0 & 0 & 0 & 0 & \rho_{12,4} & 0 & 0 & 0 & \rho_{12,8} & 0 & 0 & 0 & 0 & \rho_{12,13} & \rho_{12,14} & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_{13,5} & 0 & 0 & 0 & \rho_{13,9} & 0 & 0 & \rho_{13,12} & 0 & 0 & \rho_{13,15} \\ 0 & 0 & & 0 & 0 & 0 & \rho_{14,6} & 0 & 0 & 0 & \rho_{14,10} & 0 & \rho_{14,12} & 0 & 0 & \rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{15,7} & 0 & 0 & 0 & \rho_{15,11} & 0 & \rho_{15,13} & \rho_{15,14} & 0 \end{pmatrix}$$

(193)

$$\begin{aligned}
M_{0+} &= \langle I_{0+} \rangle = tr (I_{0+} \rho_5) \\
&= \rho_{1,0} + \rho_{3,2} + \rho_{5,2} + \rho_{7,6} + \rho_{9,8} + \rho_{11,10} + \rho_{13,12} + \rho_{15,14}
\end{aligned} \tag{194}$$

maka dengan demikian nilai magnetisasi total dari sistem adalah sebagai berikut:

$$\begin{aligned}
M_{tot} &= M_{3+} + M_{2+} + M_{1+} + M_{0+} \\
&= \rho_{8,0} + \rho_{9,1} + \rho_{10,2} + \rho_{11,3} + \rho_{12,4} + \rho_{13,5} + \rho_{14,6} + \rho_{15,7} \\
&\quad + \rho_{4,0} + \rho_{5,1} + \rho_{6,2} + \rho_{7,3} + \rho_{12,8} + \rho_{13,9} + \rho_{14,10} + \rho_{15,11} \\
&\quad + \rho_{2,0} + \rho_{3,1} + \rho_{6,4} + \rho_{7,5} + \rho_{10,8} + \rho_{11,9} + \rho_{14,12} + \rho_{15,13} \\
&\quad + \rho_{1,0} + \rho_{3,2} + \rho_{5,4} + \rho_{7,6} + \rho_{9,8} + \rho_{11,10} + \rho_{13,12} + \rho_{15,14} \\
&= e^{i(\omega_3 + \pi J_{30} + \pi J_{31} + \pi J_{32})t} + e^{i(\omega_3 - \pi J_{30} + \pi J_{31} + \pi J_{32})t} \\
&\quad + e^{i(\omega_3 + \pi J_{30} - \pi J_{31} + \pi J_{32})t} + e^{i(\omega_3 - \pi J_{30} - \pi J_{31} + \pi J_{32})t} \\
&\quad + e^{i(\omega_3 + \pi J_{30} + \pi J_{31} - \pi J_{32})t} + e^{i(\omega_3 - \pi J_{30} + \pi J_{31} - \pi J_{32})t} \\
&\quad + e^{i(\omega_3 + \pi J_{30} - \pi J_{31} - \pi J_{32})t} + e^{i(\omega_3 - \pi J_{30} - \pi J_{31} - \pi J_{32})t} \\
&\quad + e^{i(\omega_2 + \pi J_{20} + \pi J_{21} + \pi J_{32})t} + e^{i(\omega_2 - \pi J_{20} + \pi J_{21} + \pi J_{32})t} \\
&\quad + e^{i(\omega_2 + \pi J_{20} - \pi J_{21} + \pi J_{32})t} + e^{i(\omega_2 - \pi J_{20} - \pi J_{21} + \pi J_{32})t} \\
&\quad + e^{i(\omega_2 + \pi J_{20} + \pi J_{21} - \pi J_{32})t} + e^{i(\omega_2 - \pi J_{20} + \pi J_{21} - \pi J_{32})t} \\
&\quad + e^{i(\omega_2 + \pi J_{20} - \pi J_{21} - \pi J_{32})t} + e^{i(\omega_2 - \pi J_{20} - \pi J_{21} - \pi J_{32})t} \\
&\quad + e^{i(\omega_1 + \pi J_{10} + \pi J_{21} + \pi J_{31})t} + e^{i(\omega_1 - \pi J_{10} + \pi J_{21} + \pi J_{31})t} \\
&\quad + e^{i(\omega_1 + \pi J_{10} - \pi J_{21} + \pi J_{31})t} + e^{i(\omega_1 - \pi J_{10} - \pi J_{21} + \pi J_{31})t} \\
&\quad + e^{i(\omega_1 + \pi J_{21} - \pi J_{31} + \pi J_{30})t} + e^{i(\omega_1 - \pi J_{10} + \pi J_{21} - \pi J_{32})t} \\
&\quad + e^{i(\omega_1 + \pi J_{10} - \pi J_{21} - \pi J_{31})t} + e^{i(\omega_1 - \pi J_{10} - \pi J_{21} - \pi J_{31})t} \\
&\quad + e^{i(\omega_0 + \pi J_{10} + \pi J_{20} + \pi J_{30})t} + e^{i(\omega_0 - \pi J_{10} + \pi J_{20} + \pi J_{30})t} \\
&\quad + e^{i(\omega_0 + \pi J_{10} - \pi J_{20} + \pi J_{30})t} + e^{i(\omega_0 - \pi J_{10} - \pi J_{20} + \pi J_{30})t} \\
&\quad + e^{i(\omega_0 + \pi J_{10} + \pi J_{20} - \pi J_{30})t} + e^{i(\omega_0 - \pi J_{10} + \pi J_{20} - \pi J_{30})t} \\
&\quad + e^{i(\omega_0 + \pi J_{10} - \pi J_{20} - \pi J_{30})t} + e^{i(\omega_0 - \pi J_{10} - \pi J_{20} - \pi J_{30})t}
\end{aligned} \tag{195}$$

kemudian didapatkan juga untuk sinyal FID sebagai berikut:

$$\begin{aligned}
S &= M_{tot} e^{-R_2 t} \\
&= e^{-(R_2 - i(\omega_3 + \pi J_{30} + \pi J_{31} + \pi J_{32}))t} + e^{-(R_2 - i(\omega_3 - \pi J_{30} + \pi J_{31} + \pi J_{32}))t} \\
&+ e^{-(R_2 - i(\omega_3 + \pi J_{30} - \pi J_{31} + \pi J_{32}))t} + e^{-(R_2 - i(\omega_3 - \pi J_{30} - \pi J_{31} + \pi J_{32}))t} \\
&+ e^{-(R_2 - i(\omega_3 + \pi J_{30} + \pi J_{31} - \pi J_{32}))t} + e^{-(R_2 - i(\omega_3 - \pi J_{30} + \pi J_{31} - \pi J_{32}))t} \\
&+ e^{-(R_2 - i(\omega_3 + \pi J_{30} - \pi J_{31} - \pi J_{32}))t} + e^{-(R_2 - i(\omega_3 - \pi J_{30} - \pi J_{31} - \pi J_{32}))t} \\
&+ e^{-(R_2 - i(\omega_2 + \pi J_{20} + \pi J_{21} + \pi J_{32}))t} + e^{-(R_2 - i(\omega_2 - \pi J_{20} + \pi J_{21} + \pi J_{32}))t} \\
&+ e^{-(R_2 - i(\omega_2 + \pi J_{20} - \pi J_{21} + \pi J_{32}))t} + e^{-(R_2 - i(\omega_2 - \pi J_{20} - \pi J_{21} + \pi J_{32}))t} \\
&+ e^{-(R_2 - i(\omega_2 + \pi J_{20} + \pi J_{21} - \pi J_{32}))t} + e^{-(R_2 - i(\omega_2 - \pi J_{20} + \pi J_{21} - \pi J_{32}))t} \\
&+ e^{-(R_2 - i(\omega_2 + \pi J_{20} - \pi J_{21} - \pi J_{32}))t} + e^{-(R_2 - i(\omega_2 - \pi J_{20} - \pi J_{21} - \pi J_{32}))t} \quad (196) \\
&+ e^{-(R_2 - i(\omega_1 + \pi J_{10} + \pi J_{21} + \pi J_{31}))t} + e^{-(R_2 - i(\omega_1 - \pi J_{10} + \pi J_{21} + \pi J_{31}))t} \\
&+ e^{-(R_2 - i(\omega_1 + \pi J_{10} - \pi J_{21} + \pi J_{31}))t} + e^{-(R_2 - i(\omega_1 - \pi J_{10} - \pi J_{21} + \pi J_{31}))t} \\
&+ e^{-(R_2 - i(\omega_1 + \pi J_{21} - \pi J_{31} + \pi J_{30}))t} + e^{-(R_2 - i(\omega_1 - \pi J_{10} + \pi J_{21} - \pi J_{32}))t} \\
&+ e^{-(R_2 - i(\omega_1 + \pi J_{10} - \pi J_{21} - \pi J_{31}))t} + e^{-(R_2 - i(\omega_1 - \pi J_{10} - \pi J_{21} - \pi J_{31}))t} \\
&+ e^{-(R_2 - i(\omega_0 + \pi J_{10} + \pi J_{20} + \pi J_{30}))t} + e^{-(R_2 - i(\omega_0 - \pi J_{10} + \pi J_{20} + \pi J_{30}))t} \\
&+ e^{-(R_2 - i(\omega_0 + \pi J_{10} - \pi J_{20} + \pi J_{30}))t} + e^{-(R_2 - i(\omega_0 - \pi J_{10} - \pi J_{20} + \pi J_{30}))t} \\
&+ e^{-(R_2 - i(\omega_0 + \pi J_{10} + \pi J_{20} - \pi J_{30}))t} + e^{-(R_2 - i(\omega_0 - \pi J_{10} + \pi J_{20} - \pi J_{30}))t} \\
&+ e^{-(R_2 - i(\omega_0 + \pi J_{10} - \pi J_{20} - \pi J_{30}))t} + e^{-(R_2 - i(\omega_0 - \pi J_{10} - \pi J_{20} - \pi J_{30}))t}
\end{aligned}$$

dimana $R_2 = \frac{1}{T_2}$

kemudian dapat di cari terlebih dahulu untuk perkalian antara $S(t) e^{-i\omega t}$ yaitu sebagai berikut:

$$\begin{aligned}
S(t) e^{-i\omega t} = & e^{-(R_2-i(\omega_3-\omega+\pi J_{30}+\pi J_{31}+\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_3-\omega-\pi J_{30}+\pi J_{31}+\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_3-\omega+\pi J_{30}-\pi J_{31}+\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_3-\omega-\pi J_{30}-\pi J_{31}+\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_3-\omega+\pi J_{30}+\pi J_{31}-\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_3-\omega-\pi J_{30}+\pi J_{31}-\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_3-\omega+\pi J_{30}-\pi J_{31}-\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_3-\omega-\pi J_{30}-\pi J_{31}-\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_2-\omega+\pi J_{20}+\pi J_{21}+\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_2-\omega-\pi J_{20}+\pi J_{21}+\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_2-\omega+\pi J_{20}-\pi J_{21}+\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_2-\omega-\pi J_{20}-\pi J_{21}+\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_2-\omega+\pi J_{20}+\pi J_{21}-\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_2-\omega-\pi J_{20}+\pi J_{21}-\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_2-\omega+\pi J_{20}-\pi J_{21}-\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_2-\omega-\pi J_{20}-\pi J_{21}-\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_1-\omega+\pi J_{10}+\pi J_{21}+\pi J_{31}))t} \\
& + e^{-(R_2-i(\omega_1-\omega-\pi J_{10}+\pi J_{21}+\pi J_{31}))t} \\
& + e^{-(R_2-i(\omega_1-\omega+\pi J_{10}-\pi J_{21}+\pi J_{31}))t} \\
& + e^{-(R_2-i(\omega_1-\omega-\pi J_{10}-\pi J_{21}+\pi J_{31}))t}
\end{aligned}$$

$$\begin{aligned}
& + e^{-(R_2-i(\omega_1-\omega-\pi J_{10}-\pi J_{21}+\pi J_{31}))t} \\
& + e^{-(R_2-i(\omega_1-\omega+\pi J_{21}-\pi J_{31}+\pi J_{30}))t} \\
& + e^{-(R_2-i(\omega_1-\omega-\pi J_{10}+\pi J_{21}-\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_1-\omega+\pi J_{10}-\pi J_{21}-\pi J_{31}))t} \\
& + e^{-(R_2-i(\omega_1-\omega-\pi J_{10}-\pi J_{21}-\pi J_{31}))t} \\
& + e^{-(R_2-i(\omega_0-\omega+\pi J_{10}+\pi J_{20}+\pi J_{30}))t} \\
& + e^{-(R_2-i(\omega_0-\omega-\pi J_{10}+\pi J_{20}+\pi J_{30}))t} \\
& + e^{-(R_2-i(\omega_0-\omega+\pi J_{10}-\pi J_{20}+\pi J_{30}))t} \\
& + e^{-(R_2-i(\omega_0-\omega-\pi J_{10}-\pi J_{20}+\pi J_{30}))t} \\
& + e^{-(R_2-i(\omega_0-\omega+\pi J_{10}+\pi J_{20}-\pi J_{30}))t} \\
& + e^{-(R_2-i(\omega_0-\omega-\pi J_{10}+\pi J_{20}-\pi J_{30}))t} \\
& + e^{-(R_2-i(\omega_0-\omega+\pi J_{10}-\pi J_{20}-\pi J_{30}))t} \\
& + e^{-(R_2-i(\omega_0-\omega-\pi J_{10}-\pi J_{20}-\pi J_{30}))t}
\end{aligned} \tag{197}$$

Transformasi Fourirer dari sinyal FiD adalah sebagai berikut:

$$\begin{aligned}
S(\omega) &= \int_0^\infty S(t) e^{-i\omega t} dt \\
&= \frac{e^{-(R_2 - i(\omega_3 - \omega + \pi J_{30} + \pi J_{31} + \pi J_{32}))t}}{-(R_2 - i(\omega_3 - \omega + \pi J_{30} + \pi J_{31} + \pi J_{32}))} \\
&+ \frac{e^{-(R_2 - i(\omega_3 - \omega - \pi J_{30} + \pi J_{31} + \pi J_{32}))t}}{-(R_2 - i(\omega_3 - \omega - \pi J_{30} + \pi J_{31} + \pi J_{32}))} \\
&+ \frac{e^{-(R_2 - i(\omega_3 - \omega + \pi J_{30} - \pi J_{31} + \pi J_{32}))t}}{-(R_2 - i(\omega_3 - \omega + \pi J_{30} - \pi J_{31} + \pi J_{32}))} \\
&+ \frac{e^{-(R_2 - i(\omega_3 - \omega - \pi J_{30} - \pi J_{31} + \pi J_{32}))t}}{-(R_2 - i(\omega_3 - \omega - \pi J_{30} - \pi J_{31} + \pi J_{32}))} \\
&+ \frac{e^{-(R_2 - i(\omega_3 - \omega - \pi J_{30} - \pi J_{31} + \pi J_{32}))t}}{-(R_2 - i(\omega_3 - \omega - \pi J_{30} - \pi J_{31} + \pi J_{32}))} \\
&+ \frac{e^{-(R_2 - i(\omega_3 - \omega - \pi J_{30} + \pi J_{31} - \pi J_{32}))t}}{-(R_2 - i(\omega_3 - \omega - \pi J_{30} + \pi J_{31} - \pi J_{32}))} \\
&+ \frac{e^{-(R_2 - i(\omega_3 - \omega + \pi J_{30} - \pi J_{31} - \pi J_{32}))t}}{-(R_2 - i(\omega_3 - \omega + \pi J_{30} - \pi J_{31} - \pi J_{32}))} \\
&+ \frac{e^{-(R_2 - i(\omega_3 - \omega - \pi J_{30} - \pi J_{31} - \pi J_{32}))t}}{-(R_2 - i(\omega_3 - \omega - \pi J_{30} - \pi J_{31} - \pi J_{32}))} \\
&+ \frac{e^{-(R_2 - i(\omega_2 - \omega + \pi J_{20} + \pi J_{21} + \pi J_{32}))t}}{-(R_2 - i(\omega_2 - \omega + \pi J_{20} + \pi J_{21} + \pi J_{32}))} \\
&+ \frac{e^{-(R_2 - i(\omega_2 - \omega - \pi J_{20} + \pi J_{21} + \pi J_{32}))t}}{-(R_2 - i(\omega_2 - \omega - \pi J_{20} + \pi J_{21} + \pi J_{32}))} \\
&+ \frac{e^{-(R_2 - i(\omega_2 - \omega - \pi J_{20} + \pi J_{21} + \pi J_{32}))t}}{-(R_2 - i(\omega_2 - \omega - \pi J_{20} + \pi J_{21} + \pi J_{32}))} \\
&+ \frac{e^{-(R_2 - i(\omega_2 - \omega + \pi J_{20} - \pi J_{21} + \pi J_{32}))t}}{-(R_2 - i(\omega_2 - \omega + \pi J_{20} - \pi J_{21} + \pi J_{32}))} \\
&+ \frac{e^{-(R_2 - i(\omega_2 - \omega - \pi J_{20} - \pi J_{21} + \pi J_{32}))t}}{-(R_2 - i(\omega_2 - \omega - \pi J_{20} - \pi J_{21} + \pi J_{32}))} \\
&+ \frac{e^{-(R_2 - i(\omega_2 - \omega + \pi J_{20} + \pi J_{21} - \pi J_{32}))t}}{-(R_2 - i(\omega_2 - \omega + \pi J_{20} + \pi J_{21} - \pi J_{32}))} \\
&+ \frac{e^{-(R_2 - i(\omega_2 - \omega - \pi J_{20} + \pi J_{21} - \pi J_{32}))t}}{-(R_2 - i(\omega_2 - \omega - \pi J_{20} + \pi J_{21} - \pi J_{32}))} \\
&+ \frac{e^{-(R_2 - i(\omega_2 - \omega - \pi J_{20} - \pi J_{21} - \pi J_{32}))t}}{-(R_2 - i(\omega_2 - \omega - \pi J_{20} - \pi J_{21} - \pi J_{32}))} \\
&+ \frac{e^{-(R_2 - i(\omega_2 - \omega - \pi J_{20} - \pi J_{21} - \pi J_{32}))t}}{-(R_2 - i(\omega_2 - \omega - \pi J_{20} - \pi J_{21} - \pi J_{32}))}
\end{aligned}$$

$$\begin{aligned}
& + \frac{e^{-(R_2 - i(\omega_1 - \omega + \pi J_{10} + \pi J_{21} + \pi J_{31}))t}}{-(R_2 - i(\omega_1 - \omega + \pi J_{10} + \pi J_{21} + \pi J_{31}))} + \frac{e^{-(R_2 - i(\omega_1 - \omega - \pi J_{10} + \pi J_{21} + \pi J_{31}))t}}{-(R_2 - i(\omega_1 - \omega - \pi J_{10} + \pi J_{21} + \pi J_{31}))} \\
& + \frac{e^{-(R_2 - i(\omega_1 - \omega + \pi J_{10} - \pi J_{21} + \pi J_{31}))t}}{-(R_2 - i(\omega_1 - \omega + \pi J_{10} - \pi J_{21} + \pi J_{31}))} + \frac{e^{-(R_2 - i(\omega_1 - \omega - \pi J_{10} - \pi J_{21} + \pi J_{31}))t}}{-(R_2 - i(\omega_1 - \omega - \pi J_{10} - \pi J_{21} + \pi J_{31}))} \\
& + \frac{e^{-(R_2 - i(\omega_1 - \omega + \pi J_{10} - \pi J_{21} - \pi J_{31}))t}}{-(R_2 - i(\omega_1 - \omega + \pi J_{10} - \pi J_{21} - \pi J_{31}))} + \frac{e^{-(R_2 - i(\omega_1 - \omega - \pi J_{10} - \pi J_{21} - \pi J_{31}))t}}{-(R_2 - i(\omega_1 - \omega - \pi J_{10} - \pi J_{21} - \pi J_{31}))} \\
& + \frac{e^{-(R_2 - i(\omega_0 - \omega + \pi J_{10} + \pi J_{20} + \pi J_{30}))t}}{-(R_2 - i(\omega_0 - \omega + \pi J_{10} + \pi J_{20} + \pi J_{30}))} + \frac{e^{-(R_2 - i(\omega_0 - \omega - \pi J_{10} + \pi J_{20} + \pi J_{30}))t}}{-(R_2 - i(\omega_0 - \omega - \pi J_{10} + \pi J_{20} + \pi J_{30}))} \\
& + \frac{e^{-(R_2 - i(\omega_0 - \omega + \pi J_{10} - \pi J_{20} + \pi J_{30}))t}}{-(R_2 - i(\omega_0 - \omega + \pi J_{10} - \pi J_{20} + \pi J_{30}))} + \frac{e^{-(R_2 - i(\omega_0 - \omega - \pi J_{10} - \pi J_{20} + \pi J_{30}))t}}{-(R_2 - i(\omega_0 - \omega - \pi J_{10} - \pi J_{20} + \pi J_{30}))} \\
& + \frac{e^{-(R_2 - i(\omega_0 - \omega + \pi J_{10} + \pi J_{20} - \pi J_{30}))t}}{-(R_2 - i(\omega_0 - \omega + \pi J_{10} + \pi J_{20} - \pi J_{30}))} + \frac{e^{-(R_2 - i(\omega_0 - \omega - \pi J_{10} + \pi J_{20} - \pi J_{30}))t}}{-(R_2 - i(\omega_0 - \omega - \pi J_{10} + \pi J_{20} - \pi J_{30}))} \\
& + \frac{e^{-(R_2 - i(\omega_0 - \omega + \pi J_{10} - \pi J_{20} - \pi J_{30}))t}}{-(R_2 - i(\omega_0 - \omega + \pi J_{10} - \pi J_{20} - \pi J_{30}))} + \frac{e^{-(R_2 - i(\omega_0 - \omega - \pi J_{10} - \pi J_{20} - \pi J_{30}))t}}{-(R_2 - i(\omega_0 - \omega - \pi J_{10} - \pi J_{20} - \pi J_{30}))}
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{(R_2 - i(\omega_3 - \omega + \pi J_{30} + \pi J_{31} + \pi J_{32}))} + \frac{1}{(R_2 - i(\omega_3 - \omega - \pi J_{30} + \pi J_{31} + \pi J_{32}))} \\
&+ \frac{1}{(R_2 - i(\omega_3 - \omega + \pi J_{30} - \pi J_{31} + \pi J_{32}))} + \frac{1}{(R_2 - i(\omega_3 - \omega - \pi J_{30} - \pi J_{31} + \pi J_{32}))} \\
&+ \frac{1}{(R_2 - i(\omega_3 - \omega - \pi J_{30} - \pi J_{31} + \pi J_{32}))} + \frac{1}{(R_2 - i(\omega_3 - \omega - \pi J_{30} + \pi J_{31} - \pi J_{32}))} \\
&+ \frac{1}{(R_2 - i(\omega_3 - \omega + \pi J_{30} - \pi J_{31} - \pi J_{32}))} + \frac{1}{(R_2 - i(\omega_3 - \omega - \pi J_{30} - \pi J_{31} - \pi J_{32}))} \\
&+ \frac{1}{(R_2 - i(\omega_2 - \omega + \pi J_{20} + \pi J_{21} + \pi J_{32}))} + \frac{1}{(R_2 - i(\omega_2 - \omega - \pi J_{20} + \pi J_{21} + \pi J_{32}))} \\
&+ \frac{1}{(R_2 - i(\omega_2 - \omega + \pi J_{20} - \pi J_{21} + \pi J_{32}))} + \frac{1}{(R_2 - i(\omega_2 - \omega - \pi J_{20} - \pi J_{21} + \pi J_{32}))} \\
&+ \frac{1}{(R_2 - i(\omega_2 - \omega + \pi J_{20} + \pi J_{21} - \pi J_{32}))} + \frac{1}{(R_2 - i(\omega_2 - \omega - \pi J_{20} + \pi J_{21} - \pi J_{32}))} \\
&+ \frac{1}{(R_2 - i(\omega_2 - \omega + \pi J_{20} - \pi J_{21} - \pi J_{32}))} + \frac{1}{(R_2 - i(\omega_2 - \omega - \pi J_{20} - \pi J_{21} - \pi J_{32}))} \\
&+ \frac{1}{(R_2 - i(\omega_1 - \omega + \pi J_{10} + \pi J_{21} + \pi J_{31}))} + \frac{1}{(R_2 - i(\omega_1 - \omega - \pi J_{10} + \pi J_{21} + \pi J_{31}))} \\
&+ \frac{1}{(R_2 - i(\omega_1 - \omega + \pi J_{10} - \pi J_{21} + \pi J_{31}))} + \frac{1}{(R_2 - i(\omega_1 - \omega - \pi J_{10} - \pi J_{21} + \pi J_{31}))} \\
&+ \frac{1}{(R_2 - i(\omega_1 - \omega + \pi J_{21} - \pi J_{31} + \pi J_{30}))} + \frac{1}{(R_2 - i(\omega_1 - \omega - \pi J_{10} + \pi J_{21} - \pi J_{32}))} \\
&+ \frac{1}{(R_2 - i(\omega_1 - \omega + \pi J_{10} - \pi J_{21} - \pi J_{31}))} + \frac{1}{(R_2 - i(\omega_1 - \omega - \pi J_{10} - \pi J_{21} - \pi J_{31}))} \\
&+ \frac{1}{(R_2 - i(\omega_0 - \omega + \pi J_{10} + \pi J_{20} + \pi J_{30}))} + \frac{1}{(R_2 - i(\omega_0 - \omega - \pi J_{10} + \pi J_{20} + \pi J_{30}))} \\
&+ \frac{1}{(R_2 - i(\omega_0 - \omega + \pi J_{10} - \pi J_{20} + \pi J_{30}))} + \frac{1}{(R_2 - i(\omega_0 - \omega - \pi J_{10} - \pi J_{20} + \pi J_{30}))} \\
&+ \frac{1}{(R_2 - i(\omega_0 - \omega + \pi J_{10} + \pi J_{20} - \pi J_{30}))} + \frac{1}{(R_2 - i(\omega_0 - \omega - \pi J_{10} + \pi J_{20} - \pi J_{30}))} \\
&+ \frac{1}{(R_2 - i(\omega_0 - \omega + \pi J_{10} - \pi J_{20} - \pi J_{30}))} + \frac{1}{(R_2 - i(\omega_0 - \omega - \pi J_{10} - \pi J_{20} - \pi J_{30}))}
\end{aligned}$$

[illegible]

maka dengan demikian diperoleh :

$$\begin{aligned}
A(\omega) = & \frac{R_2}{R_2^2 + (\omega_3 - \omega + \pi J_{30} + \pi J_{31} + \pi J_{32})^2} + \frac{R_2}{R_2^2 + (\omega_3 - \omega - \pi J_{30} + \pi J_{31} + \pi J_{32})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_3 - \omega + \pi J_{30} - \pi J_{31} + \pi J_{32})^2} + \frac{R_2}{R_2^2 + (\omega_3 - \omega - \pi J_{30} - \pi J_{31} + \pi J_{32})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_3 - \omega - \pi J_{30} - \pi J_{31} + \pi J_{32})^2} + \frac{R_2}{R_2^2 + (\omega_3 - \omega - \pi J_{30} + \pi J_{31} - \pi J_{32})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_3 - \omega + \pi J_{30} - \pi J_{31} - \pi J_{32})^2} + \frac{R_2}{R_2^2 + (\omega_3 - \omega - \pi J_{30} - \pi J_{31} - \pi J_{32})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_2 - \omega + \pi J_{20} + \pi J_{21} + \pi J_{32})^2} + \frac{R_2}{R_2^2 + (\omega_2 - \omega - \pi J_{20} + \pi J_{21} + \pi J_{32})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_2 - \omega + \pi J_{20} - \pi J_{21} + \pi J_{32})^2} + \frac{R_2}{R_2^2 + (\omega_2 - \omega - \pi J_{20} - \pi J_{21} + \pi J_{32})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_2 - \omega + \pi J_{20} + \pi J_{21} - \pi J_{32})^2} + \frac{R_2}{R_2^2 + (\omega_2 - \omega - \pi J_{20} + \pi J_{21} - \pi J_{32})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_2 - \omega + \pi J_{20} - \pi J_{21} - \pi J_{32})^2} + \frac{R_2}{R_2^2 + (\omega_2 - \omega - \pi J_{20} - \pi J_{21} - \pi J_{32})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_1 - \omega + \pi J_{10} + \pi J_{21} + \pi J_{31})^2} + \frac{R_2}{R_2^2 + (\omega_1 - \omega - \pi J_{10} + \pi J_{21} + \pi J_{31})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_1 - \omega + \pi J_{10} - \pi J_{21} + \pi J_{31})^2} + \frac{R_2}{R_2^2 + (\omega_1 - \omega - \pi J_{10} - \pi J_{21} + \pi J_{31})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_1 - \omega + \pi J_{21} - \pi J_{31} + \pi J_{30})^2} + \frac{R_2}{R_2^2 + (\omega_1 - \omega - \pi J_{10} + \pi J_{21} - \pi J_{32})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_1 - \omega + \pi J_{10} - \pi J_{21} - \pi J_{31})^2} + \frac{R_2}{R_2^2 + (\omega_1 - \omega - \pi J_{10} - \pi J_{21} - \pi J_{31})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_0 - \omega + \pi J_{10} + \pi J_{20} + \pi J_{30})^2} + \frac{R_2}{R_2^2 + (\omega_0 - \omega - \pi J_{10} + \pi J_{20} + \pi J_{30})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_0 - \omega + \pi J_{10} - \pi J_{20} + \pi J_{30})^2} + \frac{R_2}{R_2 + (\omega_0 - \omega - \pi J_{10} - \pi J_{20} + \pi J_{30})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_0 - \omega + \pi J_{10} + \pi J_{20} - \pi J_{30})^2} + \frac{R_2}{R_2 + (\omega_0 - \omega - \pi J_{10} + \pi J_{20} - \pi J_{30})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_0 - \omega + \pi J_{10} - \pi J_{20} - \pi J_{30})^2} + \frac{R_2}{R_2^2 + (\omega_0 - \omega - \pi J_{10} - \pi J_{20} - \pi J_{30})^2}
\end{aligned} \tag{198}$$

dan diperoleh juga:

$$\begin{aligned}
D(\omega) = & \frac{i(\omega_3 - \omega + \pi J_{30} + \pi J_{31} + \pi J_{32})}{R_2^2 + (\omega_3 - \omega + \pi J_{30} + \pi J_{31} + \pi J_{32})^2} + \frac{i(\omega_3 - \omega - \pi J_{30} + \pi J_{31} + \pi J_{32})}{R_2^2 + (\omega_3 - \omega - \pi J_{30} + \pi J_{31} + \pi J_{32})^2} \\
& + \frac{i(\omega_3 - \omega + \pi J_{30} - \pi J_{31} + \pi J_{32})}{R_2^2 + (\omega_3 - \omega + \pi J_{30} - \pi J_{31} + \pi J_{32})^2} + \frac{i(\omega_3 - \omega - \pi J_{30} - \pi J_{31} + \pi J_{32})}{R_2^2 + (\omega_3 - \omega - \pi J_{30} - \pi J_{31} + \pi J_{32})^2} \\
& + \frac{i(\omega_3 - \omega - \pi J_{30} - \pi J_{31} + \pi J_{32})}{R_2^2 + (\omega_3 - \omega - \pi J_{30} - \pi J_{31} + \pi J_{32})^2} + \frac{i(\omega_3 - \omega - \pi J_{30} + \pi J_{31} - \pi J_{32})}{R_2^2 + (\omega_3 - \omega - \pi J_{30} + \pi J_{31} - \pi J_{32})^2} \\
& + \frac{i(\omega_3 - \omega + \pi J_{30} - \pi J_{31} - \pi J_{32})}{R_2^2 + (\omega_3 - \omega + \pi J_{30} - \pi J_{31} - \pi J_{32})^2} + \frac{i(\omega_3 - \omega - \pi J_{30} - \pi J_{31} - \pi J_{32})}{R_2^2 + (\omega_3 - \omega - \pi J_{30} - \pi J_{31} - \pi J_{32})^2} \\
& + \frac{i(\omega_2 - \omega + \pi J_{20} + \pi J_{21} + \pi J_{32})}{R_2^2 + (\omega_2 - \omega + \pi J_{20} + \pi J_{21} + \pi J_{32})^2} + \frac{i(\omega_2 - \omega - \pi J_{20} + \pi J_{21} + \pi J_{32})}{R_2^2 + (\omega_2 - \omega - \pi J_{20} + \pi J_{21} + \pi J_{32})^2} \\
& + \frac{i(\omega_2 - \omega + \pi J_{20} - \pi J_{21} + \pi J_{32})}{R_2^2 + (\omega_2 - \omega + \pi J_{20} - \pi J_{21} + \pi J_{32})^2} + \frac{i(\omega_2 - \omega - \pi J_{20} - \pi J_{21} + \pi J_{32})}{R_2^2 + (\omega_2 - \omega - \pi J_{20} - \pi J_{21} + \pi J_{32})^2} \\
& + \frac{i(\omega_2 - \omega + \pi J_{20} + \pi J_{21} - \pi J_{32})}{R_2^2 + (\omega_2 - \omega + \pi J_{20} + \pi J_{21} - \pi J_{32})^2} + \frac{i(\omega_2 - \omega - \pi J_{20} + \pi J_{21} - \pi J_{32})}{R_2^2 + (\omega_2 - \omega - \pi J_{20} + \pi J_{21} - \pi J_{32})^2} \\
& + \frac{i(\omega_2 - \omega + \pi J_{20} - \pi J_{21} - \pi J_{32})}{R_2^2 + (\omega_2 - \omega + \pi J_{20} - \pi J_{21} - \pi J_{32})^2} + \frac{i(\omega_2 - \omega - \pi J_{20} - \pi J_{21} - \pi J_{32})}{R_2^2 + (\omega_2 - \omega - \pi J_{20} - \pi J_{21} - \pi J_{32})^2} \\
& + \frac{i(\omega_1 - \omega + \pi J_{10} + \pi J_{21} + \pi J_{31})}{R_2^2 + (\omega_1 - \omega + \pi J_{10} + \pi J_{21} + \pi J_{31})^2} + \frac{i(\omega_1 - \omega - \pi J_{10} + \pi J_{21} + \pi J_{31})}{R_2^2 + (\omega_1 - \omega - \pi J_{10} + \pi J_{21} + \pi J_{31})^2} \\
& + \frac{i(\omega_1 - \omega + \pi J_{10} - \pi J_{21} + \pi J_{31})}{R_2^2 + (\omega_1 - \omega + \pi J_{10} - \pi J_{21} + \pi J_{31})^2} + \frac{i(\omega_1 - \omega - \pi J_{10} - \pi J_{21} + \pi J_{31})}{R_2^2 + (\omega_1 - \omega - \pi J_{10} - \pi J_{21} + \pi J_{31})^2} \\
& + \frac{i(\omega_1 - \omega + \pi J_{21} - \pi J_{31} + \pi J_{30})}{R_2^2 + (\omega_1 - \omega + \pi J_{21} - \pi J_{31} + \pi J_{30})^2} + \frac{i(\omega_1 - \omega - \pi J_{10} + \pi J_{21} - \pi J_{32})}{R_2^2 + (\omega_1 - \omega - \pi J_{10} + \pi J_{21} - \pi J_{32})^2} \\
& + \frac{i(\omega_1 - \omega + \pi J_{10} - \pi J_{21} - \pi J_{31})}{R_2^2 + (\omega_1 - \omega + \pi J_{10} - \pi J_{21} - \pi J_{31})^2} + \frac{i(\omega_1 - \omega - \pi J_{10} - \pi J_{21} - \pi J_{31})}{R_2^2 + (\omega_1 - \omega - \pi J_{10} - \pi J_{21} - \pi J_{31})^2} \\
& + \frac{i(\omega_0 - \omega + \pi J_{10} + \pi J_{20} + \pi J_{30})}{R_2^2 + (\omega_0 - \omega + \pi J_{10} + \pi J_{20} + \pi J_{30})^2} + \frac{i(\omega_0 - \omega - \pi J_{10} + \pi J_{20} + \pi J_{30})}{R_2^2 + (\omega_0 - \omega - \pi J_{10} + \pi J_{20} + \pi J_{30})^2} \\
& + \frac{i(\omega_0 - \omega + \pi J_{10} - \pi J_{20} + \pi J_{30})}{R_2^2 + (\omega_0 - \omega + \pi J_{10} - \pi J_{20} + \pi J_{30})^2} + \frac{i(\omega_0 - \omega - \pi J_{10} - \pi J_{20} + \pi J_{30})}{R_2 + (\omega_0 - \omega - \pi J_{10} - \pi J_{20} + \pi J_{30})^2} \\
& + \frac{i(\omega_0 - \omega + \pi J_{10} + \pi J_{20} - \pi J_{30})}{R_2^2 + (\omega_0 - \omega + \pi J_{10} + \pi J_{20} - \pi J_{30})^2} + \frac{i(\omega_0 - \omega - \pi J_{10} + \pi J_{20} - \pi J_{30})}{R_2 + (\omega_0 - \omega - \pi J_{10} + \pi J_{20} - \pi J_{30})^2} \\
& + \frac{i(\omega_0 - \omega + \pi J_{10} - \pi J_{20} - \pi J_{30})}{R_2^2 + (\omega_0 - \omega + \pi J_{10} - \pi J_{20} - \pi J_{30})^2} + \frac{i(\omega_0 - \omega - \pi J_{10} - \pi J_{20} - \pi J_{30})}{R_2^2 + (\omega_0 - \omega - \pi J_{10} - \pi J_{20} - \pi J_{30})^2}
\end{aligned} \tag{199}$$

kita telah memperoleh persamaan (162) yang menggambarkan nilai magnetisasi pada sumbu-x. Berikut adalah grafik daripada persamaan (162), dimana data yang dipakai untuk masing-masing qubit adalah sebesar $\omega_3 = 500MHz$, $\omega_2 = 300MHz$, $\omega_1 = 202MHz$, $\omega_0 = 126MHz$, $J_{32} = 12$, $J_{31} =$

11, $J_{21} = 10, J_{30} = 9, J_{20} = 8; J_{10} = 7, R_2 = 7$ spektrum

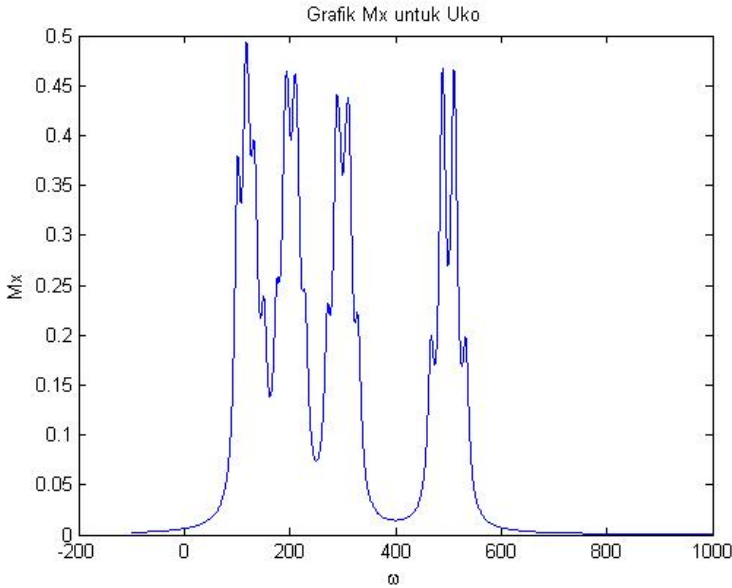


Figure 2: Grafik untuk Uko

yang terbentuk menggambarkan 4 buah puncak tertinggi dari doublet dimana setiap spektrum berisi 2 puncak tertinggi, secara berturut-turut multiplet dari kiri ke kanan merepresentasikan dari qubit-3, qubit-2, qubit-1, qubit-0. masing-masing multiplet terdiri dari 2 puncak yang mengarah keatas hal ini mengindikasikan bahwa setiap qubit adalah dalam keadaan spin up, atau keadaan $|0\rangle$, jadi pembacaan hasil algoritma berupa $|000\rangle$. Kedua puncak yang mengarah keatas dari setiap qubit menunjukkan bahwa tidak muncul keadaan terbelit dalam sistem 4 qubit apabila $U_2 = Uk0$.

apabila diambil kasus untuk $U_2 = \sigma_z \otimes I \otimes I \otimes I$ maka diperoleh untuk bentuk operator uniter totalnya adalah sebagai berikut:

$$\begin{aligned}
 U &= U_5 U_2 U_1 \\
 &= U_5 (\sigma_z \otimes I \otimes I \otimes I) U_1 \\
 &= \begin{pmatrix} e^{\beta_0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & e^{\beta_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{\beta_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{\beta_3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{\beta_4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{\beta_5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_9} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_{10}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_{11}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_{12}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_{13}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_{14}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_{15}} \end{pmatrix}
 \end{aligned}$$

[illegible]

(201)

maka dengan demikian kita dapatkan untuk nilai ρ_5 adalah

[illegible]

×

[illegible]

$$= \frac{1}{16} \begin{pmatrix} 0 & 8\rho_{0,1} & 8\rho_{0,2} & 0 & 8\rho_{0,4} & 0 & 0 & 0 & -8\rho_{0,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8\rho_{1,0} & 0 & 0 & 8\rho_{1,3} & 0 & 8\rho_{1,5} & 0 & 0 & 0 & -8\rho_{1,9} & 0 & 0 & 0 & 0 & 0 & 0 \\ 8\rho_{2,0} & 0 & 0 & 8\rho_{2,3} & 0 & 0 & 8\rho_{2,6} & 0 & 0 & 0 & -8\rho_{2,10} & 0 & 0 & 0 & 0 & 0 \\ 0 & 8\rho_{3,1} & 8\rho_{3,2} & 0 & 0 & 0 & 0 & 8\rho_{3,7} & 0 & 0 & 0 & -8\rho_{3,11} & 0 & 0 & 0 & 0 \\ 8\rho_{40} & 0 & 0 & 0 & 0 & 8\rho_{4,5} & 8\rho_{4,6} & 0 & 0 & 0 & 0 & 0 & -8\rho_{4,12} & 0 & 0 & 0 \\ 0 & 8\rho_{4,1} & 0 & 0 & 8\rho_{5,4} & 0 & 0 & 8\rho_{5,7} & 0 & 0 & 0 & 0 & 0 & -8\rho_{5,13} & 0 & 0 \\ 0 & 0 & 8\rho_{6,2} & 0 & 8\rho_{6,4} & 0 & 0 & 8\rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & -8\rho_{6,14} & 0 \\ 0 & 0 & 0 & 8\rho_{7,3} & 0 & 8\rho_{7,5} & 8\rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -8\rho_{7,15} \\ -8\rho_{8,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8\rho_{8,9} & 8\rho_{8,10} & 0 & 8\rho_{8,12} & 0 & 0 & 0 \\ 0 & 8 - \rho_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & 8\rho_{9,8} & 0 & 0 & 8\rho_{9,11} & 0 & 8\rho_{9,13} & 0 & 0 \\ 0 & 0 & -8\rho_{10,2} & 0 & 0 & 0 & 0 & 0 & 8\rho_{10,8} & 0 & 0 & 8\rho_{10,11} & 0 & 0 & 8\rho_{10,14} & 0 \\ 0 & 0 & 0 & -8\rho_{11,3} & 0 & 0 & 0 & 0 & 0 & 8\rho_{11,9} & 8\rho_{11,10} & 0 & 0 & 0 & 0 & 8\rho_{11,15} \\ 0 & 0 & 0 & 0 & -8\rho_{12,4} & 0 & 0 & 0 & 8\rho_{12,8} & 0 & 0 & 0 & 0 & 8\rho_{12,13} & 8\rho_{12,14} & 0 \\ 0 & 0 & 0 & 0 & 0 & -8\rho_{13,5} & 0 & 0 & 0 & 8\rho_{13,9} & 0 & 0 & 8\rho_{13,12} & 0 & 0 & 8\rho_{13,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & -8\rho_{14,6} & 0 & 0 & 0 & 8\rho_{14,10} & 0 & 8\rho_{14,12} & 0 & 0 & 8\rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -8\rho_{15,7} & 0 & 0 & 0 & 8\rho_{15,11} & 0 & 8\rho_{15,13} & 8\rho_{15,14} & 0 \end{pmatrix} \quad (202)$$

$$= \frac{1}{2} \begin{pmatrix} 0 & \rho_{0,1} & \rho_{0,2} & 0 & \rho_{0,4} & 0 & 0 & 0 & -\rho_{0,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho_{1,0} & 0 & 0 & \rho_{1,3} & 0 & \rho_{1,5} & 0 & 0 & 0 & -\rho_{1,9} & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho_{2,0} & 0 & 0 & \rho_{2,3} & 0 & 0 & \rho_{2,6} & 0 & 0 & 0 & -\rho_{2,10} & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_{3,1} & \rho_{3,2} & 0 & 0 & 0 & 0 & \rho_{3,7} & 0 & 0 & 0 & -\rho_{3,11} & 0 & 0 & 0 & 0 \\ \rho_{40} & 0 & 0 & 0 & 0 & \rho_{4,5} & \rho_{4,6} & 0 & 0 & 0 & 0 & 0 & -\rho_{4,12} & 0 & 0 & 0 \\ 0 & \rho_{4,1} & 0 & 0 & \rho_{5,4} & 0 & 0 & \rho_{5,7} & 0 & 0 & 0 & 0 & 0 & -\rho_{5,13} & 0 & 0 \\ 0 & 0 & \rho_{6,2} & 0 & \rho_{6,4} & 0 & 0 & \rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{6,14} & 0 \\ 0 & 0 & 0 & \rho_{7,3} & 0 & \rho_{7,5} & \rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{7,15} \\ -\rho_{8,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{8,9} & \rho_{8,10} & 0 & \rho_{8,12} & 0 & 0 & 0 \\ 0 & -\rho_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{9,8} & 0 & 0 & \rho_{9,11} & 0 & \rho_{9,13} & 0 & 0 \\ 0 & 0 & -\rho_{10,2} & 0 & 0 & 0 & 0 & 0 & \rho_{10,8} & 0 & 0 & \rho_{10,11} & 0 & 0 & \rho_{10,14} & 0 \\ 0 & 0 & 0 & -\rho_{11,3} & 0 & 0 & 0 & 0 & 0 & \rho_{11,9} & \rho_{11,10} & 0 & 0 & 0 & 0 & \rho_{11,15} \\ 0 & 0 & 0 & 0 & -\rho_{12,4} & 0 & 0 & 0 & \rho_{12,8} & 0 & 0 & 0 & 0 & \rho_{12,13} & \rho_{12,14} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\rho_{13,5} & 0 & 0 & 0 & \rho_{13,9} & 0 & 0 & \rho_{13,12} & 0 & 0 & \rho_{13,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{14,6} & 0 & 0 & 0 & \rho_{14,10} & 0 & \rho_{14,12} & 0 & 0 & \rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{15,7} & 0 & 0 & 0 & \rho_{15,11} & 0 & \rho_{15,13} & \rho_{15,14} & 0 \end{pmatrix} \quad (203)$$

dengan

$$\begin{aligned}
\rho_{0,1} &= \rho_{1,0}^* = e^{\beta_0 - \beta_1} & \rho_{6,7} &= \rho_{7,6}^* = e^{\beta_6 - \beta_7} \\
\rho_{0,2} &= \rho_{2,0}^* = e^{\beta_0 - \beta_2} & \rho_{6,14} &= \rho_{14,6}^* = e^{\beta_6 - \beta_{14}} \\
\rho_{0,8} &= \rho_{8,0}^* = e^{\beta_8 - \beta_1} & \rho_{8,9} &= \rho_{9,8}^* = e^{\beta_8 - \beta_9} \\
\rho_{1,3} &= \rho_{3,1}^* = e^{\beta_1 - \beta_3} & \rho_{8,10} &= \rho_{10,8}^* = e^{\beta_8 - \beta_{10}} \\
\rho_{1,5} &= \rho_{5,1}^* = e^{\beta_1 - \beta_5} & \rho_{8,12} &= \rho_{12,8}^* = e^{\beta_8 - \beta_{12}} \\
\rho_{1,9} &= \rho_{9,1}^* = e^{\beta_1 - \beta_9} & \rho_{9,11} &= \rho_{11,9}^* = e^{\beta_9 - \beta_{11}} \\
\rho_{2,3} &= \rho_{3,2}^* = e^{\beta_2 - \beta_3} & \rho_{9,13} &= \rho_{13,9}^* = e^{\beta_9 - \beta_{13}} \\
\rho_{2,6} &= \rho_{6,2}^* = e^{\beta_2 - \beta_6} & \rho_{10,11} &= \rho_{11,10}^* = e^{\beta_{10} - \beta_{11}} \\
\rho_{2,10} &= \rho_{10,2}^* = e^{\beta_2 - \beta_{10}} & \rho_{10,14} &= \rho_{14,10}^* = e^{\beta_{10} - \beta_{14}} \\
\rho_{3,7} &= \rho_{7,3}^* = e^{\beta_3 - \beta_7} & \rho_{12,13} &= \rho_{13,12}^* = e^{\beta_{12} - \beta_{13}} \\
\rho_{3,11} &= \rho_{11,3}^* = e^{\beta_3 - \beta_{11}} & \rho_{12,14} &= \rho_{14,12}^* = e^{\beta_{12} - \beta_{14}} \\
\rho_{4,5} &= \rho_{5,4}^* = e^{\beta_4 - \beta_5} & \rho_{11,15} &= \rho_{15,11}^* = e^{\beta_{11} - \beta_{15}} \\
\rho_{4,6} &= \rho_{6,4}^* = e^{\beta_4 - \beta_6} & \rho_{13,15} &= \rho_{15,13}^* = e^{\beta_{13} - \beta_{15}} \\
\rho_{4,12} &= \rho_{12,4}^* = e^{\beta_4 - \beta_{12}} & \rho_{14,15} &= \rho_{15,14}^* = e^{\beta_{14} - \beta_{15}} \\
\rho_{5,7} &= \rho_{7,5}^* = e^{\beta_5 - \beta_7} & \rho_{15,7} &= \rho_{7,15}^* = e^{\beta_7 - \beta_{15}} \\
\rho_{5,13} &= \rho_{13,5}^* = e^{\beta_5 - \beta_{13}} & \rho_{0,4} &= \rho_{4,0}^* = e^{\beta_0 - \beta_4}
\end{aligned} \tag{204}$$

dan

$$\begin{array}{ll}
\beta_0 - \beta_1 = -i(E_{4,0} - E_{4,1})t & \beta_0 - \beta_2 = -i(E_{4,0} - E_{4,2})t \\
\beta_0 - \beta_4 = -i(E_{4,0} - E_{4,4})t & \beta_0 - \beta_8 = -i(E_{4,0} - E_{4,8})t \\
\beta_1 - \beta_3 = -i(E_{4,1} - E_{4,3})t & \beta_1 - \beta_5 = -i(E_{4,1} - E_{4,5})t \\
\beta_1 - \beta_9 = -i(E_{4,1} - E_{4,9})t & \beta_2 - \beta_3 = -i(E_{4,2} - E_{4,3})t \\
\beta_2 - \beta_6 = -i(E_{4,2} - E_{4,6})t & \beta_2 - \beta_{10} = -i(E_{4,2} - E_{4,10})t \\
\beta_3 - \beta_7 = -i(E_{4,3} - E_{4,7})t & \beta_3 - \beta_{11} = -i(E_{4,3} - E_{4,11})t \\
\beta_4 - \beta_5 = -i(E_{4,4} - E_{4,5})t & \beta_4 - \beta_6 = -i(E_{4,4} - E_{4,6})t \\
\beta_4 - \beta_{12} = -i(E_{4,4} - E_{4,12})t & \beta_5 - \beta_7 = -i(E_{4,5} - E_{4,7})t \\
\beta_5 - \beta_{13} = -i(E_{4,5} - E_{4,13})t & \beta_6 - \beta_7 = -i(E_{4,6} - E_{4,7})t \\
\beta_6 - \beta_{14} = -i(E_{4,6} - E_{4,14})t & \beta_8 - \beta_9 = -i(E_{4,8} - E_{4,9})t \\
\beta_8 - \beta_{10} = -i(E_{4,8} - E_{4,10})t & \beta_8 - \beta_{12} = -i(E_{4,8} - E_{4,12})t \\
\beta_9 - \beta_{11} = -i(E_{4,9} - E_{4,11})t & \beta_9 - \beta_{13} = -i(E_{4,9} - E_{4,13})t \\
\beta_{10} - \beta_{11} = -i(E_{4,10} - E_{4,11})t & \beta_{10} - \beta_{14} = -i(E_{4,10} - E_{4,14})t \\
\beta_{12} - \beta_{13} = -i(E_{4,12} - E_{4,13})t & \beta_{12} - \beta_{14} = -i(E_{4,12} - E_{4,14})t \\
\beta_{11} - \beta_{15} = -i(E_{4,11} - E_{4,15})t & \beta_{13} - \beta_{15} = -i(E_{4,13} - E_{4,15})t \\
\beta_{14} - \beta_{15} = -i(E_{4,14} - E_{4,15})t & \beta_{15} - \beta_7 = -i(E_{4,15} - E_{4,7})t
\end{array} \quad (205)$$

dan

$$\begin{aligned} E_{4,0} - E_{4,1} &= \omega_0 + \pi J_{10} + \pi J_{20} + \pi J_{30} & E_{4,0} - E_{4,2} &= \omega_1 + \pi J_{10} + \pi J_{21} + \pi J_{31} \\ E_{4,0} - E_{4,4} &= \omega_2 + \pi J_{20} + \pi J_{21} + \pi J_{32} & E_{4,0} - E_{4,8} &= \omega_3 + \pi J_{30} + \pi J_{31} + \pi J_{32} \\ E_{4,1} - E_{4,3} &= \omega_1 - \pi J_{10} + \pi J_{21} + \pi J_{31} & E_{4,1} - E_{4,5} &= \omega_2 - \pi J_{20} + \pi J_{21} + \pi J_{32} \\ E_{4,1} - E_{4,9} &= \omega_3 - \pi J_{30} + \pi J_{310} + \pi J_{32} & E_{4,2} - E_{4,3} &= \omega_0 - \pi J_{10} + \pi J_{20} + \pi J_{30} \\ E_{4,2} - E_{4,6} &= \omega_2 + \pi J_{20} - \pi J_{21} + \pi J_{32} & E_{4,2} - E_{4,10} &= \omega_3 + \pi J_{30} - \pi J_{31} + \pi J_{32} \\ E_{4,3} - E_{4,7} &= \omega_2 - \pi J_{20} - \pi J_{21} + \pi J_{32} & E_{4,3} - E_{4,11} &= \omega_3 - \pi J_{30} - \pi J_{31} + \pi J_{32} \\ E_{4,4} - E_{4,5} &= \omega_0 + \pi J_{10} - \pi J_{20} + \pi J_{30} & E_{4,4} - E_{4,6} &= \omega_1 + \pi J_{10} - \pi J_{21} + \pi J_{31} \\ E_{4,4} - E_{4,12} &= \omega_3 + \pi J_{30} + \pi J_{31} - \pi J_{32} & E_{4,5} - E_{4,7} &= \omega_1 - \pi J_{10} - \pi J_{21} + \pi J_{31} \\ E_{4,5} - E_{4,13} &= \omega_3 - \pi J_{30} + \pi J_{31} - \pi J_{32} & E_{4,6} - E_{4,7} &= \omega_0 - \pi J_{10} - \pi J_{20} + \pi J_{30} \\ E_{4,6} - E_{4,14} &= \omega_3 + \pi J_{30} - \pi J_{31} - \pi J_{32} & E_{4,8} - E_{4,9} &= \omega_0 + \pi J_{10} + \pi J_{20} - \pi J_{30} \\ E_{4,8} - E_{4,10} &= \omega_1 + \pi J_{21} - \pi J_{31} + \pi J_{30} & E_{4,8} - E_{4,12} &= \omega_2 + \pi J_{20} + \pi J_{21} - \pi J_{32} \\ E_{4,9} - E_{4,11} &= \omega_1 - \pi J_{10} + \pi J_{21} - \pi J_{32} & E_{4,10} - E_{4,11} &= \omega_0 - \pi J_{10} + \pi J_{20} - \pi J_{30} \\ E_{4,10} - E_{4,14} &= \omega_2 + \pi J_{20} - \pi J_{21} - \pi J_{32} & E_{4,12} - E_{4,13} &= \omega_0 + \pi J_{10} - \pi J_{20} - \pi J_{30} \\ E_{4,12} - E_{4,14} &= \omega_1 + \pi J_{10} - \pi J_{21} - \pi J_{31} & E_{4,11} - E_{4,15} &= \omega_2 - \pi J_{20} - \pi J_{21} - \pi J_{32} \\ E_{4,13} - E_{4,15} &= \omega_1 - \pi J_{10} - \pi J_{21} - \pi J_{31} & E_{4,14} - E_{4,15} &= \omega_0 - \pi J_{10} - \pi J_{20} - \pi J_{30} \\ E_{4,9} - E_{4,13} &= \omega_2 - \pi J_{20} + \pi J_{21} - \pi J_{32} & E_{4,15} - E_{4,7} &= \omega_3 + \pi J_{30} + \pi J_{31} + \pi J_{32} \end{aligned} \tag{206}$$

maka dengan demikian suku-suku tidak nol dari matriks ρ_5 menjadi:

$$\begin{aligned}
\rho_{0,1} &= \rho_{1,0}^* = e^{-i(\omega_0 + \pi J_{10} + \pi J_{20} + \pi J_{30})t} \\
\rho_{0,2} &= \rho_{2,0}^* = e^{-i(\omega_1 + \pi J_{10} + \pi J_{21} + \pi J_{31})t} \\
\rho_{0,4} &= \rho_{4,0}^* = e^{-i(\omega_2 + \pi J_{20} + \pi J_{21} + \pi J_{32})t} \\
\rho_{0,8} &= \rho_{8,0}^* = e^{-i(\omega_3 + \pi J_{30} + \pi J_{31} + \pi J_{32})t} \\
\rho_{1,3} &= \rho_{3,1}^* = e^{-i(\omega_1 - \pi J_{10} + \pi J_{21} + \pi J_{31})t} \\
\rho_{1,5} &= \rho_{5,1}^* = e^{-i(\omega_2 - \pi J_{20} + \pi J_{21} + \pi J_{32})t} \\
\rho_{1,9} &= \rho_{9,1}^* = e^{-i(\omega_3 - \pi J_{30} + \pi J_{31} + \pi J_{32})t} \\
\rho_{2,3} &= \rho_{3,2}^* = e^{-i(\omega_0 - \pi J_{10} + \pi J_{20} + \pi J_{30})t} \\
\rho_{2,6} &= \rho_{6,2}^* = e^{-i(\omega_2 + \pi J_{20} - \pi J_{21} + \pi J_{32})t} \\
\rho_{2,10} &= \rho_{10,2}^* = e^{-i(\omega_3 + \pi J_{30} - \pi J_{31} + \pi J_{32})t} \\
\rho_{3,7} &= \rho_{7,3}^* = e^{-i(\omega_2 - \pi J_{20} - \pi J_{21} + \pi J_{32})t} \\
\rho_{3,11} &= \rho_{11,3}^* = e^{-i(\omega_3 - \pi J_{30} - \pi J_{31} + \pi J_{32})t} \\
\rho_{4,5} &= \rho_{5,4}^* = e^{-i(\omega_0 + \pi J_{10} - \pi J_{20} + \pi J_{30})t} \\
\rho_{4,6} &= \rho_{6,4}^* = e^{-i(\omega_1 + \pi J_{10} - \pi J_{21} + \pi J_{31})t} \\
\rho_{4,12} &= \rho_{12,4}^* = e^{-i(\omega_3 + \pi J_{30} + \pi J_{31} - \pi J_{32})t} \\
\rho_{5,7} &= \rho_{7,5}^* = e^{-i(\omega_1 - \pi J_{10} - \pi J_{21} + \pi J_{31})t} \\
\rho_{5,13} &= \rho_{13,5}^* = e^{-i(\omega_3 - \pi J_{30} + \pi J_{31} - \pi J_{32})t} \\
\rho_{6,7} &= \rho_{7,6}^* = e^{-i(\omega_0 - \pi J_{10} - \pi J_{20} + \pi J_{30})t} \\
\rho_{6,14} &= \rho_{14,6}^* = e^{-i(\omega_3 + \pi J_{30} - \pi J_{31} - \pi J_{32})t} \\
\rho_{8,9} &= \rho_{9,8}^* = e^{-i(\omega_0 + \pi J_{10} + \pi J_{20} - \pi J_{30})t} \\
\rho_{8,10} &= \rho_{10,8}^* = e^{-i(\omega_1 + \pi J_{21} - \pi J_{31} + \pi J_{30})t} \\
\rho_{8,12} &= \rho_{12,8}^* = e^{-i(\omega_2 + \pi J_{20} + \pi J_{21} - \pi J_{32})t} \\
\rho_{9,11} &= \rho_{11,9}^* = e^{-i(\omega_1 - \pi J_{10} + \pi J_{21} - \pi J_{32})t} \\
\rho_{10,11} &= \rho_{11,10}^* = e^{-i(\omega_0 - \pi J_{10} + \pi J_{20} - \pi J_{30})t} \\
\rho_{10,14} &= \rho_{14,10}^* = e^{-i(\omega_2 + \pi J_{20} - \pi J_{21} - \pi J_{32})t} \\
\rho_{12,13} &= \rho_{13,12}^* = e^{-i(\omega_0 + \pi J_{10} - \pi J_{20} - \pi J_{30})t} \\
\rho_{12,14} &= \rho_{14,12}^* = e^{-i(\omega_1 + \pi J_{10} - \pi J_{21} - \pi J_{31})t} \\
\rho_{11,15} &= \rho_{15,11}^* = e^{-i(\omega_2 - \pi J_{20} - \pi J_{21} - \pi J_{32})t} \\
\rho_{7,15} &= \rho_{15,7}^* = e^{-i(\omega_3 - \pi J_{30} - \pi J_{31} - \pi J_{32})t} \\
\rho_{13,15} &= \rho_{15,13}^* = e^{-i(\omega_1 - \pi J_{10} - \pi J_{21} - \pi J_{31})t} \\
\rho_{14,15} &= \rho_{15,14}^* = e^{-i(\omega_0 - \pi J_{10} - \pi J_{20} - \pi J_{30})t} \\
\rho_{9,13} &= \rho_{13,9}^* = e^{-i(\omega_2 - \pi J_{20} + \pi J_{21} - \pi J_{32})t}
\end{aligned} \tag{207}$$

maka dapat ditentukan nilai magnetisasi dari masing-masing qubitnya yaitu sebagai berikut:

$$(I_{3+}\rho_5) =$$

$$\times \frac{1}{2} \begin{pmatrix} 0 & \rho_{0,1} & \rho_{0,2} & 0 & \rho_{0,4} & 0 & 0 & 0 & -\rho_{0,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho_{1,0} & 0 & 0 & \rho_{1,3} & 0 & \rho_{1,5} & 0 & 0 & 0 & -\rho_{1,9} & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho_{2,0} & 0 & 0 & \rho_{2,3} & 0 & 0 & \rho_{2,6} & 0 & 0 & 0 & -\rho_{2,10} & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_{3,1} & \rho_{3,2} & 0 & 0 & 0 & 0 & \rho_{3,7} & 0 & 0 & -\rho_{3,11} & 0 & 0 & 0 & 0 & 0 \\ \rho_{40} & 0 & 0 & 0 & 0 & \rho_{4,5} & \rho_{4,6} & 0 & 0 & 0 & 0 & 0 & -\rho_{4,12} & 0 & 0 & 0 \\ 0 & \rho_{4,1} & 0 & 0 & \rho_{5,4} & 0 & 0 & \rho_{5,7} & 0 & 0 & 0 & 0 & 0 & -\rho_{5,13} & 0 & 0 \\ 0 & 0 & \rho_{6,2} & 0 & \rho_{6,4} & 0 & 0 & \rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{6,14} & 0 \\ 0 & 0 & 0 & \rho_{7,3} & 0 & \rho_{7,5} & \rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{7,15} \\ -\rho_{8,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{8,9} & \rho_{8,10} & 0 & \rho_{8,12} & 0 & 0 & 0 \\ 0 & -\rho_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{9,8} & 0 & 0 & \rho_{9,11} & 0 & \rho_{9,13} & 0 & 0 \\ 0 & 0 & -\rho_{10,2} & 0 & 0 & 0 & 0 & 0 & \rho_{10,8} & 0 & 0 & \rho_{10,11} & 0 & 0 & \rho_{10,14} & 0 \\ 0 & 0 & 0 & -\rho_{11,3} & 0 & 0 & 0 & 0 & 0 & \rho_{11,9} & \rho_{11,10} & 0 & 0 & 0 & 0 & \rho_{11,15} \\ 0 & 0 & 0 & 0 & -\rho_{12,4} & 0 & 0 & 0 & \rho_{12,8} & 0 & 0 & 0 & 0 & \rho_{12,13} & \rho_{12,14} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\rho_{13,5} & 0 & 0 & 0 & \rho_{13,9} & 0 & 0 & \rho_{13,12} & 0 & 0 & \rho_{13,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{14,6} & 0 & 0 & 0 & \rho_{14,10} & 0 & \rho_{14,12} & 0 & 0 & \rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{15,7} & 0 & 0 & 0 & \rho_{15,11} & 0 & \rho_{15,13} & \rho_{15,14} & 0 \end{pmatrix}$$

[illegible]

$$\begin{aligned}
M_{3+} &= \langle I_{3+} \rangle = tr (I_{3+} \rho_5) \\
&= -(\rho_{8,0} + \rho_{9,1} + \rho_{10,2} + \rho_{11,3} + \rho_{12,4} + \rho_{13,5} + \rho_{14,6} + \rho_{15,7}) \quad (208)
\end{aligned}$$

dan

$$(I_{2+} \rho_5) = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\times \frac{1}{2} \begin{pmatrix} 0 & \rho_{0,1} & \rho_{0,2} & 0 & \rho_{0,4} & 0 & 0 & 0 & -\rho_{0,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho_{1,0} & 0 & 0 & \rho_{1,3} & 0 & \rho_{1,5} & 0 & 0 & 0 & -\rho_{1,9} & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho_{2,0} & 0 & 0 & \rho_{2,3} & 0 & 0 & \rho_{2,6} & 0 & 0 & 0 & -\rho_{2,10} & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_{3,1} & \rho_{3,2} & 0 & 0 & 0 & 0 & \rho_{3,7} & 0 & 0 & -\rho_{3,11} & 0 & 0 & 0 & 0 & 0 \\ \rho_{40} & 0 & 0 & 0 & 0 & \rho_{4,5} & \rho_{4,6} & 0 & 0 & 0 & 0 & 0 & -\rho_{4,12} & 0 & 0 & 0 \\ 0 & \rho_{4,1} & 0 & 0 & \rho_{5,4} & 0 & 0 & \rho_{5,7} & 0 & 0 & 0 & 0 & 0 & -\rho_{5,13} & 0 & 0 \\ 0 & 0 & \rho_{6,2} & 0 & \rho_{6,4} & 0 & 0 & \rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{6,14} & 0 \\ 0 & 0 & 0 & \rho_{7,3} & 0 & \rho_{7,5} & \rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{7,15} \\ -\rho_{8,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{8,9} & \rho_{8,10} & 0 & \rho_{8,12} & 0 & 0 & 0 \\ 0 & -\rho_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{9,8} & 0 & 0 & \rho_{9,11} & 0 & \rho_{9,13} & 0 & 0 \\ 0 & 0 & -\rho_{10,2} & 0 & 0 & 0 & 0 & 0 & \rho_{10,8} & 0 & 0 & \rho_{10,11} & 0 & 0 & \rho_{10,14} & 0 \\ 0 & 0 & 0 & -\rho_{11,3} & 0 & 0 & 0 & 0 & 0 & \rho_{11,9} & \rho_{11,10} & 0 & 0 & 0 & 0 & \rho_{11,15} \\ 0 & 0 & 0 & 0 & -\rho_{12,4} & 0 & 0 & 0 & \rho_{12,8} & 0 & 0 & 0 & 0 & \rho_{12,13} & \rho_{12,14} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\rho_{13,5} & 0 & 0 & 0 & \rho_{13,9} & 0 & 0 & \rho_{13,12} & 0 & 0 & \rho_{13,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{14,6} & 0 & 0 & 0 & \rho_{14,10} & 0 & \rho_{14,12} & 0 & 0 & \rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{15,7} & 0 & 0 & 0 & \rho_{15,11} & 0 & \rho_{15,13} & \rho_{15,14} & 0 \end{pmatrix}$$

[illegible]

$$\begin{aligned}
M_{2+} &= \langle I_{2+} \rangle = tr(I_{2+} \rho_5) \\
&= \rho_{4,0} + \rho_{5,1} + \rho_{6,2} + \rho_{7,3} + \rho_{12,8} + \rho_{13,9} + \rho_{14,10} + \rho_{15,11}
\end{aligned} \tag{210}$$

dan

$$(I_{1+} \rho_5) = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\times \frac{1}{2} \begin{pmatrix} 0 & \rho_{0,1} & \rho_{0,2} & 0 & \rho_{0,4} & 0 & 0 & 0 & -\rho_{0,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho_{1,0} & 0 & 0 & \rho_{1,3} & 0 & \rho_{1,5} & 0 & 0 & 0 & -\rho_{1,9} & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho_{2,0} & 0 & 0 & \rho_{2,3} & 0 & 0 & \rho_{2,6} & 0 & 0 & 0 & -\rho_{2,10} & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_{3,1} & \rho_{3,2} & 0 & 0 & 0 & 0 & \rho_{3,7} & 0 & 0 & -\rho_{3,11} & 0 & 0 & 0 & 0 & 0 \\ \rho_{4,0} & 0 & 0 & 0 & 0 & \rho_{4,5} & \rho_{4,6} & 0 & 0 & 0 & 0 & 0 & -\rho_{4,12} & 0 & 0 & 0 \\ 0 & \rho_{4,1} & 0 & 0 & \rho_{5,4} & 0 & 0 & \rho_{5,7} & 0 & 0 & 0 & 0 & 0 & -\rho_{5,13} & 0 & 0 \\ 0 & 0 & \rho_{6,2} & 0 & \rho_{6,4} & 0 & 0 & \rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{6,14} & 0 \\ 0 & 0 & 0 & \rho_{7,3} & 0 & \rho_{7,5} & \rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{7,15} \\ -\rho_{8,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{8,9} & \rho_{8,10} & 0 & \rho_{8,12} & 0 & 0 & 0 \\ 0 & -\rho_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{9,8} & 0 & 0 & \rho_{9,11} & 0 & \rho_{9,13} & 0 & 0 \\ 0 & 0 & -\rho_{10,2} & 0 & 0 & 0 & 0 & 0 & \rho_{10,8} & 0 & 0 & \rho_{10,11} & 0 & 0 & \rho_{10,14} & 0 \\ 0 & 0 & 0 & -\rho_{11,3} & 0 & 0 & 0 & 0 & 0 & \rho_{11,9} & \rho_{11,10} & 0 & 0 & 0 & 0 & \rho_{11,15} \\ 0 & 0 & 0 & 0 & -\rho_{12,4} & 0 & 0 & 0 & \rho_{12,8} & 0 & 0 & 0 & 0 & \rho_{12,13} & \rho_{12,14} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\rho_{13,5} & 0 & 0 & 0 & \rho_{13,9} & 0 & 0 & \rho_{13,12} & 0 & 0 & \rho_{13,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{14,6} & 0 & 0 & 0 & \rho_{14,10} & 0 & \rho_{14,12} & 0 & 0 & \rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{15,7} & 0 & 0 & 0 & \rho_{15,11} & 0 & \rho_{15,13} & \rho_{15,14} & 0 \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} \rho_{2,0} & 0 & 0 & \rho_{2,3} & 0 & 0 & \rho_{2,6} & 0 & 0 & 0 & -\rho_{2,10} & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_{3,1} & \rho_{3,2} & 0 & 0 & 0 & 0 & \rho_{3,7} & 0 & 0 & -\rho_{3,11} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_{6,2} & 0 & \rho_{6,4} & 0 & 0 & \rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{6,14} & 0 \\ 0 & 0 & 0 & \rho_{7,3} & 0 & \rho_{7,5} & \rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{7,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\rho_{10,2} & 0 & 0 & 0 & 0 & 0 & \rho_{10,8} & 0 & 0 & \rho_{10,11} & 0 & 0 & \rho_{10,14} & 0 \\ 0 & 0 & 0 & -\rho_{11,3} & 0 & 0 & 0 & 0 & 0 & \rho_{11,9} & \rho_{11,10} & 0 & 0 & 0 & 0 & \rho_{11,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{14,6} & 0 & 0 & 0 & \rho_{14,10} & 0 & \rho_{14,12} & 0 & 0 & \rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{15,7} & 0 & 0 & 0 & \rho_{15,11} & 0 & \rho_{15,13} & \rho_{15,14} & 0 \end{pmatrix} \quad (211)$$

$$\begin{aligned}
M_{1+} &= \langle I_{2+} \rangle = tr(I_{2+} \rho_5) \\
&= \rho_{2,0} + \rho_{3,1} + \rho_{6,4} + \rho_{7,5} + \rho_{10,8} + \rho_{11,9} + \rho_{14,12} + \rho_{15,13}
\end{aligned} \tag{212}$$

dan

$$(I_{0+} \rho_5) = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\times \frac{1}{2} \begin{pmatrix} 0 & \rho_{0,1} & \rho_{0,2} & 0 & \rho_{0,4} & 0 & 0 & 0 & -\rho_{0,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho_{1,0} & 0 & 0 & \rho_{1,3} & 0 & \rho_{1,5} & 0 & 0 & 0 & -\rho_{1,9} & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho_{2,0} & 0 & 0 & \rho_{2,3} & 0 & 0 & \rho_{2,6} & 0 & 0 & 0 & -\rho_{2,10} & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_{3,1} & \rho_{3,2} & 0 & 0 & 0 & 0 & \rho_{3,7} & 0 & 0 & -\rho_{3,11} & 0 & 0 & 0 & 0 & 0 \\ \rho_{4,0} & 0 & 0 & 0 & 0 & \rho_{4,5} & \rho_{4,6} & 0 & 0 & 0 & 0 & 0 & -\rho_{4,12} & 0 & 0 & 0 \\ 0 & \rho_{4,1} & 0 & 0 & \rho_{5,4} & 0 & 0 & \rho_{5,7} & 0 & 0 & 0 & 0 & 0 & -\rho_{5,13} & 0 & 0 \\ 0 & 0 & \rho_{6,2} & 0 & \rho_{6,4} & 0 & 0 & \rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{6,14} & 0 \\ 0 & 0 & 0 & \rho_{7,3} & 0 & \rho_{7,5} & \rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{7,15} \\ -\rho_{8,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{8,9} & \rho_{8,10} & 0 & \rho_{8,12} & 0 & 0 & 0 \\ 0 & -\rho_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{9,8} & 0 & 0 & \rho_{9,11} & 0 & \rho_{9,13} & 0 & 0 \\ 0 & 0 & -\rho_{10,2} & 0 & 0 & 0 & 0 & 0 & \rho_{10,8} & 0 & 0 & \rho_{10,11} & 0 & 0 & \rho_{10,14} & 0 \\ 0 & 0 & 0 & -\rho_{11,3} & 0 & 0 & 0 & 0 & 0 & \rho_{11,9} & \rho_{11,10} & 0 & 0 & 0 & 0 & \rho_{11,15} \\ 0 & 0 & 0 & 0 & -\rho_{12,4} & 0 & 0 & 0 & \rho_{12,8} & 0 & 0 & 0 & 0 & \rho_{12,13} & \rho_{12,14} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\rho_{13,5} & 0 & 0 & 0 & \rho_{13,9} & 0 & 0 & \rho_{13,12} & 0 & 0 & \rho_{13,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{14,6} & 0 & 0 & 0 & \rho_{14,10} & 0 & \rho_{14,12} & 0 & 0 & \rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{15,7} & 0 & 0 & 0 & \rho_{15,11} & 0 & \rho_{15,13} & \rho_{15,14} & 0 \end{pmatrix}$$

$$(213)$$

$$\begin{aligned}
M_{0+} &= \langle I_{0+} \rangle = tr (I_{0+} \rho_5) \\
&= \rho_{1,0} + \rho_{3,2} + \rho_{5,2} + \rho_{7,6} + \rho_{9,8} + \rho_{11,10} + \rho_{13,12} + \rho_{15,14}
\end{aligned} \tag{214}$$

maka dengan demikian nilai magnetisasi total dari sistem adalah sebagai berikut:

$$\begin{aligned}
M_{tot} &= M_{3+} + M_{2+} + M_{1+} + M_{0+} \\
&= -\rho_{8,0} - \rho_{9,1} - \rho_{10,2} - \rho_{11,3} - \rho_{12,4} - \rho_{13,5} - \rho_{14,6} - \rho_{15,7} \\
&\quad + \rho_{4,0} + \rho_{5,1} + \rho_{6,2} + \rho_{7,3} + \rho_{12,8} + \rho_{13,9} + \rho_{14,10} + \rho_{15,11} \\
&\quad + \rho_{2,0} + \rho_{3,1} + \rho_{6,4} + \rho_{7,5} + \rho_{10,8} + \rho_{11,9} + \rho_{14,12} + \rho_{15,13} \\
&\quad + \rho_{1,0} + \rho_{3,2} + \rho_{5,4} + \rho_{7,6} + \rho_{9,8} + \rho_{11,10} + \rho_{13,12} + \rho_{15,14} \\
&= -e^{i(\omega_3 + \pi J_{30} + \pi J_{31} + \pi J_{32})t} - e^{i(\omega_3 - \pi J_{30} + \pi J_{31} + \pi J_{32})t} \\
&\quad - e^{i(\omega_3 + \pi J_{30} - \pi J_{31} + \pi J_{32})t} - e^{i(\omega_3 - \pi J_{30} - \pi J_{31} + \pi J_{32})t} \\
&\quad - e^{i(\omega_3 + \pi J_{30} + \pi J_{31} - \pi J_{32})t} - e^{i(\omega_3 - \pi J_{30} + \pi J_{31} - \pi J_{32})t} \\
&\quad - e^{i(\omega_3 + \pi J_{30} - \pi J_{31} - \pi J_{32})t} - e^{i(\omega_3 - \pi J_{30} - \pi J_{31} - \pi J_{32})t} \\
&\quad + e^{i(\omega_2 + \pi J_{20} + \pi J_{21} + \pi J_{32})t} + e^{i(\omega_2 - \pi J_{20} + \pi J_{21} + \pi J_{32})t} \\
&\quad + e^{i(\omega_2 + \pi J_{20} - \pi J_{21} + \pi J_{32})t} + e^{i(\omega_2 - \pi J_{20} - \pi J_{21} + \pi J_{32})t} \\
&\quad + e^{i(\omega_2 + \pi J_{20} + \pi J_{21} - \pi J_{32})t} + e^{i(\omega_2 - \pi J_{20} + \pi J_{21} - \pi J_{32})t} \\
&\quad + e^{i(\omega_2 + \pi J_{20} - \pi J_{21} - \pi J_{32})t} + e^{i(\omega_2 - \pi J_{20} - \pi J_{21} - \pi J_{32})t} \\
&\quad + e^{i(\omega_1 + \pi J_{10} + \pi J_{21} + \pi J_{31})t} + e^{i(\omega_1 - \pi J_{10} + \pi J_{21} + \pi J_{31})t} \\
&\quad + e^{i(\omega_1 + \pi J_{10} - \pi J_{21} + \pi J_{31})t} + e^{i(\omega_1 - \pi J_{10} - \pi J_{21} + \pi J_{31})t} \\
&\quad + e^{i(\omega_1 + \pi J_{21} - \pi J_{31} + \pi J_{30})t} + e^{i(\omega_1 - \pi J_{10} + \pi J_{21} - \pi J_{32})t} \\
&\quad + e^{i(\omega_1 + \pi J_{10} - \pi J_{21} - \pi J_{31})t} + e^{i(\omega_1 - \pi J_{10} - \pi J_{21} - \pi J_{31})t} \\
&\quad + e^{i(\omega_0 + \pi J_{10} + \pi J_{20} + \pi J_{30})t} + e^{i(\omega_0 - \pi J_{10} + \pi J_{20} + \pi J_{30})t} \\
&\quad + e^{i(\omega_0 + \pi J_{10} - \pi J_{20} + \pi J_{30})t} + e^{i(\omega_0 - \pi J_{10} - \pi J_{20} + \pi J_{30})t} \\
&\quad + e^{i(\omega_0 + \pi J_{10} + \pi J_{20} - \pi J_{30})t} + e^{i(\omega_0 - \pi J_{10} + \pi J_{20} - \pi J_{30})t} \\
&\quad + e^{i(\omega_0 + \pi J_{10} - \pi J_{20} - \pi J_{30})t} + e^{i(\omega_0 - \pi J_{10} - \pi J_{20} - \pi J_{30})t}
\end{aligned} \tag{215}$$

kemudian didapatkan juga untuk sinyal FID sebagai berikut:

$$\begin{aligned}
S &= M_{tot} e^{-R_2 t} \\
&= -e^{-(R_2 - i(\omega_3 + \pi J_{30} + \pi J_{31} + \pi J_{32}))t} - e^{-(R_2 - i(\omega_3 - \pi J_{30} + \pi J_{31} + \pi J_{32}))t} \\
&- e^{-(R_2 - i(\omega_3 + \pi J_{30} - \pi J_{31} + \pi J_{32}))t} - e^{-(R_2 - i(\omega_3 - \pi J_{30} - \pi J_{31} + \pi J_{32}))t} \\
&- e^{-(R_2 - i(\omega_3 + \pi J_{30} + \pi J_{31} - \pi J_{32}))t} - e^{-(R_2 - i(\omega_3 - \pi J_{30} + \pi J_{31} - \pi J_{32}))t} \\
&- e^{-(R_2 - i(\omega_3 + \pi J_{30} - \pi J_{31} - \pi J_{32}))t} - e^{-(R_2 - i(\omega_3 - \pi J_{30} - \pi J_{31} - \pi J_{32}))t} \\
&+ e^{-(R_2 - i(\omega_2 + \pi J_{20} + \pi J_{21} + \pi J_{32}))t} + e^{-(R_2 - i(\omega_2 - \pi J_{20} + \pi J_{21} + \pi J_{32}))t} \\
&+ e^{-(R_2 - i(\omega_2 + \pi J_{20} - \pi J_{21} + \pi J_{32}))t} + e^{-(R_2 - i(\omega_2 - \pi J_{20} - \pi J_{21} + \pi J_{32}))t} \\
&+ e^{-(R_2 - i(\omega_2 + \pi J_{20} + \pi J_{21} - \pi J_{32}))t} + e^{-(R_2 - i(\omega_2 - \pi J_{20} + \pi J_{21} - \pi J_{32}))t} \\
&+ e^{-(R_2 - i(\omega_2 + \pi J_{20} - \pi J_{21} - \pi J_{32}))t} + e^{-(R_2 - i(\omega_2 - \pi J_{20} - \pi J_{21} - \pi J_{32}))t} \quad (216) \\
&+ e^{-(R_2 - i(\omega_1 + \pi J_{10} + \pi J_{21} + \pi J_{31}))t} + e^{-(R_2 - i(\omega_1 - \pi J_{10} + \pi J_{21} + \pi J_{31}))t} \\
&+ e^{-(R_2 - i(\omega_1 + \pi J_{10} - \pi J_{21} + \pi J_{31}))t} + e^{-(R_2 - i(\omega_1 - \pi J_{10} - \pi J_{21} + \pi J_{31}))t} \\
&+ e^{-(R_2 - i(\omega_1 + \pi J_{21} - \pi J_{31} + \pi J_{30}))t} + e^{-(R_2 - i(\omega_1 - \pi J_{10} + \pi J_{21} - \pi J_{32}))t} \\
&+ e^{-(R_2 - i(\omega_1 + \pi J_{10} - \pi J_{21} - \pi J_{31}))t} + e^{-(R_2 - i(\omega_1 - \pi J_{10} - \pi J_{21} - \pi J_{31}))t} \\
&+ e^{-(R_2 - i(\omega_0 + \pi J_{10} + \pi J_{20} + \pi J_{30}))t} + e^{-(R_2 - i(\omega_0 - \pi J_{10} + \pi J_{20} + \pi J_{30}))t} \\
&+ e^{-(R_2 - i(\omega_0 + \pi J_{10} - \pi J_{20} + \pi J_{30}))t} + e^{-(R_2 - i(\omega_0 - \pi J_{10} - \pi J_{20} + \pi J_{30}))t} \\
&+ e^{-(R_2 - i(\omega_0 + \pi J_{10} + \pi J_{20} - \pi J_{30}))t} + e^{-(R_2 - i(\omega_0 - \pi J_{10} + \pi J_{20} - \pi J_{30}))t} \\
&+ e^{-(R_2 - i(\omega_0 + \pi J_{10} - \pi J_{20} - \pi J_{30}))t} + e^{-(R_2 - i(\omega_0 - \pi J_{10} - \pi J_{20} - \pi J_{30}))t}
\end{aligned}$$

dimana $R_2 = \frac{1}{T_2}$

kemudian dapat di cari terlebih dahulu untuk perkalian antara $S(t) e^{-i\omega t}$ yaitu sebagai berikut:

$$\begin{aligned}
S(t) e^{-i\omega t} = & -e^{-(R_2-i(\omega_3-\omega+\pi J_{30}+\pi J_{31}+\pi J_{32}))t} - e^{-(R_2-i(\omega_3-\omega-\pi J_{30}+\pi J_{31}+\pi J_{32}))t} \\
& - e^{-(R_2-i(\omega_3-\omega+\pi J_{30}-\pi J_{31}+\pi J_{32}))t} - e^{-(R_2-i(\omega_3-\omega-\pi J_{30}-\pi J_{31}+\pi J_{32}))t} \\
& - e^{-(R_2-i(\omega_3-\omega+\pi J_{30}+\pi J_{31}-\pi J_{32}))t} - e^{-(R_2-i(\omega_3-\omega-\pi J_{30}+\pi J_{31}-\pi J_{32}))t} \\
& - e^{-(R_2-i(\omega_3-\omega+\pi J_{30}-\pi J_{31}-\pi J_{32}))t} - e^{-(R_2-i(\omega_3-\omega-\pi J_{30}-\pi J_{31}-\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_2-\omega+\pi J_{20}+\pi J_{21}+\pi J_{32}))t} + e^{-(R_2-i(\omega_2-\omega-\pi J_{20}+\pi J_{21}+\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_2-\omega+\pi J_{20}-\pi J_{21}+\pi J_{32}))t} + e^{-(R_2-i(\omega_2-\omega-\pi J_{20}-\pi J_{21}+\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_2-\omega+\pi J_{20}+\pi J_{21}-\pi J_{32}))t} + e^{-(R_2-i(\omega_2-\omega-\pi J_{20}+\pi J_{21}-\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_2-\omega+\pi J_{20}-\pi J_{21}-\pi J_{32}))t} + e^{-(R_2-i(\omega_2-\omega-\pi J_{20}-\pi J_{21}-\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_1-\omega+\pi J_{10}+\pi J_{21}+\pi J_{31}))t} + e^{-(R_2-i(\omega_1-\omega-\pi J_{10}+\pi J_{21}+\pi J_{31}))t} \\
& + e^{-(R_2-i(\omega_1-\omega+\pi J_{10}-\pi J_{21}+\pi J_{31}))t} + e^{-(R_2-i(\omega_1-\omega-\pi J_{10}-\pi J_{21}+\pi J_{31}))t} \\
& + e^{-(R_2-i(\omega_1-\omega+\pi J_{21}-\pi J_{31}+\pi J_{30}))t} + e^{-(R_2-i(\omega_1-\omega-\pi J_{10}+\pi J_{21}-\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_1-\omega+\pi J_{10}-\pi J_{21}-\pi J_{31}))t} + e^{-(R_2-i(\omega_1-\omega-\pi J_{10}-\pi J_{21}-\pi J_{31}))t} \\
& + e^{-(R_2-i(\omega_0-\omega+\pi J_{10}+\pi J_{20}+\pi J_{30}))t} + e^{-(R_2-i(\omega_0-\omega-\pi J_{10}+\pi J_{20}+\pi J_{30}))t} \\
& + e^{-(R_2-i(\omega_0-\omega+\pi J_{10}-\pi J_{20}+\pi J_{30}))t} + e^{-(R_2-i(\omega_0-\omega-\pi J_{10}-\pi J_{20}+\pi J_{30}))t} \\
& + e^{-(R_2-i(\omega_0-\omega+\pi J_{10}+\pi J_{20}-\pi J_{30}))t} + e^{-(R_2-i(\omega_0-\omega-\pi J_{10}+\pi J_{20}-\pi J_{30}))t} \\
& + e^{-(R_2-i(\omega_0-\omega+\pi J_{10}-\pi J_{20}-\pi J_{30}))t} + e^{-(R_2-i(\omega_0-\omega-\pi J_{10}-\pi J_{20}-\pi J_{30}))t}
\end{aligned} \tag{217}$$

Transformasi Fourier dari sinyal FiD adalah sebagai berikut:

[illegible]

$$\begin{aligned}
&= \frac{-1}{(R_2 - i(\omega_3 - \omega + \pi J_{30} + \pi J_{31} + \pi J_{32}))} - \frac{1}{(R_2 - i(\omega_3 - \omega - \pi J_{30} + \pi J_{31} + \pi J_{32}))} \\
&- \frac{1}{(R_2 - i(\omega_3 - \omega + \pi J_{30} - \pi J_{31} + \pi J_{32}))} - \frac{1}{(R_2 - i(\omega_3 - \omega - \pi J_{30} - \pi J_{31} + \pi J_{32}))} \\
&- \frac{1}{(R_2 - i(\omega_3 - \omega - \pi J_{30} + \pi J_{31} - \pi J_{32}))} - \frac{1}{(R_2 - i(\omega_3 - \omega - \pi J_{30} - \pi J_{31} - \pi J_{32}))} \\
&- \frac{1}{(R_2 - i(\omega_3 - \omega + \pi J_{30} - \pi J_{31} - \pi J_{32}))} - \frac{1}{(R_2 - i(\omega_3 - \omega - \pi J_{30} - \pi J_{31} - \pi J_{32}))} \\
&+ \frac{1}{(R_2 - i(\omega_2 - \omega + \pi J_{20} + \pi J_{21} + \pi J_{32}))} + \frac{1}{(R_2 - i(\omega_2 - \omega - \pi J_{20} + \pi J_{21} + \pi J_{32}))} \\
&+ \frac{1}{(R_2 - i(\omega_2 - \omega + \pi J_{20} - \pi J_{21} + \pi J_{32}))} + \frac{1}{(R_2 - i(\omega_2 - \omega - \pi J_{20} - \pi J_{21} + \pi J_{32}))} \\
&+ \frac{1}{(R_2 - i(\omega_2 - \omega + \pi J_{20} + \pi J_{21} - \pi J_{32}))} + \frac{1}{(R_2 - i(\omega_2 - \omega - \pi J_{20} + \pi J_{21} - \pi J_{32}))} \\
&+ \frac{1}{(R_2 - i(\omega_2 - \omega + \pi J_{20} - \pi J_{21} - \pi J_{32}))} + \frac{1}{(R_2 - i(\omega_2 - \omega - \pi J_{20} - \pi J_{21} - \pi J_{32}))} \\
&+ \frac{1}{(R_2 - i(\omega_1 - \omega + \pi J_{10} + \pi J_{21} + \pi J_{31}))} + \frac{1}{(R_2 - i(\omega_1 - \omega - \pi J_{10} + \pi J_{21} + \pi J_{31}))} \\
&+ \frac{1}{(R_2 - i(\omega_1 - \omega + \pi J_{10} - \pi J_{21} + \pi J_{31}))} + \frac{1}{(R_2 - i(\omega_1 - \omega - \pi J_{10} - \pi J_{21} + \pi J_{31}))} \\
&+ \frac{1}{(R_2 - i(\omega_1 - \omega + \pi J_{21} - \pi J_{31} + \pi J_{30}))} + \frac{1}{(R_2 - i(\omega_1 - \omega - \pi J_{10} + \pi J_{21} - \pi J_{32}))} \\
&+ \frac{1}{(R_2 - i(\omega_1 - \omega + \pi J_{10} - \pi J_{21} - \pi J_{31}))} + \frac{1}{(R_2 - i(\omega_1 - \omega - \pi J_{10} - \pi J_{21} - \pi J_{31}))} \\
&+ \frac{1}{(R_2 - i(\omega_0 - \omega + \pi J_{10} + \pi J_{20} + \pi J_{30}))} + \frac{1}{(R_2 - i(\omega_0 - \omega - \pi J_{10} + \pi J_{20} + \pi J_{30}))} \\
&+ \frac{1}{(R_2 - i(\omega_0 - \omega + \pi J_{10} - \pi J_{20} + \pi J_{30}))} + \frac{1}{(R_2 - i(\omega_0 - \omega - \pi J_{10} - \pi J_{20} + \pi J_{30}))} \\
&+ \frac{1}{(R_2 - i(\omega_0 - \omega + \pi J_{10} + \pi J_{20} - \pi J_{30}))} + \frac{1}{(R_2 - i(\omega_0 - \omega - \pi J_{10} + \pi J_{20} - \pi J_{30}))} \\
&+ \frac{1}{(R_2 - i(\omega_0 - \omega + \pi J_{10} - \pi J_{20} - \pi J_{30}))} + \frac{1}{(R_2 - i(\omega_0 - \omega - \pi J_{10} - \pi J_{20} - \pi J_{30}))}
\end{aligned}$$

[illegible]

maka dengan demikian diperoleh :

$$\begin{aligned}
A(\omega) = & -\frac{R_2}{R_2^2 + (\omega_3 - \omega + \pi J_{30} + \pi J_{31} + \pi J_{32})^2} - \frac{R_2}{R_2^2 + (\omega_3 - \omega - \pi J_{30} + \pi J_{31} + \pi J_{32})^2} \\
& - \frac{R_2}{R_2^2 - (\omega_3 - \omega + \pi J_{30} - \pi J_{31} + \pi J_{32})^2} - \frac{R_2}{R_2^2 + (\omega_3 - \omega - \pi J_{30} - \pi J_{31} + \pi J_{32})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_3 - \omega - \pi J_{30} - \pi J_{31} + \pi J_{32})^2} - \frac{R_2}{R_2^2 + (\omega_3 - \omega - \pi J_{30} + \pi J_{31} - \pi J_{32})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_3 - \omega + \pi J_{30} - \pi J_{31} - \pi J_{32})^2} - \frac{R_2}{R_2^2 + (\omega_3 - \omega - \pi J_{30} - \pi J_{31} - \pi J_{32})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_2 - \omega + \pi J_{20} + \pi J_{21} + \pi J_{32})^2} + \frac{R_2}{R_2^2 + (\omega_2 - \omega - \pi J_{20} + \pi J_{21} + \pi J_{32})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_2 - \omega + \pi J_{20} - \pi J_{21} + \pi J_{32})^2} + \frac{R_2}{R_2^2 + (\omega_2 - \omega - \pi J_{20} - \pi J_{21} + \pi J_{32})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_2 - \omega + \pi J_{20} + \pi J_{21} - \pi J_{32})^2} + \frac{R_2}{R_2^2 + (\omega_2 - \omega - \pi J_{20} + \pi J_{21} - \pi J_{32})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_2 - \omega + \pi J_{20} - \pi J_{21} - \pi J_{32})^2} + \frac{R_2}{R_2^2 + (\omega_2 - \omega - \pi J_{20} - \pi J_{21} - \pi J_{32})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_1 - \omega + \pi J_{10} + \pi J_{21} + \pi J_{31})^2} + \frac{R_2}{R_2^2 + (\omega_1 - \omega - \pi J_{10} + \pi J_{21} + \pi J_{31})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_1 - \omega + \pi J_{10} - \pi J_{21} + \pi J_{31})^2} + \frac{R_2}{R_2^2 + (\omega_1 - \omega - \pi J_{10} - \pi J_{21} + \pi J_{31})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_1 - \omega + \pi J_{21} - \pi J_{31} + \pi J_{30})^2} + \frac{R_2}{R_2^2 + (\omega_1 - \omega - \pi J_{10} + \pi J_{21} - \pi J_{32})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_1 - \omega + \pi J_{10} - \pi J_{21} - \pi J_{31})^2} + \frac{R_2}{R_2^2 + (\omega_1 - \omega - \pi J_{10} - \pi J_{21} - \pi J_{31})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_0 - \omega + \pi J_{10} + \pi J_{20} + \pi J_{30})^2} + \frac{R_2}{R_2^2 + (\omega_0 - \omega - \pi J_{10} + \pi J_{20} + \pi J_{30})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_0 - \omega + \pi J_{10} - \pi J_{20} + \pi J_{30})^2} + \frac{R_2}{R_2^2 + (\omega_0 - \omega - \pi J_{10} - \pi J_{20} + \pi J_{30})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_0 - \omega + \pi J_{10} + \pi J_{20} - \pi J_{30})^2} + \frac{R_2}{R_2^2 + (\omega_0 - \omega - \pi J_{10} + \pi J_{20} - \pi J_{30})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_0 - \omega + \pi J_{10} - \pi J_{20} - \pi J_{30})^2} + \frac{R_2}{R_2^2 + (\omega_0 - \omega - \pi J_{10} - \pi J_{20} - \pi J_{30})^2}
\end{aligned} \tag{218}$$

dan diperoleh juga:

[illegible]

berdasarkan persamaan (182) maka didapatkan grafik sebagai berikut:

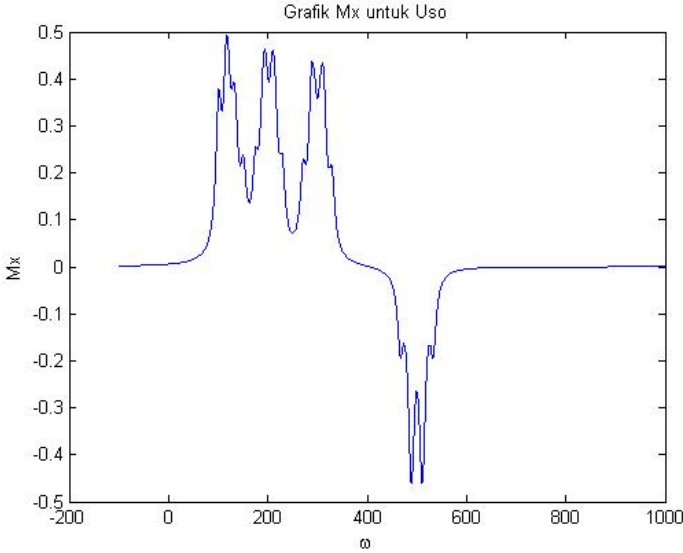


Figure 3: Grafik untuk U_{s0}

data yang dipakai untuk masing-masing qubit adalah sebesar $\omega_3 = 500MHz$, $\omega_2 = 300MHz$, $\omega_1 = 202MHz$, $\omega_0 = 126MHz$, $J_{32} = 12$, $J_{31} = 11$, $J_{21} = 10$, $J_{30} = 9$, $J_{20} = 8$, $J_{10} = 7$, $R_2 = 7$. untuk $u_2 = U_{s0} = \sigma_z \otimes I \otimes I \otimes I$ berdasarkan grafik M_x untuk U_{s0} maka didapatkan ada 3 multiplet mengarah keatas dan 1 multiplet mengarah ke bawah dan dapat diketahui bahwa multiplet yang mengarah keatas adalah qubit-3, qubit-2, dan qubit-1 sedangkan multiplet yang mengarah ke bawah adalah qubit-0, sehingga hal ini mengindikasikan keadaan down-up-up-up atau $|0001\rangle$. berdasarkan grafik tersebut diketahui bahwa 2 puncak mengarah mengikuti arah 4 puncak multiplet sehingga tidak terjadi keadaan terbelit untuk $U_2 = U_{s0}$

Dengan cara yang serupa dihasilkan komponen magnetisasi terhadap sumbu-x untuk fungsi-fungsi lainnya, berikut

adalah grafik-grafiknya:

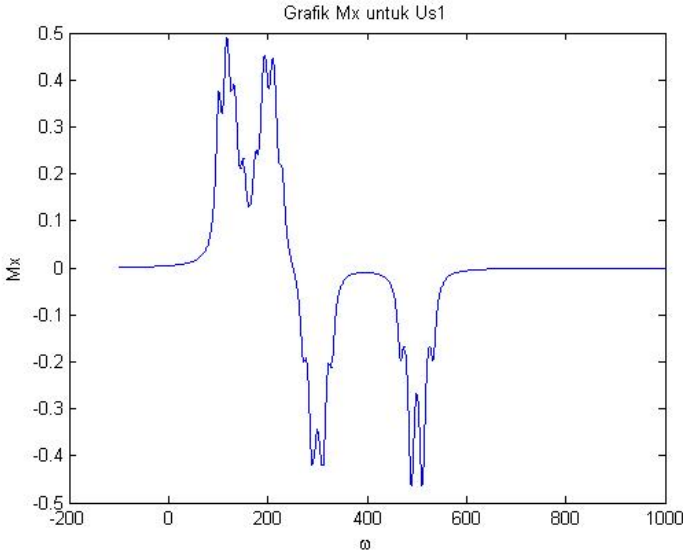


Figure 4: Grafik untuk U_{s1}

untuk $u_2 = U_{s1} = I \otimes I \otimes \sigma_z \otimes \sigma_z$ dapat diketahui berdasarkan grafik diatas bahwa ada 2 multiplet yang mengarah keatas dan 2 yang lain mengarah kebawah. Sehingga didapatkan keadaan up-up-down-down, secara berturut-turut berikut adalah qubit yang mengalami keadaan tersebut yaitu: qubit-3(up), qubit-2(up), qubit-1(down) dan qubit-0 (down), sehingga keadaan dari sistem tersebut adalah $|0011\rangle$. Berdasarkan arah 2 puncak pada masing-masing multiplet menunjukkan keadaan terbelit juga belum terjadi untuk $U_2 = U_{s1}$

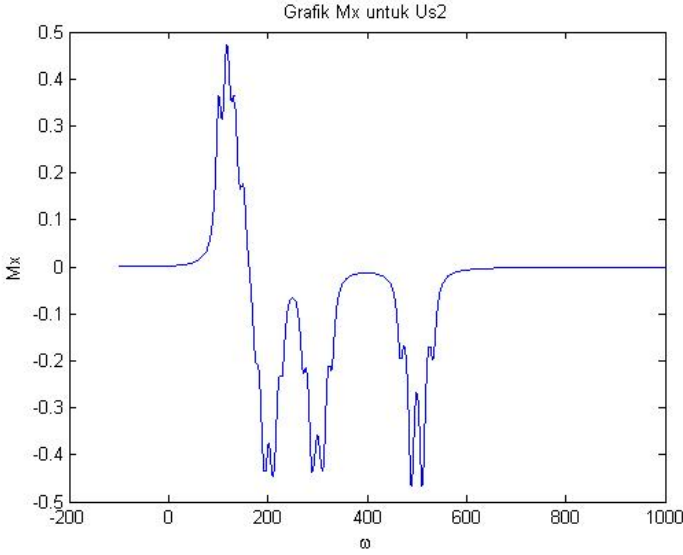


Figure 5: Grafik untuk U_{s2}

berdasarkan Grafik M_x untuk U_{s2} maka dapat diketahui bahwa ada 1 buah qubit dalam keadaan up dan 3 qubit yang lainnya dalam keadaan down, sehingga secara berturut-turut qubit-qubit tersebut adalah qubit-3(up), qubit-2(down), qubit-1(down), qubit-0(down). Maka dengan demikian grafik tersebut menunjukkan sistem dengan keadaan $|0111\rangle$, Selain itu berdasarkan arah 2 puncak dalam masing-masing multipletnya menunjukkan arah yang sama dengan arah puncak multipletnya, maka diketahui belum terjadi keadaan terbelit untuk sistem $u_2 = U_{s2} = I \otimes \sigma_z \otimes \sigma_z \otimes \sigma_z$

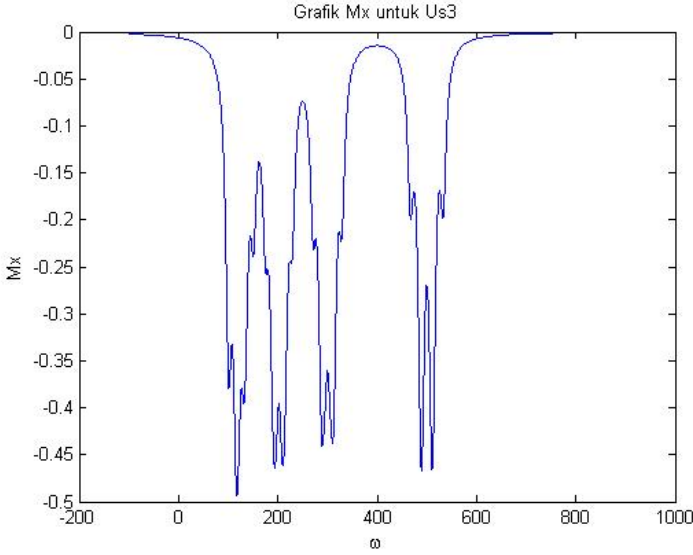


Figure 6: Grafik untuk U_{s3}

berdasarkan Grafik M_x untuk U_{s3} dapat diketahui bahwa keempat puncak multiblet berada di bawah nol, hal ini menunjukkan bahwa semua qubit mengalami keadaan down, sehingga sistem ini menunjukkan keadaan $|1111\rangle$, dan berdasarkan arah 2 puncak dari masing-masing multipletnya mengikuti arah puncak multiplet sehingga tidak terjadi keadaan terbelit untuk keadaan $u_2 = U_{s3} = \sigma_z \otimes \sigma_z \otimes \sigma_z \otimes \sigma_z$

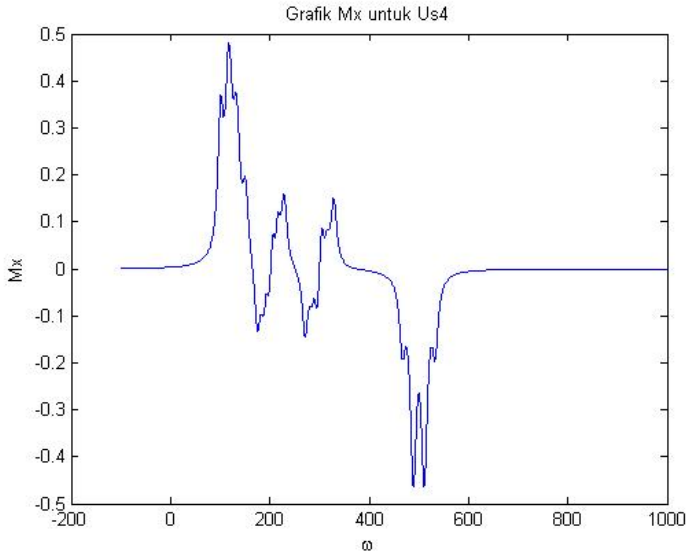


Figure 7: Grafik untuk U_{s4}

berdasarkan Grafik M_x untuk U_{s4} dapat diketahui bahwa 1 buah qubit dalam keadaan up, 2 buah qubit dalam keadaan yang tercampur, dan 1 buah qubit dalam keadaan down. untuk 2 buah qubit yang dalam keadaan campuran adalah qubit-2 dan qubit-1, sehingga keadaan terbelit sudah terjadi untuk sistem dengan $U_{s4} = I \otimes ([I \otimes \sigma_z] \oplus [\sigma_z \otimes \sigma_z])$

”Halaman ini sengaja dikosongkan”

BAB VI

KESIMPULAN

Berdasarkan Analisa Penerapan Algoritma Deutsch-Jozsa dalam sistem 4 qubit diperoleh kesimpulan sebagai berikut:

1. Didapatkan spektrum dlam sistem 4 qubit terdiri dari 4 puncak
2. setiap puncak menggambarkan keadaan dari masing-masing qubit, dari kiri ke kanan secara berurutan merupakan puncak qubit ke 3,2,1 dan 0.
3. karakterisasi puncak up dan down menggambarkan keadaan spin up dan spin down.
4. berdasarkan Grafik Mx untuk $U_{k0}, U_{s1}, U_{s2}, U_{s3}, U_{s4}$ menggambarkan keadaan dari ψ_{out} dari masing-masing fungsi yaitu sebagai berikut:

$$\begin{aligned}
 |\psi_{k0}\rangle &= \frac{1}{\sqrt{2}} |0000\rangle [|0\rangle - |1\rangle] \\
 |\psi_{s0}\rangle &= \frac{1}{\sqrt{2}} |1000\rangle [|0\rangle - |1\rangle] \\
 |\psi_{s1}\rangle &= \frac{1}{\sqrt{2}} |0011\rangle [|0\rangle - |1\rangle] \\
 |\psi_{s2}\rangle &= \frac{1}{\sqrt{2}} |0111\rangle [|0\rangle - |1\rangle] \\
 |\psi_{s3}\rangle &= \frac{1}{\sqrt{2}} |1111\rangle [|0\rangle - |1\rangle]
 \end{aligned} \tag{220}$$

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lampiran

7.1 Penerapan Algoritma Deutsch jozsa 4 qubit untuk $U_2 = U_5$

apabila diambil kasus untuk $U_2 = I \otimes I \otimes \sigma_z \otimes \sigma_z$ maka diperoleh untuk bentuk operator uniter totalnya adalah sebagai berikut:

$$U = U_5 U_2 U_1$$

$$= U_5 (\sigma_z \otimes I \otimes I \otimes I) U_1$$

$$= \begin{pmatrix} e^{\beta_0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & e^{\beta_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{\beta_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{\beta_3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{\beta_4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{\beta_5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_9} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_{10}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_{11}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_{12}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_{13}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_{14}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_{15}} \end{pmatrix}$$

[illegible]

[illegible]

maka dengan demikian kita dapatkan untuk nilai ρ_5 adalah

[illegible]

×

[illegible]

$$= \frac{1}{16} \begin{pmatrix} 0 & -8\rho_{0,1} & -8\rho_{0,2} & 0 & 8\rho_{0,4} & 0 & 0 & 0 & -8\rho_{0,8} & 0 & 0 & 0 & 0 & 0 & 0 \\ -8\rho_{1,0} & 0 & 0 & -8\rho_{1,3} & 0 & 8\rho_{1,5} & 0 & 0 & 0 & -8\rho_{1,9} & 0 & 0 & 0 & 0 & 0 \\ -8\rho_{2,0} & 0 & 0 & -8\rho_{2,3} & 0 & 0 & 8\rho_{2,6} & 0 & 0 & 0 & -8\rho_{2,10} & 0 & 0 & 0 & 0 \\ 0 & -8\rho_{3,1} & -8\rho_{3,2} & 0 & 0 & 0 & 0 & 8\rho_{3,7} & 0 & 0 & 0 & -8\rho_{3,11} & 0 & 0 & 0 \\ 8\rho_{40} & 0 & 0 & 0 & 0 & -8\rho_{4,5} & -8\rho_{4,6} & 0 & 0 & 0 & 0 & 0 & 8\rho_{4,12} & 0 & 0 \\ 0 & 8\rho_{4,1} & 0 & 0 & -8\rho_{5,4} & 0 & 0 & -8\rho_{5,7} & 0 & 0 & 0 & 0 & 0 & 8\rho_{5,13} & 0 \\ 0 & 0 & 8\rho_{6,2} & 0 & -8\rho_{6,4} & 0 & 0 & -8\rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & 8\rho_{6,14} \\ 0 & 0 & 0 & 8\rho_{7,3} & 0 & -8\rho_{7,5} & -8\rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -8\rho_{7,15} \\ -8\rho_{8,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -8\rho_{8,9} & -8\rho_{8,10} & 0 & 8\rho_{8,12} & 0 & 0 \\ 0 & -8\rho_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & -8\rho_{9,8} & 0 & 0 & -8\rho_{9,11} & 0 & 8\rho_{9,13} & 0 \\ 0 & 0 & -8\rho_{10,2} & 0 & 0 & 0 & 0 & 0 & -8\rho_{10,8} & 0 & 0 & -8\rho_{10,11} & 0 & 0 & 8\rho_{10,14} \\ 0 & 0 & 0 & -8\rho_{11,3} & 0 & 0 & 0 & 0 & 0 & -8\rho_{11,9} & -8\rho_{11,10} & 0 & 0 & 0 & 8\rho_{11,15} \\ 0 & 0 & 0 & 0 & -8\rho_{12,4} & 0 & 0 & 0 & 8\rho_{12,8} & 0 & 0 & 0 & 0 & -8\rho_{12,13} & -8\rho_{12,14} \\ 0 & 0 & 0 & 0 & 0 & -8\rho_{13,5} & 0 & 0 & 0 & 8\rho_{13,9} & 0 & 0 & -8\rho_{13,12} & 0 & -8\rho_{13,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & -8\rho_{14,6} & 0 & 0 & 0 & 8\rho_{14,10} & 0 & -8\rho_{14,12} & 0 & -8\rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -8\rho_{15,7} & 0 & 0 & 0 & 8\rho_{15,11} & 0 & -8\rho_{15,13} & -8\rho_{15,14} \end{pmatrix} \quad (22)$$

$$= \frac{1}{2} \begin{pmatrix} 0 & -\rho_{0,1} & -\rho_{0,2} & 0 & \rho_{0,4} & 0 & 0 & 0 & -\rho_{0,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\rho_{1,0} & 0 & 0 & -\rho_{1,3} & 0 & \rho_{1,5} & 0 & 0 & 0 & -\rho_{1,9} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\rho_{2,0} & 0 & 0 & -\rho_{2,3} & 0 & 0 & \rho_{2,6} & 0 & 0 & 0 & -\rho_{2,10} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\rho_{3,1} & -\rho_{3,2} & 0 & 0 & 0 & 0 & \rho_{3,7} & 0 & 0 & 0 & -\rho_{3,11} & 0 & 0 & 0 & 0 \\ \rho_{40} & 0 & 0 & 0 & 0 & -\rho_{4,5} & -\rho_{4,6} & 0 & 0 & 0 & 0 & 0 & \rho_{4,12} & 0 & 0 & 0 \\ 0 & \rho_{4,1} & 0 & 0 & -\rho_{5,4} & 0 & 0 & -\rho_{5,7} & 0 & 0 & 0 & 0 & 0 & \rho_{5,13} & 0 & 0 \\ 0 & 0 & \rho_{6,2} & 0 & -\rho_{6,4} & 0 & 0 & -\rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{6,14} & 0 \\ 0 & 0 & 0 & \rho_{7,3} & 0 & -\rho_{7,5} & -\rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{7,15} \\ -\rho_{8,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{8,9} & -\rho_{8,10} & 0 & \rho_{8,12} & 0 & 0 & 0 \\ 0 & -\rho_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{9,8} & 0 & 0 & -\rho_{9,11} & 0 & \rho_{9,13} & 0 & 0 \\ 0 & 0 & -\rho_{10,2} & 0 & 0 & 0 & 0 & 0 & -\rho_{10,8} & 0 & 0 & -\rho_{10,11} & 0 & 0 & \rho_{10,14} & 0 \\ 0 & 0 & 0 & -\rho_{11,3} & 0 & 0 & 0 & 0 & 0 & -\rho_{11,9} & -\rho_{11,10} & 0 & 0 & 0 & 0 & \rho_{11,15} \\ 0 & 0 & 0 & 0 & -\rho_{12,4} & 0 & 0 & 0 & \rho_{12,8} & 0 & 0 & 0 & 0 & -\rho_{12,13} & -\rho_{12,14} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\rho_{13,5} & 0 & 0 & 0 & \rho_{13,9} & 0 & 0 & -\rho_{13,12} & 0 & 0 & -\rho_{13,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{14,6} & 0 & 0 & 0 & \rho_{14,10} & 0 & -\rho_{14,12} & 0 & 0 & -\rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{15,7} & 0 & 0 & 0 & \rho_{15,11} & 0 & -\rho_{15,13} & -\rho_{15,14} & 0 \end{pmatrix} \quad (224)$$

dengan

$$\begin{aligned}
\rho_{0,1} &= \rho_{1,0}^* = e^{\beta_0 - \beta_1} & \rho_{6,7} &= \rho_{7,6}^* = e^{\beta_6 - \beta_7} \\
\rho_{0,2} &= \rho_{2,0}^* = e^{\beta_0 - \beta_2} & \rho_{6,14} &= \rho_{14,6}^* = e^{\beta_6 - \beta_{14}} \\
\rho_{0,8} &= \rho_{8,0}^* = e^{\beta_8 - \beta_1} & \rho_{8,9} &= \rho_{9,8}^* = e^{\beta_8 - \beta_9} \\
\rho_{1,3} &= \rho_{3,1}^* = e^{\beta_1 - \beta_3} & \rho_{8,10} &= \rho_{10,8}^* = e^{\beta_8 - \beta_{10}} \\
\rho_{1,5} &= \rho_{5,1}^* = e^{\beta_1 - \beta_5} & \rho_{8,12} &= \rho_{12,8}^* = e^{\beta_8 - \beta_{12}} \\
\rho_{1,9} &= \rho_{9,1}^* = e^{\beta_1 - \beta_9} & \rho_{9,11} &= \rho_{11,9}^* = e^{\beta_9 - \beta_{11}} \\
\rho_{2,3} &= \rho_{3,2}^* = e^{\beta_2 - \beta_3} & \rho_{9,13} &= \rho_{13,9}^* = e^{\beta_9 - \beta_{13}} \\
\rho_{2,6} &= \rho_{6,2}^* = e^{\beta_2 - \beta_6} & \rho_{10,11} &= \rho_{11,10}^* = e^{\beta_{10} - \beta_{11}} \\
\rho_{2,10} &= \rho_{10,2}^* = e^{\beta_2 - \beta_{10}} & \rho_{10,14} &= \rho_{14,10}^* = e^{\beta_{10} - \beta_{14}} \\
\rho_{3,7} &= \rho_{7,3}^* = e^{\beta_3 - \beta_7} & \rho_{12,13} &= \rho_{13,12}^* = e^{\beta_{12} - \beta_{13}} \\
\rho_{3,11} &= \rho_{11,3}^* = e^{\beta_3 - \beta_{11}} & \rho_{12,14} &= \rho_{14,12}^* = e^{\beta_{12} - \beta_{14}} \\
\rho_{4,5} &= \rho_{5,4}^* = e^{\beta_4 - \beta_5} & \rho_{11,15} &= \rho_{15,11}^* = e^{\beta_{11} - \beta_{15}} \\
\rho_{4,6} &= \rho_{6,4}^* = e^{\beta_4 - \beta_6} & \rho_{13,15} &= \rho_{15,13}^* = e^{\beta_{13} - \beta_{15}} \\
\rho_{4,12} &= \rho_{12,4}^* = e^{\beta_4 - \beta_{12}} & \rho_{14,15} &= \rho_{15,14}^* = e^{\beta_{14} - \beta_{15}} \\
\rho_{5,7} &= \rho_{7,5}^* = e^{\beta_5 - \beta_7} & \rho_{15,7} &= \rho_{7,15}^* = e^{\beta_7 - \beta_{15}} \\
\rho_{5,13} &= \rho_{13,5}^* = e^{\beta_5 - \beta_{13}} & \rho_{0,4} &= \rho_{4,0}^* = e^{\beta_0 - \beta_4}
\end{aligned} \tag{225}$$

dan

$$\begin{aligned}
\beta_0 - \beta_1 &= -i(E_{4,0} - E_{4,1})t & \beta_0 - \beta_2 &= -i(E_{4,0} - E_{4,2})t \\
\beta_0 - \beta_4 &= -i(E_{4,0} - E_{4,4})t & \beta_0 - \beta_8 &= -i(E_{4,0} - E_{4,8})t \\
\beta_1 - \beta_3 &= -i(E_{4,1} - E_{4,3})t & \beta_1 - \beta_5 &= -i(E_{4,1} - E_{4,5})t \\
\beta_1 - \beta_9 &= -i(E_{4,1} - E_{4,9})t & \beta_2 - \beta_3 &= -i(E_{4,2} - E_{4,3})t \\
\beta_2 - \beta_6 &= -i(E_{4,2} - E_{4,6})t & \beta_2 - \beta_{10} &= -i(E_{4,2} - E_{4,10})t \\
\beta_3 - \beta_7 &= -i(E_{4,3} - E_{4,7})t & \beta_3 - \beta_{11} &= -i(E_{4,3} - E_{4,11})t \\
\beta_4 - \beta_5 &= -i(E_{4,4} - E_{4,5})t & \beta_4 - \beta_6 &= -i(E_{4,4} - E_{4,6})t \\
\beta_4 - \beta_{12} &= -i(E_{4,4} - E_{4,12})t & \beta_5 - \beta_7 &= -i(E_{4,5} - E_{4,7})t \\
\beta_5 - \beta_{13} &= -i(E_{4,5} - E_{4,13})t & \beta_6 - \beta_7 &= -i(E_{4,6} - E_{4,7})t \\
\beta_6 - \beta_{14} &= -i(E_{4,6} - E_{4,14})t & \beta_8 - \beta_9 &= -i(E_{4,8} - E_{4,9})t \\
\beta_8 - \beta_{10} &= -i(E_{4,8} - E_{4,10})t & \beta_8 - \beta_{12} &= -i(E_{4,8} - E_{4,12})t \\
\beta_9 - \beta_{11} &= -i(E_{4,9} - E_{4,11})t & \beta_9 - \beta_{13} &= -i(E_{4,9} - E_{4,13})t \\
\beta_{10} - \beta_{11} &= -i(E_{4,10} - E_{4,11})t & \beta_{10} - \beta_{14} &= -i(E_{4,10} - E_{4,14})t \\
\beta_{12} - \beta_{13} &= -i(E_{4,12} - E_{4,13})t & \beta_{12} - \beta_{14} &= -i(E_{4,12} - E_{4,14})t \\
\beta_{11} - \beta_{15} &= -i(E_{4,11} - E_{4,15})t & \beta_{13} - \beta_{15} &= -i(E_{4,13} - E_{4,15})t \\
\beta_{14} - \beta_{15} &= -i(E_{4,14} - E_{4,15})t & \beta_{15} - \beta_7 &= -i(E_{4,15} - E_{4,7})t
\end{aligned} \tag{226}$$

dan

$$\begin{aligned}
E_{4,0} - E_{4,1} &= \omega_0 + \pi J_{10} + \pi J_{20} + \pi J_{30} & E_{4,0} - E_{4,2} &= \omega_1 + \pi J_{10} + \pi J_{21} + \pi J_{31} \\
E_{4,0} - E_{4,4} &= \omega_2 + \pi J_{20} + \pi J_{21} + \pi J_{32} & E_{4,0} - E_{4,8} &= \omega_3 + \pi J_{30} + \pi J_{31} + \pi J_{32} \\
E_{4,1} - E_{4,3} &= \omega_1 - \pi J_{10} + \pi J_{21} + \pi J_{31} & E_{4,1} - E_{4,5} &= \omega_2 - \pi J_{20} + \pi J_{21} + \pi J_{32} \\
E_{4,1} - E_{4,9} &= \omega_3 - \pi J_{30} + \pi J_{310} + \pi J_{32} & E_{4,2} - E_{4,3} &= \omega_0 - \pi J_{10} + \pi J_{20} + \pi J_{30} \\
E_{4,2} - E_{4,6} &= \omega_2 + \pi J_{20} - \pi J_{21} + \pi J_{32} & E_{4,2} - E_{4,10} &= \omega_3 + \pi J_{30} - \pi J_{31} + \pi J_{32} \\
E_{4,3} - E_{4,7} &= \omega_2 - \pi J_{20} - \pi J_{21} + \pi J_{32} & E_{4,3} - E_{4,11} &= \omega_3 - \pi J_{30} - \pi J_{31} + \pi J_{32} \\
E_{4,4} - E_{4,5} &= \omega_0 + \pi J_{10} - \pi J_{20} + \pi J_{30} & E_{4,4} - E_{4,6} &= \omega_1 + \pi J_{10} - \pi J_{21} + \pi J_{31} \\
E_{4,4} - E_{4,12} &= \omega_3 + \pi J_{30} + \pi J_{31} - \pi J_{32} & E_{4,5} - E_{4,7} &= \omega_1 - \pi J_{10} - \pi J_{21} + \pi J_{31} \\
E_{4,5} - E_{4,13} &= \omega_3 - \pi J_{30} + \pi J_{31} - \pi J_{32} & E_{4,6} - E_{4,7} &= \omega_0 - \pi J_{10} - \pi J_{20} + \pi J_{30} \\
E_{4,6} - E_{4,14} &= \omega_3 + \pi J_{30} - \pi J_{31} - \pi J_{32} & E_{4,8} - E_{4,9} &= \omega_0 + \pi J_{10} + \pi J_{20} - \pi J_{30} \\
E_{4,8} - E_{4,10} &= \omega_1 + \pi J_{21} - \pi J_{31} + \pi J_{30} & E_{4,8} - E_{4,12} &= \omega_2 + \pi J_{20} + \pi J_{21} - \pi J_{32} \\
E_{4,9} - E_{4,11} &= \omega_1 - \pi J_{10} + \pi J_{21} - \pi J_{32} & E_{4,10} - E_{4,11} &= \omega_0 - \pi J_{10} + \pi J_{20} - \pi J_{30} \\
E_{4,10} - E_{4,14} &= \omega_2 + \pi J_{20} - \pi J_{21} - \pi J_{32} & E_{4,12} - E_{4,13} &= \omega_0 + \pi J_{10} - \pi J_{20} - \pi J_{30} \\
E_{4,12} - E_{4,14} &= \omega_1 + \pi J_{10} - \pi J_{21} - \pi J_{31} & E_{4,11} - E_{4,15} &= \omega_2 - \pi J_{20} - \pi J_{21} - \pi J_{32} \\
E_{4,13} - E_{4,15} &= \omega_1 - \pi J_{10} - \pi J_{21} - \pi J_{31} & E_{4,14} - E_{4,15} &= \omega_0 - \pi J_{10} - \pi J_{20} - \pi J_{30} \\
E_{4,9} - E_{4,13} &= \omega_2 - \pi J_{20} + \pi J_{21} - \pi J_{32} & E_{4,15} - E_{4,7} &= \omega_3 + \pi J_{30} + \pi J_{31} + \pi J_{32}
\end{aligned} \tag{227}$$

maka dapat ditentukan nilai magnetisasi dari masing-masing qubitnya yaitu sebagai berikut:

$$(I_{3+}\rho_5) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\times \frac{1}{2} \begin{pmatrix} 0 & -\rho_{0,1} & -\rho_{0,2} & 0 & \rho_{0,4} & 0 & 0 & 0 & -\rho_{0,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\rho_{1,0} & 0 & 0 & -\rho_{1,3} & 0 & \rho_{1,5} & 0 & 0 & 0 & -\rho_{1,9} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\rho_{2,0} & 0 & 0 & -\rho_{2,3} & 0 & 0 & \rho_{2,6} & 0 & 0 & 0 & -\rho_{2,10} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\rho_{3,1} & -\rho_{3,2} & 0 & 0 & 0 & 0 & \rho_{3,7} & 0 & 0 & 0 & -\rho_{3,11} & 0 & 0 & 0 & 0 \\ \rho_{40} & 0 & 0 & 0 & 0 & -\rho_{4,5} & -\rho_{4,6} & 0 & 0 & 0 & 0 & 0 & \rho_{4,12} & 0 & 0 & 0 \\ 0 & \rho_{4,1} & 0 & 0 & -\rho_{5,4} & 0 & 0 & -\rho_{5,7} & 0 & 0 & 0 & 0 & 0 & \rho_{5,13} & 0 & 0 \\ 0 & 0 & \rho_{6,2} & 0 & -\rho_{6,4} & 0 & 0 & -\rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{6,14} & 0 \\ 0 & 0 & 0 & \rho_{7,3} & 0 & -\rho_{7,5} & -\rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{7,15} \\ -\rho_{8,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{8,9} & -\rho_{8,10} & 0 & \rho_{8,12} & 0 & 0 & 0 \\ 0 & -\rho_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{9,8} & 0 & 0 & -\rho_{9,11} & 0 & \rho_{9,13} & 0 & 0 \\ 0 & 0 & -\rho_{10,2} & 0 & 0 & 0 & 0 & 0 & -\rho_{10,8} & 0 & 0 & -\rho_{10,11} & 0 & 0 & \rho_{10,14} & 0 \\ 0 & 0 & 0 & -\rho_{11,3} & 0 & 0 & 0 & 0 & 0 & -\rho_{11,9} & -\rho_{11,10} & 0 & 0 & 0 & 0 & \rho_{11,15} \\ 0 & 0 & 0 & 0 & -\rho_{12,4} & 0 & 0 & 0 & \rho_{12,8} & 0 & 0 & 0 & 0 & -\rho_{12,13} & -\rho_{12,14} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\rho_{13,5} & 0 & 0 & 0 & \rho_{13,9} & 0 & 0 & -\rho_{13,12} & 0 & 0 & -\rho_{13,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{14,6} & 0 & 0 & 0 & \rho_{14,10} & 0 & -\rho_{14,12} & 0 & 0 & -\rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{15,7} & 0 & 0 & 0 & \rho_{15,11} & 0 & -\rho_{15,13} & -\rho_{15,14} & 0 \end{pmatrix}$$

[illegible]

$$\begin{aligned}
M_{3+} &= \langle I_{3+} \rangle = \text{tr} (I_{3+} \rho_5) \\
&= \rho_{8,0} + \rho_{9,1} + \rho_{10,2} + \rho_{11,3} + \rho_{12,4} + \rho_{13,5} + \rho_{14,6} + \rho_{15,7}
\end{aligned} \tag{228}$$

dan

$$(I_{2+} \rho_5) = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\times \frac{1}{2} \begin{pmatrix} 0 & -\rho_{0,1} & -\rho_{0,2} & 0 & \rho_{0,4} & 0 & 0 & 0 & -\rho_{0,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\rho_{1,0} & 0 & 0 & -\rho_{1,3} & 0 & \rho_{1,5} & 0 & 0 & 0 & -\rho_{1,9} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\rho_{2,0} & 0 & 0 & -\rho_{2,3} & 0 & 0 & \rho_{2,6} & 0 & 0 & 0 & -\rho_{2,10} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\rho_{3,1} & -\rho_{3,2} & 0 & 0 & 0 & 0 & \rho_{3,7} & 0 & 0 & 0 & -\rho_{3,11} & 0 & 0 & 0 & 0 \\ \rho_{40} & 0 & 0 & 0 & 0 & -\rho_{4,5} & -\rho_{4,6} & 0 & 0 & 0 & 0 & 0 & \rho_{4,12} & 0 & 0 & 0 \\ 0 & \rho_{4,1} & 0 & 0 & -\rho_{5,4} & 0 & 0 & -\rho_{5,7} & 0 & 0 & 0 & 0 & 0 & \rho_{5,13} & 0 & 0 \\ 0 & 0 & \rho_{6,2} & 0 & -\rho_{6,4} & 0 & 0 & -\rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{6,14} & 0 \\ 0 & 0 & 0 & \rho_{7,3} & 0 & -\rho_{7,5} & -\rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{7,15} \\ -\rho_{8,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{8,9} & -\rho_{8,10} & 0 & \rho_{8,12} & 0 & 0 & 0 \\ 0 & -\rho_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{9,8} & 0 & 0 & -\rho_{9,11} & 0 & \rho_{9,13} & 0 & 0 \\ 0 & 0 & -\rho_{10,2} & 0 & 0 & 0 & 0 & 0 & -\rho_{10,8} & 0 & 0 & -\rho_{10,11} & 0 & 0 & \rho_{10,14} & 0 \\ 0 & 0 & 0 & -\rho_{11,3} & 0 & 0 & 0 & 0 & 0 & -\rho_{11,9} & -\rho_{11,10} & 0 & 0 & 0 & 0 & \rho_{11,15} \\ 0 & 0 & 0 & 0 & -\rho_{12,4} & 0 & 0 & 0 & \rho_{12,8} & 0 & 0 & 0 & 0 & -\rho_{12,13} & -\rho_{12,14} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\rho_{13,5} & 0 & 0 & 0 & \rho_{13,9} & 0 & 0 & -\rho_{13,12} & 0 & 0 & -\rho_{13,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{14,6} & 0 & 0 & 0 & \rho_{14,10} & 0 & -\rho_{14,12} & 0 & 0 & -\rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{15,7} & 0 & 0 & 0 & \rho_{15,11} & 0 & -\rho_{15,13} & -\rho_{15,14} & 0 \end{pmatrix}$$

[illegible]

$$\begin{aligned}
M_{2+} &= \langle I_{2+} \rangle = tr(I_{2+} \rho_5) \\
&= \rho_{4,0} + \rho_{5,1} + \rho_{6,2} + \rho_{7,3} + \rho_{12,8} + \rho_{13,9} + \rho_{14,10} + \rho_{15,11}
\end{aligned} \tag{230}$$

dan

$$(I_{1+} \rho_5) = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\times \frac{1}{2} \begin{pmatrix} 0 & -\rho_{0,1} & -\rho_{0,2} & 0 & \rho_{0,4} & 0 & 0 & 0 & -\rho_{0,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\rho_{1,0} & 0 & 0 & -\rho_{1,3} & 0 & \rho_{1,5} & 0 & 0 & 0 & -\rho_{1,9} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\rho_{2,0} & 0 & 0 & -\rho_{2,3} & 0 & 0 & \rho_{2,6} & 0 & 0 & 0 & -\rho_{2,10} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\rho_{3,1} & -\rho_{3,2} & 0 & 0 & 0 & 0 & \rho_{3,7} & 0 & 0 & 0 & -\rho_{3,11} & 0 & 0 & 0 & 0 \\ \rho_{40} & 0 & 0 & 0 & 0 & -\rho_{4,5} & -\rho_{4,6} & 0 & 0 & 0 & 0 & 0 & \rho_{4,12} & 0 & 0 & 0 \\ 0 & \rho_{4,1} & 0 & 0 & -\rho_{5,4} & 0 & 0 & -\rho_{5,7} & 0 & 0 & 0 & 0 & 0 & \rho_{5,13} & 0 & 0 \\ 0 & 0 & \rho_{6,2} & 0 & -\rho_{6,4} & 0 & 0 & -\rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{6,14} & 0 \\ 0 & 0 & 0 & \rho_{7,3} & 0 & -\rho_{7,5} & -\rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{7,15} \\ -\rho_{8,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{8,9} & -\rho_{8,10} & 0 & \rho_{8,12} & 0 & 0 & 0 \\ 0 & -\rho_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{9,8} & 0 & 0 & -\rho_{9,11} & 0 & \rho_{9,13} & 0 & 0 \\ 0 & 0 & -\rho_{10,2} & 0 & 0 & 0 & 0 & 0 & -\rho_{10,8} & 0 & 0 & -\rho_{10,11} & 0 & 0 & \rho_{10,14} & 0 \\ 0 & 0 & 0 & -\rho_{11,3} & 0 & 0 & 0 & 0 & 0 & -\rho_{11,9} & -\rho_{11,10} & 0 & 0 & 0 & 0 & \rho_{11,15} \\ 0 & 0 & 0 & 0 & -\rho_{12,4} & 0 & 0 & 0 & \rho_{12,8} & 0 & 0 & 0 & 0 & -\rho_{12,13} & -\rho_{12,14} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\rho_{13,5} & 0 & 0 & 0 & \rho_{13,9} & 0 & 0 & -\rho_{13,12} & 0 & 0 & -\rho_{13,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{14,6} & 0 & 0 & 0 & \rho_{14,10} & 0 & -\rho_{14,12} & 0 & 0 & -\rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{15,7} & 0 & 0 & 0 & \rho_{15,11} & 0 & -\rho_{15,13} & -\rho_{15,14} & 0 \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} -\rho_{2,0} & 0 & 0 & -\rho_{2,3} & 0 & 0 & \rho_{2,6} & 0 & 0 & 0 & \rho_{2,10} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\rho_{3,1} & -\rho_{3,2} & 0 & 0 & 0 & 0 & \rho_{3,7} & 0 & 0 & \rho_{3,11} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_{6,2} & 0 & -\rho_{6,4} & 0 & 0 & -\rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{6,14} & 0 \\ 0 & 0 & 0 & \rho_{7,3} & 0 & -\rho_{7,5} & -\rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{7,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_{10,2} & 0 & 0 & 0 & 0 & 0 & -\rho_{10,8} & 0 & 0 & -\rho_{10,11} & 0 & 0 & \rho_{10,14} & 0 \\ 0 & 0 & 0 & \rho_{11,3} & 0 & 0 & 0 & 0 & 0 & -\rho_{11,9} & -\rho_{11,10} & 0 & 0 & 0 & 0 & \rho_{11,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho_{14,6} & 0 & 0 & 0 & \rho_{14,10} & 0 & -\rho_{14,12} & 0 & 0 & -\rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{15,7} & 0 & 0 & 0 & \rho_{15,11} & 0 & -\rho_{15,13} & -\rho_{15,14} & 0 \end{pmatrix} \quad (231)$$

$$\begin{aligned}
M_{1+} &= \langle I_{1+} \rangle = tr(I_{1+} \rho_5) \\
&= -(\rho_{2,0} + \rho_{3,1} + \rho_{6,4} + \rho_{7,5} + \rho_{10,8} + \rho_{11,9} + \rho_{14,12} + \rho_{15,13}) \quad (232)
\end{aligned}$$

dan

$$(I_{0+} \rho_5) = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\times \frac{1}{2} \begin{pmatrix} 0 & -\rho_{0,1} & -\rho_{0,2} & 0 & \rho_{0,4} & 0 & 0 & 0 & -\rho_{0,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\rho_{1,0} & 0 & 0 & -\rho_{1,3} & 0 & \rho_{1,5} & 0 & 0 & 0 & -\rho_{1,9} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\rho_{2,0} & 0 & 0 & -\rho_{2,3} & 0 & 0 & \rho_{2,6} & 0 & 0 & 0 & -\rho_{2,10} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\rho_{3,1} & -\rho_{3,2} & 0 & 0 & 0 & 0 & \rho_{3,7} & 0 & 0 & 0 & -\rho_{3,11} & 0 & 0 & 0 & 0 \\ \rho_{40} & 0 & 0 & 0 & 0 & -\rho_{4,5} & -\rho_{4,6} & 0 & 0 & 0 & 0 & 0 & \rho_{4,12} & 0 & 0 & 0 \\ 0 & \rho_{4,1} & 0 & 0 & -\rho_{5,4} & 0 & 0 & -\rho_{5,7} & 0 & 0 & 0 & 0 & 0 & \rho_{5,13} & 0 & 0 \\ 0 & 0 & \rho_{6,2} & 0 & -\rho_{6,4} & 0 & 0 & -\rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{6,14} & 0 \\ 0 & 0 & 0 & \rho_{7,3} & 0 & -\rho_{7,5} & -\rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{7,15} \\ -\rho_{8,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{8,9} & -\rho_{8,10} & 0 & \rho_{8,12} & 0 & 0 & 0 \\ 0 & -\rho_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{9,8} & 0 & 0 & -\rho_{9,11} & 0 & \rho_{9,13} & 0 & 0 \\ 0 & 0 & -\rho_{10,2} & 0 & 0 & 0 & 0 & 0 & -\rho_{10,8} & 0 & 0 & -\rho_{10,11} & 0 & 0 & \rho_{10,14} & 0 \\ 0 & 0 & 0 & -\rho_{11,3} & 0 & 0 & 0 & 0 & 0 & -\rho_{11,9} & -\rho_{11,10} & 0 & 0 & 0 & 0 & \rho_{11,15} \\ 0 & 0 & 0 & 0 & -\rho_{12,4} & 0 & 0 & 0 & \rho_{12,8} & 0 & 0 & 0 & 0 & -\rho_{12,13} & -\rho_{12,14} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\rho_{13,5} & 0 & 0 & 0 & \rho_{13,9} & 0 & 0 & -\rho_{13,12} & 0 & 0 & -\rho_{13,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{14,6} & 0 & 0 & 0 & \rho_{14,10} & 0 & -\rho_{14,12} & 0 & 0 & -\rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{15,7} & 0 & 0 & 0 & \rho_{15,11} & 0 & -\rho_{15,13} & -\rho_{15,14} & 0 \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} -\rho_{1,0} & 0 & 0 & -\rho_{1,3} & 0 & \rho_{1,5} & 0 & 0 & 0 & \rho_{1,9} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\rho_{3,1} & -\rho_{3,2} & 0 & 0 & 0 & 0 & \rho_{3,7} & 0 & 0 & \rho_{3,11} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_{5,1} & 0 & 0 & -\rho_{5,4} & 0 & 0 & -\rho_{5,7} & 0 & 0 & 0 & 0 & 0 & \rho_{5,13} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho_{7,3} & 0 & -\rho_{7,5} & -\rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{7,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{9,8} & 0 & 0 & -\rho_{9,11} & 0 & \rho_{9,13} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\rho_{11,3} & 0 & 0 & 0 & 0 & 0 & -\rho_{11,9} & -\rho_{11,10} & 0 & 0 & 0 & 0 & \rho_{11,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\rho_{13,5} & 0 & 0 & 0 & \rho_{13,9} & 0 & 0 & -\rho_{13,12} & 0 & 0 & -\rho_{13,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{15,7} & 0 & 0 & 0 & \rho_{15,11} & 0 & -\rho_{15,13} & -\rho_{15,14} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

(233)

$$M_{0+} = \langle I_{0+} \rangle = tr (I_{0+} \rho_5) \quad (234)$$

$$= -(\rho_{1,0} + \rho_{3,2} + \rho_{5,2} + \rho_{7,6} + \rho_{9,8} + \rho_{11,10} + \rho_{13,12} + \rho_{15,14})$$

maka dengan demikian nilai magnetisasi total dari sistem adalah sebagai berikut:

$$M_{tot} = M_{3+} + M_{2+} + M_{1+} + M_{0+}$$

$$= \rho_{8,0} + \rho_{9,1} + \rho_{10,2} + \rho_{11,3} + \rho_{12,4} + \rho_{13,5} + \rho_{14,6} + \rho_{15,7}$$

$$+ \rho_{4,0} + \rho_{5,1} + \rho_{6,2} + \rho_{7,3} + \rho_{12,8} + \rho_{13,9} + \rho_{14,10} + \rho_{15,11}$$

$$- (\rho_{2,0} + \rho_{3,1} + \rho_{6,4} + \rho_{7,5} + \rho_{10,8} + \rho_{11,9} + \rho_{14,12} + \rho_{15,13})$$

$$- (\rho_{1,0} + \rho_{3,2} + \rho_{5,4} + \rho_{7,6} + \rho_{9,8} + \rho_{11,10} + \rho_{13,12} + \rho_{15,14})$$

$$= + e^{i(\omega_3 + \pi J_{30} + \pi J_{31} + \pi J_{32})t} + e^{i(\omega_3 - \pi J_{30} + \pi J_{31} + \pi J_{32})t}$$

$$+ e^{i(\omega_3 + \pi J_{30} - \pi J_{31} + \pi J_{32})t} + e^{i(\omega_3 - \pi J_{30} - \pi J_{31} + \pi J_{32})t}$$

$$+ e^{i(\omega_3 + \pi J_{30} + \pi J_{31} - \pi J_{32})t} + e^{i(\omega_3 - \pi J_{30} + \pi J_{31} - \pi J_{32})t}$$

$$+ e^{i(\omega_3 + \pi J_{30} - \pi J_{31} - \pi J_{32})t} + e^{i(\omega_3 - \pi J_{30} - \pi J_{31} - \pi J_{32})t}$$

$$+ e^{i(\omega_2 + \pi J_{20} + \pi J_{21} + \pi J_{32})t} + e^{i(\omega_2 - \pi J_{20} + \pi J_{21} + \pi J_{32})t}$$

$$+ e^{i(\omega_2 + \pi J_{20} - \pi J_{21} + \pi J_{32})t} + e^{i(\omega_2 - \pi J_{20} - \pi J_{21} + \pi J_{32})t} \quad (235)$$

$$+ e^{i(\omega_2 + \pi J_{20} + \pi J_{21} - \pi J_{32})t} + e^{i(\omega_2 - \pi J_{20} + \pi J_{21} - \pi J_{32})t}$$

$$+ e^{i(\omega_2 + \pi J_{20} - \pi J_{21} - \pi J_{32})t} + e^{i(\omega_2 - \pi J_{20} - \pi J_{21} - \pi J_{32})t}$$

$$- e^{i(\omega_1 + \pi J_{10} + \pi J_{21} + \pi J_{31})t} - e^{i(\omega_1 - \pi J_{10} + \pi J_{21} + \pi J_{31})t}$$

$$- e^{i(\omega_1 + \pi J_{10} - \pi J_{21} + \pi J_{31})t} - e^{i(\omega_1 - \pi J_{10} - \pi J_{21} + \pi J_{31})t}$$

$$- e^{i(\omega_1 + \pi J_{21} - \pi J_{31} + \pi J_{30})t} - e^{i(\omega_1 - \pi J_{10} + \pi J_{21} - \pi J_{32})t}$$

$$- e^{i(\omega_1 + \pi J_{10} - \pi J_{21} - \pi J_{31})t} - e^{i(\omega_1 - \pi J_{10} - \pi J_{21} - \pi J_{31})t}$$

$$- e^{i(\omega_0 + \pi J_{10} + \pi J_{20} + \pi J_{30})t} - e^{i(\omega_0 - \pi J_{10} + \pi J_{20} + \pi J_{30})t}$$

$$- e^{i(\omega_0 + \pi J_{10} - \pi J_{20} + \pi J_{30})t} - e^{i(\omega_0 - \pi J_{10} - \pi J_{20} + \pi J_{30})t}$$

$$- e^{i(\omega_0 + \pi J_{10} + \pi J_{20} - \pi J_{30})t} - e^{i(\omega_0 - \pi J_{10} + \pi J_{20} - \pi J_{30})t}$$

$$- e^{i(\omega_0 + \pi J_{10} - \pi J_{20} - \pi J_{30})t} - e^{i(\omega_0 - \pi J_{10} - \pi J_{20} - \pi J_{30})t}$$

kemudian didapatkan juga untuk sinyal FID sebagai berikut:

$$\begin{aligned}
S = M_{tot} e^{-R_2 t} &= e^{-(R_2 - i(\omega_3 + \pi J_{30} + \pi J_{31} + \pi J_{32}))t} + e^{-(R_2 - i(\omega_3 - \pi J_{30} + \pi J_{31} + \pi J_{32}))t} \\
&+ e^{-(R_2 - i(\omega_3 + \pi J_{30} - \pi J_{31} + \pi J_{32}))t} + e^{-(R_2 - i(\omega_3 - \pi J_{30} - \pi J_{31} + \pi J_{32}))t} \\
&+ e^{-(R_2 - i(\omega_3 + \pi J_{30} + \pi J_{31} - \pi J_{32}))t} + e^{-(R_2 - i(\omega_3 - \pi J_{30} + \pi J_{31} - \pi J_{32}))t} \\
&e^{-(R_2 - i(\omega_3 + \pi J_{30} - \pi J_{31} - \pi J_{32}))t} + e^{-(R_2 - i(\omega_3 - \pi J_{30} - \pi J_{31} - \pi J_{32}))t} \\
&+ e^{-(R_2 - i(\omega_2 + \pi J_{20} + \pi J_{21} + \pi J_{32}))t} + e^{-(R_2 - i(\omega_2 - \pi J_{20} + \pi J_{21} + \pi J_{32}))t} \\
&+ e^{-(R_2 - i(\omega_2 + \pi J_{20} - \pi J_{21} + \pi J_{32}))t} + e^{-(R_2 - i(\omega_2 - \pi J_{20} - \pi J_{21} + \pi J_{32}))t} \\
&+ e^{-(R_2 - i(\omega_2 + \pi J_{20} + \pi J_{21} - \pi J_{32}))t} + e^{-(R_2 - i(\omega_2 - \pi J_{20} + \pi J_{21} - \pi J_{32}))t} \\
&+ e^{-(R_2 - i(\omega_2 + \pi J_{20} - \pi J_{21} - \pi J_{32}))t} + e^{-(R_2 - i(\omega_2 - \pi J_{20} - \pi J_{21} - \pi J_{32}))t} \quad (236) \\
&- e^{-(R_2 - i(\omega_1 + \pi J_{10} + \pi J_{21} + \pi J_{31}))t} - e^{-(R_2 - i(\omega_1 - \pi J_{10} + \pi J_{21} + \pi J_{31}))t} \\
&- e^{-(R_2 - i(\omega_1 + \pi J_{10} - \pi J_{21} + \pi J_{31}))t} - e^{-(R_2 - i(\omega_1 - \pi J_{10} - \pi J_{21} + \pi J_{31}))t} \\
&- e^{-(R_2 - i(\omega_1 + \pi J_{21} - \pi J_{31} + \pi J_{30}))t} - e^{-(R_2 - i(\omega_1 - \pi J_{10} + \pi J_{21} - \pi J_{32}))t} \\
&- e^{-(R_2 - i(\omega_1 + \pi J_{10} - \pi J_{21} - \pi J_{31}))t} - e^{-(R_2 - i(\omega_1 - \pi J_{10} - \pi J_{21} - \pi J_{31}))t} \\
&- e^{-(R_2 - i(\omega_0 + \pi J_{10} + \pi J_{20} + \pi J_{30}))t} - e^{-(R_2 - i(\omega_0 - \pi J_{10} + \pi J_{20} + \pi J_{30}))t} \\
&- e^{-(R_2 - i(\omega_0 + \pi J_{10} - \pi J_{20} + \pi J_{30}))t} - e^{-(R_2 - i(\omega_0 - \pi J_{10} - \pi J_{20} + \pi J_{30}))t} \\
&- e^{-(R_2 - i(\omega_0 + \pi J_{10} + \pi J_{20} - \pi J_{30}))t} - e^{-(R_2 - i(\omega_0 - \pi J_{10} + \pi J_{20} - \pi J_{30}))t} \\
&- e^{-(R_2 - i(\omega_0 + \pi J_{10} - \pi J_{20} - \pi J_{30}))t} - e^{-(R_2 - i(\omega_0 - \pi J_{10} - \pi J_{20} - \pi J_{30}))t}
\end{aligned}$$

dimana $R_2 = \frac{1}{T_2}$

kemudian dapat di cari terlebih dahulu untuk perkalian antara $S(t) e^{-i\omega t}$ yaitu sebagai berikut:

$$\begin{aligned}
S(t) e^{-i\omega t} = & e^{-(R_2-i(\omega_3-\omega+\pi J_{30}+\pi J_{31}+\pi J_{32}))t} + e^{-(R_2-i(\omega_3-\omega-\pi J_{30}+\pi J_{31}+\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_3-\omega+\pi J_{30}-\pi J_{31}+\pi J_{32}))t} + e^{-(R_2-i(\omega_3-\omega-\pi J_{30}-\pi J_{31}+\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_3-\omega+\pi J_{30}+\pi J_{31}-\pi J_{32}))t} + e^{-(R_2-i(\omega_3-\omega-\pi J_{30}+\pi J_{31}-\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_3-\omega+\pi J_{30}-\pi J_{31}-\pi J_{32}))t} + e^{-(R_2-i(\omega_3-\omega-\pi J_{30}-\pi J_{31}-\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_2-\omega+\pi J_{20}+\pi J_{21}+\pi J_{32}))t} + e^{-(R_2-i(\omega_2-\omega-\pi J_{20}+\pi J_{21}+\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_2-\omega+\pi J_{20}-\pi J_{21}+\pi J_{32}))t} + e^{-(R_2-i(\omega_2-\omega-\pi J_{20}-\pi J_{21}+\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_2-\omega+\pi J_{20}+\pi J_{21}-\pi J_{32}))t} + e^{-(R_2-i(\omega_2-\omega-\pi J_{20}+\pi J_{21}-\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_2-\omega+\pi J_{20}-\pi J_{21}-\pi J_{32}))t} + e^{-(R_2-i(\omega_2-\omega-\pi J_{20}-\pi J_{21}-\pi J_{32}))t} \\
& - e^{-(R_2-i(\omega_1-\omega+\pi J_{10}+\pi J_{21}+\pi J_{31}))t} - e^{-(R_2-i(\omega_1-\omega-\pi J_{10}+\pi J_{21}+\pi J_{31}))t} \\
& - e^{-(R_2-i(\omega_1-\omega+\pi J_{10}-\pi J_{21}+\pi J_{31}))t} - e^{-(R_2-i(\omega_1-\omega-\pi J_{10}-\pi J_{21}+\pi J_{31}))t} \\
& - e^{-(R_2-i(\omega_1-\omega+\pi J_{21}-\pi J_{31}+\pi J_{30}))t} - e^{-(R_2-i(\omega_1-\omega-\pi J_{10}+\pi J_{21}-\pi J_{32}))t} \\
& - e^{-(R_2-i(\omega_1-\omega+\pi J_{10}-\pi J_{21}-\pi J_{31}))t} - e^{-(R_2-i(\omega_1-\omega-\pi J_{10}-\pi J_{21}-\pi J_{31}))t} \\
& - e^{-(R_2-i(\omega_0-\omega+\pi J_{10}+\pi J_{20}+\pi J_{30}))t} - e^{-(R_2-i(\omega_0-\omega-\pi J_{10}+\pi J_{20}+\pi J_{30}))t} \\
& - e^{-(R_2-i(\omega_0-\omega+\pi J_{10}-\pi J_{20}+\pi J_{30}))t} - e^{-(R_2-i(\omega_0-\omega-\pi J_{10}-\pi J_{20}+\pi J_{30}))t} \\
& - e^{-(R_2-i(\omega_0-\omega+\pi J_{10}+\pi J_{20}-\pi J_{30}))t} - e^{-(R_2-i(\omega_0-\omega-\pi J_{10}+\pi J_{20}-\pi J_{30}))t} \\
& - e^{-(R_2-i(\omega_0-\omega+\pi J_{10}-\pi J_{20}-\pi J_{30}))t} - e^{-(R_2-i(\omega_0-\omega-\pi J_{10}-\pi J_{20}-\pi J_{30}))t}
\end{aligned} \tag{237}$$

Transformasi Fourier dari sinyal FiD adalah sebagai berikut:

[illegible]

$$\begin{aligned}
&= \frac{-1}{(R_2 - i(\omega_3 - \omega + \pi J_{30} + \pi J_{31} + \pi J_{32}))} - \frac{1}{(R_2 - i(\omega_3 - \omega - \pi J_{30} + \pi J_{31} + \pi J_{32}))} \\
&- \frac{1}{(R_2 - i(\omega_3 - \omega + \pi J_{30} - \pi J_{31} + \pi J_{32}))} - \frac{1}{(R_2 - i(\omega_3 - \omega - \pi J_{30} - \pi J_{31} + \pi J_{32}))} \\
&- \frac{1}{(R_2 - i(\omega_3 - \omega - \pi J_{30} + \pi J_{31} - \pi J_{32}))} - \frac{1}{(R_2 - i(\omega_3 - \omega - \pi J_{30} - \pi J_{31} - \pi J_{32}))} \\
&- \frac{1}{(R_2 - i(\omega_3 - \omega + \pi J_{30} - \pi J_{31} - \pi J_{32}))} - \frac{1}{(R_2 - i(\omega_3 - \omega - \pi J_{30} - \pi J_{31} - \pi J_{32}))} \\
&+ \frac{1}{(R_2 - i(\omega_2 - \omega + \pi J_{20} + \pi J_{21} + \pi J_{32}))} + \frac{1}{(R_2 - i(\omega_2 - \omega - \pi J_{20} + \pi J_{21} + \pi J_{32}))} \\
&+ \frac{1}{(R_2 - i(\omega_2 - \omega + \pi J_{20} - \pi J_{21} + \pi J_{32}))} + \frac{1}{(R_2 - i(\omega_2 - \omega - \pi J_{20} - \pi J_{21} + \pi J_{32}))} \\
&+ \frac{1}{(R_2 - i(\omega_2 - \omega + \pi J_{20} + \pi J_{21} - \pi J_{32}))} + \frac{1}{(R_2 - i(\omega_2 - \omega - \pi J_{20} + \pi J_{21} - \pi J_{32}))} \\
&+ \frac{1}{(R_2 - i(\omega_2 - \omega + \pi J_{20} - \pi J_{21} - \pi J_{32}))} + \frac{1}{(R_2 - i(\omega_2 - \omega - \pi J_{20} - \pi J_{21} - \pi J_{32}))} \\
&+ \frac{1}{(R_2 - i(\omega_1 - \omega + \pi J_{10} + \pi J_{21} + \pi J_{31}))} + \frac{1}{(R_2 - i(\omega_1 - \omega - \pi J_{10} + \pi J_{21} + \pi J_{31}))} \\
&+ \frac{1}{(R_2 - i(\omega_1 - \omega + \pi J_{10} - \pi J_{21} + \pi J_{31}))} + \frac{1}{(R_2 - i(\omega_1 - \omega - \pi J_{10} - \pi J_{21} + \pi J_{31}))} \\
&+ \frac{1}{(R_2 - i(\omega_1 - \omega + \pi J_{21} - \pi J_{31} + \pi J_{30}))} + \frac{1}{(R_2 - i(\omega_1 - \omega - \pi J_{10} + \pi J_{21} - \pi J_{32}))} \\
&+ \frac{1}{(R_2 - i(\omega_1 - \omega + \pi J_{10} - \pi J_{21} - \pi J_{31}))} + \frac{1}{(R_2 - i(\omega_1 - \omega - \pi J_{10} - \pi J_{21} - \pi J_{31}))} \\
&+ \frac{1}{(R_2 - i(\omega_0 - \omega + \pi J_{10} + \pi J_{20} + \pi J_{30}))} + \frac{1}{(R_2 - i(\omega_0 - \omega - \pi J_{10} + \pi J_{20} + \pi J_{30}))} \\
&+ \frac{1}{(R_2 - i(\omega_0 - \omega + \pi J_{10} - \pi J_{20} + \pi J_{30}))} + \frac{1}{(R_2 - i(\omega_0 - \omega - \pi J_{10} - \pi J_{20} + \pi J_{30}))} \\
&+ \frac{1}{(R_2 - i(\omega_0 - \omega + \pi J_{10} + \pi J_{20} - \pi J_{30}))} + \frac{1}{(R_2 - i(\omega_0 - \omega - \pi J_{10} + \pi J_{20} - \pi J_{30}))} \\
&+ \frac{1}{(R_2 - i(\omega_0 - \omega + \pi J_{10} - \pi J_{20} - \pi J_{30}))} + \frac{1}{(R_2 - i(\omega_0 - \omega - \pi J_{10} - \pi J_{20} - \pi J_{30}))}
\end{aligned}$$

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maka dengan demikian diperoleh :

$$\begin{aligned}
A(\omega) = & \frac{R_2}{R_2^2 + (\omega_3 - \omega + \pi J_{30} + \pi J_{31} + \pi J_{32})^2} + \frac{R_2}{R_2^2 + (\omega_3 - \omega - \pi J_{30} + \pi J_{31} + \pi J_{32})^2} \\
& + \frac{R_2}{R_2^2 - (\omega_3 - \omega + \pi J_{30} - \pi J_{31} + \pi J_{32})^2} + \frac{R_2}{R_2^2 + (\omega_3 - \omega - \pi J_{30} - \pi J_{31} + \pi J_{32})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_3 - \omega - \pi J_{30} - \pi J_{31} + \pi J_{32})^2} + \frac{R_2}{R_2^2 + (\omega_3 - \omega - \pi J_{30} + \pi J_{31} - \pi J_{32})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_3 - \omega + \pi J_{30} - \pi J_{31} - \pi J_{32})^2} + \frac{R_2}{R_2^2 + (\omega_3 - \omega - \pi J_{30} - \pi J_{31} - \pi J_{32})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_2 - \omega + \pi J_{20} + \pi J_{21} + \pi J_{32})^2} + \frac{R_2}{R_2^2 + (\omega_2 - \omega - \pi J_{20} + \pi J_{21} + \pi J_{32})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_2 - \omega + \pi J_{20} - \pi J_{21} + \pi J_{32})^2} + \frac{R_2}{R_2^2 + (\omega_2 - \omega - \pi J_{20} - \pi J_{21} + \pi J_{32})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_2 - \omega + \pi J_{20} + \pi J_{21} - \pi J_{32})^2} + \frac{R_2}{R_2^2 + (\omega_2 - \omega - \pi J_{20} + \pi J_{21} - \pi J_{32})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_2 - \omega + \pi J_{20} - \pi J_{21} - \pi J_{32})^2} + \frac{R_2}{R_2^2 + (\omega_2 - \omega - \pi J_{20} - \pi J_{21} - \pi J_{32})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_1 - \omega + \pi J_{10} + \pi J_{21} + \pi J_{31})^2} - \frac{R_2}{R_2^2 + (\omega_1 - \omega - \pi J_{10} + \pi J_{21} + \pi J_{31})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_1 - \omega + \pi J_{10} - \pi J_{21} + \pi J_{31})^2} - \frac{R_2}{R_2^2 + (\omega_1 - \omega - \pi J_{10} - \pi J_{21} + \pi J_{31})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_1 - \omega + \pi J_{21} - \pi J_{31} + \pi J_{30})^2} - \frac{R_2}{R_2^2 + (\omega_1 - \omega - \pi J_{10} + \pi J_{21} - \pi J_{32})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_1 - \omega + \pi J_{10} - \pi J_{21} - \pi J_{31})^2} - \frac{R_2}{R_2^2 + (\omega_1 - \omega - \pi J_{10} - \pi J_{21} - \pi J_{31})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_0 - \omega + \pi J_{10} + \pi J_{20} + \pi J_{30})^2} - \frac{R_2}{R_2^2 + (\omega_0 - \omega - \pi J_{10} + \pi J_{20} + \pi J_{30})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_0 - \omega + \pi J_{10} - \pi J_{20} + \pi J_{30})^2} - \frac{R_2}{R_2^2 + (\omega_0 - \omega - \pi J_{10} - \pi J_{20} + \pi J_{30})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_0 - \omega + \pi J_{10} + \pi J_{20} - \pi J_{30})^2} - \frac{R_2}{R_2^2 + (\omega_0 - \omega - \pi J_{10} + \pi J_{20} - \pi J_{30})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_0 - \omega + \pi J_{10} - \pi J_{20} - \pi J_{30})^2} - \frac{R_2}{R_2^2 + (\omega_0 - \omega - \pi J_{10} - \pi J_{20} - \pi J_{30})^2}
\end{aligned} \tag{238}$$

dan diperoleh juga:

[illegible]

7.2 Penerapan Algoritma Deutsch jozsa 4 qubit untuk $U_2 = U_{s2}$

apabila diambil kasus untuk $U_2 = I \otimes \sigma_z \otimes \sigma_z \otimes \sigma_z$ maka diperoleh untuk bentuk operator uniter totalnya adalah sebagai berikut:

$$\begin{aligned}
 U &= U_5 U_2 U_1 \\
 &= U_5 (\sigma_z \otimes I \otimes I \otimes I) U_1 \\
 &= \begin{pmatrix} e^{\beta_0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & e^{\beta_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{\beta_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{\beta_3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{\beta_4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{\beta_5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_9} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_{10}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_{11}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_{12}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_{13}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_{14}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_{15}} \end{pmatrix}
 \end{aligned}$$

[illegible]

[illegible]

maka dengan demikian kita dapatkan untuk nilai ρ_5 adalah

[illegible]

[illegible]

$$= \frac{1}{16} \begin{pmatrix} 0 & -8\rho_{0,1} & -8\rho_{0,2} & 0 & -8\rho_{0,4} & 0 & 0 & 0 & 8\rho_{0,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -8\rho_{1,0} & 0 & 0 & -8\rho_{1,3} & 0 & -8\rho_{1,5} & 0 & 0 & 0 & 8\rho_{1,9} & 0 & 0 & 0 & 0 & 0 & 0 \\ -8\rho_{2,0} & 0 & 0 & -8\rho_{2,3} & 0 & 0 & -8\rho_{2,6} & 0 & 0 & 0 & 8\rho_{2,10} & 0 & 0 & 0 & 0 & 0 \\ 0 & -8\rho_{3,1} & -8\rho_{3,2} & 0 & 0 & 0 & 0 & -8\rho_{3,7} & 0 & 0 & 0 & 8\rho_{3,11} & 0 & 0 & 0 & 0 \\ -8\rho_{4,0} & 0 & 0 & 0 & 0 & -8\rho_{4,5} & -8\rho_{4,6} & 0 & 0 & 0 & 0 & 0 & 8\rho_{4,12} & 0 & 0 & 0 \\ 0 & -8\rho_{4,1} & 0 & 0 & -8\rho_{5,4} & 0 & 0 & -8\rho_{5,7} & 0 & 0 & 0 & 0 & 0 & 8\rho_{5,13} & 0 & 0 \\ 0 & 0 & -8\rho_{6,2} & 0 & -8\rho_{6,4} & 0 & 0 & -8\rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & 8\rho_{6,14} & 0 \\ 0 & 0 & 0 & -8\rho_{7,3} & 0 & -8\rho_{7,5} & -8\rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8\rho_{7,15} \\ 8\rho_{8,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -8\rho_{8,9} & -8\rho_{8,10} & 0 & -8\rho_{8,12} & 0 & 0 & 0 \\ 0 & 8\rho_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & -8\rho_{9,8} & 0 & 0 & -8\rho_{9,11} & 0 & -8\rho_{9,13} & 0 & 0 \\ 0 & 0 & 8\rho_{10,2} & 0 & 0 & 0 & 0 & 0 & -8\rho_{10,8} & 0 & 0 & -8\rho_{10,11} & 0 & 0 & -8\rho_{10,14} & 0 \\ 0 & 0 & 0 & 8\rho_{11,3} & 0 & 0 & 0 & 0 & 0 & -8\rho_{11,9} & -8\rho_{11,10} & 0 & 0 & 0 & 0 & -8\rho_{11,15} \\ 0 & 0 & 0 & 0 & 8\rho_{12,4} & 0 & 0 & 0 & -8\rho_{12,8} & 0 & 0 & 0 & 0 & -8\rho_{12,13} & -8\rho_{12,14} & 0 \\ 0 & 0 & 0 & 0 & 0 & 8\rho_{13,5} & 0 & 0 & 0 & -8\rho_{13,9} & 0 & 0 & -8\rho_{13,12} & 0 & 0 & -8\rho_{13,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 8\rho_{14,6} & 0 & 0 & 0 & -8\rho_{14,10} & 0 & -8\rho_{14,12} & 0 & 0 & -8\rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8\rho_{15,7} & 0 & 0 & 0 & -8\rho_{15,11} & 0 & -8\rho_{15,13} & -8\rho_{15,14} & 0 \end{pmatrix} \quad (242)$$

$$= \frac{1}{2} \begin{pmatrix} 0 & -\rho_{0,1} & -\rho_{0,2} & 0 & -\rho_{0,4} & 0 & 0 & 0 & \rho_{0,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\rho_{1,0} & 0 & 0 & -\rho_{1,3} & 0 & -\rho_{1,5} & 0 & 0 & 0 & \rho_{1,9} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\rho_{2,0} & 0 & 0 & -\rho_{2,3} & 0 & 0 & -\rho_{2,6} & 0 & 0 & 0 & \rho_{2,10} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\rho_{3,1} & -\rho_{3,2} & 0 & 0 & 0 & 0 & -\rho_{3,7} & 0 & 0 & 0 & \rho_{3,11} & 0 & 0 & 0 & 0 \\ -\rho_{4,0} & 0 & 0 & 0 & 0 & -\rho_{4,5} & -\rho_{4,6} & 0 & 0 & 0 & 0 & 0 & \rho_{4,12} & 0 & 0 & 0 \\ 0 & -\rho_{4,1} & 0 & 0 & -\rho_{5,4} & 0 & 0 & -\rho_{5,7} & 0 & 0 & 0 & 0 & 0 & \rho_{5,13} & 0 & 0 \\ 0 & 0 & -\rho_{6,2} & 0 & -\rho_{6,4} & 0 & 0 & -\rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{6,14} & 0 \\ 0 & 0 & 0 & -\rho_{7,3} & 0 & -\rho_{7,5} & -\rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{7,15} \\ \rho_{8,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{8,9} & -\rho_{8,10} & 0 & -\rho_{8,12} & 0 & 0 & 0 \\ 0 & \rho_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{9,8} & 0 & 0 & -\rho_{9,11} & 0 & -\rho_{9,13} & 0 & 0 \\ 0 & 0 & \rho_{10,2} & 0 & 0 & 0 & 0 & 0 & -\rho_{10,8} & 0 & 0 & -\rho_{10,11} & 0 & 0 & -\rho_{10,14} & 0 \\ 0 & 0 & 0 & \rho_{11,3} & 0 & 0 & 0 & 0 & 0 & -\rho_{11,9} & -\rho_{11,10} & 0 & 0 & 0 & 0 & -\rho_{11,15} \\ 0 & 0 & 0 & 0 & \rho_{12,4} & 0 & 0 & 0 & -\rho_{12,8} & 0 & 0 & 0 & 0 & -\rho_{12,13} & -\rho_{12,14} & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_{13,5} & 0 & 0 & 0 & -\rho_{13,9} & 0 & 0 & -\rho_{13,12} & 0 & 0 & -\rho_{13,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho_{14,6} & 0 & 0 & 0 & -\rho_{14,10} & 0 & -\rho_{14,12} & 0 & 0 & -\rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{15,7} & 0 & 0 & 0 & -\rho_{15,11} & 0 & -\rho_{15,13} & -\rho_{15,14} & 0 \end{pmatrix} \quad (243)$$

dengan

$$\begin{aligned}
\rho_{0,1} &= \rho_{1,0}^* = e^{\beta_0 - \beta_1} & \rho_{6,7} &= \rho_{7,6}^* = e^{\beta_6 - \beta_7} \\
\rho_{0,2} &= \rho_{2,0}^* = e^{\beta_0 - \beta_2} & \rho_{6,14} &= \rho_{14,6}^* = e^{\beta_6 - \beta_{14}} \\
\rho_{0,8} &= \rho_{8,0}^* = e^{\beta_8 - \beta_1} & \rho_{8,9} &= \rho_{9,8}^* = e^{\beta_8 - \beta_9} \\
\rho_{1,3} &= \rho_{3,1}^* = e^{\beta_1 - \beta_3} & \rho_{8,10} &= \rho_{10,8}^* = e^{\beta_8 - \beta_{10}} \\
\rho_{1,5} &= \rho_{5,1}^* = e^{\beta_1 - \beta_5} & \rho_{8,12} &= \rho_{12,8}^* = e^{\beta_8 - \beta_{12}} \\
\rho_{1,9} &= \rho_{9,1}^* = e^{\beta_1 - \beta_9} & \rho_{9,11} &= \rho_{11,9}^* = e^{\beta_9 - \beta_{11}} \\
\rho_{2,3} &= \rho_{3,2}^* = e^{\beta_2 - \beta_3} & \rho_{9,13} &= \rho_{13,9}^* = e^{\beta_9 - \beta_{13}} \\
\rho_{2,6} &= \rho_{6,2}^* = e^{\beta_2 - \beta_6} & \rho_{10,11} &= \rho_{11,10}^* = e^{\beta_{10} - \beta_{11}} \\
\rho_{2,10} &= \rho_{10,2}^* = e^{\beta_2 - \beta_{10}} & \rho_{10,14} &= \rho_{14,10}^* = e^{\beta_{10} - \beta_{14}} \\
\rho_{3,7} &= \rho_{7,3}^* = e^{\beta_3 - \beta_7} & \rho_{12,13} &= \rho_{13,12}^* = e^{\beta_{12} - \beta_{13}} \\
\rho_{3,11} &= \rho_{11,3}^* = e^{\beta_3 - \beta_{11}} & \rho_{12,14} &= \rho_{14,12}^* = e^{\beta_{12} - \beta_{14}} \\
\rho_{4,5} &= \rho_{5,4}^* = e^{\beta_4 - \beta_5} & \rho_{11,15} &= \rho_{15,11}^* = e^{\beta_{11} - \beta_{15}} \\
\rho_{4,6} &= \rho_{6,4}^* = e^{\beta_4 - \beta_6} & \rho_{13,15} &= \rho_{15,13}^* = e^{\beta_{13} - \beta_{15}} \\
\rho_{4,12} &= \rho_{12,4}^* = e^{\beta_4 - \beta_{12}} & \rho_{14,15} &= \rho_{15,14}^* = e^{\beta_{14} - \beta_{15}} \\
\rho_{5,7} &= \rho_{7,5}^* = e^{\beta_5 - \beta_7} & \rho_{15,7} &= \rho_{7,15}^* = e^{\beta_7 - \beta_{15}} \\
\rho_{5,13} &= \rho_{13,5}^* = e^{\beta_5 - \beta_{13}} & \rho_{0,4} &= \rho_{4,0}^* = e^{\beta_0 - \beta_4}
\end{aligned} \tag{244}$$

dan

$$\begin{array}{ll}
\beta_0 - \beta_1 = -i(E_{4,0} - E_{4,1})t & \beta_0 - \beta_2 = -i(E_{4,0} - E_{4,2})t \\
\beta_0 - \beta_4 = -i(E_{4,0} - E_{4,4})t & \beta_0 - \beta_8 = -i(E_{4,0} - E_{4,8})t \\
\beta_1 - \beta_3 = -i(E_{4,1} - E_{4,3})t & \beta_1 - \beta_5 = -i(E_{4,1} - E_{4,5})t \\
\beta_1 - \beta_9 = -i(E_{4,1} - E_{4,9})t & \beta_2 - \beta_3 = -i(E_{4,2} - E_{4,3})t \\
\beta_2 - \beta_6 = -i(E_{4,2} - E_{4,6})t & \beta_2 - \beta_{10} = -i(E_{4,2} - E_{4,10})t \\
\beta_3 - \beta_7 = -i(E_{4,3} - E_{4,7})t & \beta_3 - \beta_{11} = -i(E_{4,3} - E_{4,11})t \\
\beta_4 - \beta_5 = -i(E_{4,4} - E_{4,5})t & \beta_4 - \beta_6 = -i(E_{4,4} - E_{4,6})t \\
\beta_4 - \beta_{12} = -i(E_{4,4} - E_{4,12})t & \beta_5 - \beta_7 = -i(E_{4,5} - E_{4,7})t \\
\beta_5 - \beta_{13} = -i(E_{4,5} - E_{4,13})t & \beta_6 - \beta_7 = -i(E_{4,6} - E_{4,7})t \\
\beta_6 - \beta_{14} = -i(E_{4,6} - E_{4,14})t & \beta_8 - \beta_9 = -i(E_{4,8} - E_{4,9})t \\
\beta_8 - \beta_{10} = -i(E_{4,8} - E_{4,10})t & \beta_8 - \beta_{12} = -i(E_{4,8} - E_{4,12})t \\
\beta_9 - \beta_{11} = -i(E_{4,9} - E_{4,11})t & \beta_9 - \beta_{13} = -i(E_{4,9} - E_{4,13})t \\
\beta_{10} - \beta_{11} = -i(E_{4,10} - E_{4,11})t & \beta_{10} - \beta_{14} = -i(E_{4,10} - E_{4,14})t \\
\beta_{12} - \beta_{13} = -i(E_{4,12} - E_{4,13})t & \beta_{12} - \beta_{14} = -i(E_{4,12} - E_{4,14})t \\
\beta_{11} - \beta_{15} = -i(E_{4,11} - E_{4,15})t & \beta_{13} - \beta_{15} = -i(E_{4,13} - E_{4,15})t \\
\beta_{14} - \beta_{15} = -i(E_{4,14} - E_{4,15})t & \beta_{15} - \beta_7 = -i(E_{4,15} - E_{4,7})t
\end{array} \quad (245)$$

dan

$$\begin{aligned}
E_{4,0} - E_{4,1} &= \omega_0 + \pi J_{10} + \pi J_{20} + \pi J_{30} & E_{4,0} - E_{4,2} &= \omega_1 + \pi J_{10} + \pi J_{21} + \pi J_{31} \\
E_{4,0} - E_{4,4} &= \omega_2 + \pi J_{20} + \pi J_{21} + \pi J_{32} & E_{4,0} - E_{4,8} &= \omega_3 + \pi J_{30} + \pi J_{31} + \pi J_{32} \\
E_{4,1} - E_{4,3} &= \omega_1 - \pi J_{10} + \pi J_{21} + \pi J_{31} & E_{4,1} - E_{4,5} &= \omega_2 - \pi J_{20} + \pi J_{21} + \pi J_{32} \\
E_{4,1} - E_{4,9} &= \omega_3 - \pi J_{30} + \pi J_{310} + \pi J_{32} & E_{4,2} - E_{4,3} &= \omega_0 - \pi J_{10} + \pi J_{20} + \pi J_{30} \\
E_{4,2} - E_{4,6} &= \omega_2 + \pi J_{20} - \pi J_{21} + \pi J_{32} & E_{4,2} - E_{4,10} &= \omega_3 + \pi J_{30} - \pi J_{31} + \pi J_{32} \\
E_{4,3} - E_{4,7} &= \omega_2 - \pi J_{20} - \pi J_{21} + \pi J_{32} & E_{4,3} - E_{4,11} &= \omega_3 - \pi J_{30} - \pi J_{31} + \pi J_{32} \\
E_{4,4} - E_{4,5} &= \omega_0 + \pi J_{10} - \pi J_{20} + \pi J_{30} & E_{4,4} - E_{4,6} &= \omega_1 + \pi J_{10} - \pi J_{21} + \pi J_{31} \\
E_{4,4} - E_{4,12} &= \omega_3 + \pi J_{30} + \pi J_{31} - \pi J_{32} & E_{4,5} - E_{4,7} &= \omega_1 - \pi J_{10} - \pi J_{21} + \pi J_{31} \\
E_{4,5} - E_{4,13} &= \omega_3 - \pi J_{30} + \pi J_{31} - \pi J_{32} & E_{4,6} - E_{4,7} &= \omega_0 - \pi J_{10} - \pi J_{20} + \pi J_{30} \\
E_{4,6} - E_{4,14} &= \omega_3 + \pi J_{30} - \pi J_{31} - \pi J_{32} & E_{4,8} - E_{4,9} &= \omega_0 + \pi J_{10} + \pi J_{20} - \pi J_{30} \\
E_{4,8} - E_{4,10} &= \omega_1 + \pi J_{21} - \pi J_{31} + \pi J_{30} & E_{4,8} - E_{4,12} &= \omega_2 + \pi J_{20} + \pi J_{21} - \pi J_{32} \\
E_{4,9} - E_{4,11} &= \omega_1 - \pi J_{10} + \pi J_{21} - \pi J_{32} & E_{4,10} - E_{4,11} &= \omega_0 - \pi J_{10} + \pi J_{20} - \pi J_{30} \\
E_{4,10} - E_{4,14} &= \omega_2 + \pi J_{20} - \pi J_{21} - \pi J_{32} & E_{4,12} - E_{4,13} &= \omega_0 + \pi J_{10} - \pi J_{20} - \pi J_{30} \\
E_{4,12} - E_{4,14} &= \omega_1 + \pi J_{10} - \pi J_{21} - \pi J_{31} & E_{4,11} - E_{4,15} &= \omega_2 - \pi J_{20} - \pi J_{21} - \pi J_{32} \\
E_{4,13} - E_{4,15} &= \omega_1 - \pi J_{10} - \pi J_{21} - \pi J_{31} & E_{4,14} - E_{4,15} &= \omega_0 - \pi J_{10} - \pi J_{20} - \pi J_{30} \\
E_{4,9} - E_{4,13} &= \omega_2 - \pi J_{20} + \pi J_{21} - \pi J_{32} & E_{4,15} - E_{4,7} &= \omega_3 + \pi J_{30} + \pi J_{31} + \pi J_{32}
\end{aligned} \tag{246}$$

maka dapat ditentukan nilai magnetisasi dari masing-masing qubitnya yaitu sebagai berikut:

$$(I_{3+}\rho_5) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\times \frac{1}{2} \begin{pmatrix} 0 & -\rho_{0,1} & -\rho_{0,2} & 0 & -\rho_{0,4} & 0 & 0 & 0 & \rho_{0,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\rho_{1,0} & 0 & 0 & -\rho_{1,3} & 0 & -\rho_{1,5} & 0 & 0 & 0 & \rho_{1,9} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\rho_{2,0} & 0 & 0 & -\rho_{2,3} & 0 & 0 & -\rho_{2,6} & 0 & 0 & 0 & \rho_{2,10} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\rho_{3,1} & -\rho_{3,2} & 0 & 0 & 0 & 0 & -\rho_{3,7} & 0 & 0 & 0 & \rho_{3,11} & 0 & 0 & 0 & 0 \\ -\rho_{4,0} & 0 & 0 & 0 & 0 & -\rho_{4,5} & -\rho_{4,6} & 0 & 0 & 0 & 0 & 0 & \rho_{4,12} & 0 & 0 & 0 \\ 0 & -\rho_{4,1} & 0 & 0 & -\rho_{5,4} & 0 & 0 & -\rho_{5,7} & 0 & 0 & 0 & 0 & 0 & \rho_{5,13} & 0 & 0 \\ 0 & 0 & -\rho_{6,2} & 0 & -\rho_{6,4} & 0 & 0 & -\rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{6,14} & 0 \\ 0 & 0 & 0 & -\rho_{7,3} & 0 & -\rho_{7,5} & -\rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{7,15} \\ \rho_{8,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{8,9} & -\rho_{8,10} & 0 & -\rho_{8,12} & 0 & 0 & 0 \\ 0 & \rho_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{9,8} & 0 & 0 & -\rho_{9,11} & 0 & -\rho_{9,13} & 0 & 0 \\ 0 & 0 & \rho_{10,2} & 0 & 0 & 0 & 0 & 0 & -\rho_{10,8} & 0 & 0 & -\rho_{10,11} & 0 & 0 & -\rho_{10,14} & 0 \\ 0 & 0 & 0 & \rho_{11,3} & 0 & 0 & 0 & 0 & 0 & -\rho_{11,9} & -\rho_{11,10} & 0 & 0 & 0 & 0 & -\rho_{11,15} \\ 0 & 0 & 0 & 0 & \rho_{12,4} & 0 & 0 & 0 & -\rho_{12,8} & 0 & 0 & 0 & 0 & -\rho_{12,13} & -\rho_{12,14} & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_{13,5} & 0 & 0 & 0 & -\rho_{13,9} & 0 & 0 & -\rho_{13,12} & 0 & 0 & -\rho_{13,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho_{14,6} & 0 & 0 & 0 & -\rho_{14,10} & 0 & -\rho_{14,12} & 0 & 0 & -\rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{15,7} & 0 & 0 & 0 & -\rho_{15,11} & 0 & -\rho_{15,13} & -\rho_{15,14} & 0 \end{pmatrix}$$

[illegible]

$$\begin{aligned}
M_{3+} &= \langle I_{3+} \rangle = \text{tr} (I_{3+} \rho_5) \\
&= \rho_{8,0} + \rho_{9,1} + \rho_{10,2} + \rho_{11,3} + \rho_{12,4} + \rho_{13,5} + \rho_{14,6} + \rho_{15,7}
\end{aligned} \tag{247}$$

dan

$$(I_{2+} \rho_5) = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\times \frac{1}{2} \begin{pmatrix} 0 & -\rho_{0,1} & -\rho_{0,2} & 0 & -\rho_{0,4} & 0 & 0 & 0 & \rho_{0,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\rho_{1,0} & 0 & 0 & -\rho_{1,3} & 0 & -\rho_{1,5} & 0 & 0 & 0 & \rho_{1,9} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\rho_{2,0} & 0 & 0 & -\rho_{2,3} & 0 & 0 & -\rho_{2,6} & 0 & 0 & 0 & \rho_{2,10} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\rho_{3,1} & -\rho_{3,2} & 0 & 0 & 0 & 0 & -\rho_{3,7} & 0 & 0 & 0 & \rho_{3,11} & 0 & 0 & 0 & 0 \\ -\rho_{4,0} & 0 & 0 & 0 & 0 & -\rho_{4,5} & -\rho_{4,6} & 0 & 0 & 0 & 0 & 0 & \rho_{4,12} & 0 & 0 & 0 \\ 0 & -\rho_{4,1} & 0 & 0 & -\rho_{5,4} & 0 & 0 & -\rho_{5,7} & 0 & 0 & 0 & 0 & 0 & \rho_{5,13} & 0 & 0 \\ 0 & 0 & -\rho_{6,2} & 0 & -\rho_{6,4} & 0 & 0 & -\rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{6,14} & 0 \\ 0 & 0 & 0 & -\rho_{7,3} & 0 & -\rho_{7,5} & -\rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{7,15} \\ \rho_{8,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{8,9} & -\rho_{8,10} & 0 & -\rho_{8,12} & 0 & 0 & 0 \\ 0 & \rho_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{9,8} & 0 & 0 & -\rho_{9,11} & 0 & -\rho_{9,13} & 0 & 0 \\ 0 & 0 & \rho_{10,2} & 0 & 0 & 0 & 0 & 0 & -\rho_{10,8} & 0 & 0 & -\rho_{10,11} & 0 & 0 & -\rho_{10,14} & 0 \\ 0 & 0 & 0 & \rho_{11,3} & 0 & 0 & 0 & 0 & 0 & -\rho_{11,9} & -\rho_{11,10} & 0 & 0 & 0 & 0 & -\rho_{11,15} \\ 0 & 0 & 0 & 0 & \rho_{12,4} & 0 & 0 & 0 & -\rho_{12,8} & 0 & 0 & 0 & 0 & -\rho_{12,13} & -\rho_{12,14} & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_{13,5} & 0 & 0 & 0 & -\rho_{13,9} & 0 & 0 & -\rho_{13,12} & 0 & 0 & -\rho_{13,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho_{14,6} & 0 & 0 & 0 & -\rho_{14,10} & 0 & -\rho_{14,12} & 0 & 0 & -\rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{15,7} & 0 & 0 & 0 & -\rho_{15,11} & 0 & -\rho_{15,13} & -\rho_{15,14} & 0 \end{pmatrix}$$

[illegible]

$$\begin{aligned}
M_{2+} &= \langle I_{2+} \rangle = tr(I_{2+} \rho_5) \\
&= -(\rho_{4,0} + \rho_{5,1} + \rho_{6,2} + \rho_{7,3} + \rho_{12,8} + \rho_{13,9} + \rho_{14,10} + \rho_{15,11}) \quad (249)
\end{aligned}$$

dan

$$(I_{1+} \rho_5) = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\times \frac{1}{2} \begin{pmatrix} 0 & -\rho_{0,1} & -\rho_{0,2} & 0 & -\rho_{0,4} & 0 & 0 & 0 & \rho_{0,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\rho_{1,0} & 0 & 0 & -\rho_{1,3} & 0 & -\rho_{1,5} & 0 & 0 & 0 & \rho_{1,9} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\rho_{2,0} & 0 & 0 & -\rho_{2,3} & 0 & 0 & -\rho_{2,6} & 0 & 0 & 0 & \rho_{2,10} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\rho_{3,1} & -\rho_{3,2} & 0 & 0 & 0 & 0 & -\rho_{3,7} & 0 & 0 & 0 & \rho_{3,11} & 0 & 0 & 0 & 0 \\ -\rho_{4,0} & 0 & 0 & 0 & 0 & -\rho_{4,5} & -\rho_{4,6} & 0 & 0 & 0 & 0 & 0 & \rho_{4,12} & 0 & 0 & 0 \\ 0 & -\rho_{4,1} & 0 & 0 & -\rho_{5,4} & 0 & 0 & -\rho_{5,7} & 0 & 0 & 0 & 0 & 0 & \rho_{5,13} & 0 & 0 \\ 0 & 0 & -\rho_{6,2} & 0 & -\rho_{6,4} & 0 & 0 & -\rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{6,14} & 0 \\ 0 & 0 & 0 & -\rho_{7,3} & 0 & -\rho_{7,5} & -\rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{7,15} \\ \rho_{8,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{8,9} & -\rho_{8,10} & 0 & -\rho_{8,12} & 0 & 0 & 0 \\ 0 & \rho_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{9,8} & 0 & 0 & -\rho_{9,11} & 0 & -\rho_{9,13} & 0 & 0 \\ 0 & 0 & \rho_{10,2} & 0 & 0 & 0 & 0 & 0 & -\rho_{10,8} & 0 & 0 & -\rho_{10,11} & 0 & 0 & -\rho_{10,14} & 0 \\ 0 & 0 & 0 & \rho_{11,3} & 0 & 0 & 0 & 0 & 0 & -\rho_{11,9} & -\rho_{11,10} & 0 & 0 & 0 & 0 & -\rho_{11,15} \\ 0 & 0 & 0 & 0 & \rho_{12,4} & 0 & 0 & 0 & -\rho_{12,8} & 0 & 0 & 0 & 0 & -\rho_{12,13} & -\rho_{12,14} & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_{13,5} & 0 & 0 & 0 & -\rho_{13,9} & 0 & 0 & -\rho_{13,12} & 0 & 0 & -\rho_{13,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho_{14,6} & 0 & 0 & 0 & -\rho_{14,10} & 0 & -\rho_{14,12} & 0 & 0 & -\rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{15,7} & 0 & 0 & 0 & -\rho_{15,11} & 0 & -\rho_{15,13} & -\rho_{15,14} & 0 \end{pmatrix}$$

[illegible]

$$\begin{aligned}
M_{1+} &= \langle I_{1+} \rangle = tr(I_{1+}\rho_5) \\
&= -(\rho_{2,0} + \rho_{3,1} + \rho_{6,4} + \rho_{7,5} + \rho_{10,8} + \rho_{11,9} + \rho_{14,12} + \rho_{15,13}) \quad (251)
\end{aligned}$$

dan

$$(I_{0+}\rho_5) = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\times \frac{1}{2} \begin{pmatrix} 0 & -\rho_{0,1} & -\rho_{0,2} & 0 & -\rho_{0,4} & 0 & 0 & 0 & \rho_{0,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\rho_{1,0} & 0 & 0 & -\rho_{1,3} & 0 & -\rho_{1,5} & 0 & 0 & 0 & \rho_{1,9} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\rho_{2,0} & 0 & 0 & -\rho_{2,3} & 0 & 0 & -\rho_{2,6} & 0 & 0 & 0 & \rho_{2,10} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\rho_{3,1} & -\rho_{3,2} & 0 & 0 & 0 & 0 & -\rho_{3,7} & 0 & 0 & 0 & \rho_{3,11} & 0 & 0 & 0 & 0 \\ -\rho_{4,0} & 0 & 0 & 0 & 0 & -\rho_{4,5} & -\rho_{4,6} & 0 & 0 & 0 & 0 & 0 & \rho_{4,12} & 0 & 0 & 0 \\ 0 & -\rho_{4,1} & 0 & 0 & -\rho_{5,4} & 0 & 0 & -\rho_{5,7} & 0 & 0 & 0 & 0 & 0 & \rho_{5,13} & 0 & 0 \\ 0 & 0 & -\rho_{6,2} & 0 & -\rho_{6,4} & 0 & 0 & -\rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{6,14} & 0 \\ 0 & 0 & 0 & -\rho_{7,3} & 0 & -\rho_{7,5} & -\rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{7,15} \\ \rho_{8,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{8,9} & -\rho_{8,10} & 0 & -\rho_{8,12} & 0 & 0 & 0 \\ 0 & \rho_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{9,8} & 0 & 0 & -\rho_{9,11} & 0 & -\rho_{9,13} & 0 & 0 \\ 0 & 0 & \rho_{10,2} & 0 & 0 & 0 & 0 & 0 & -\rho_{10,8} & 0 & 0 & -\rho_{10,11} & 0 & 0 & -\rho_{10,14} & 0 \\ 0 & 0 & 0 & \rho_{11,3} & 0 & 0 & 0 & 0 & 0 & -\rho_{11,9} & -\rho_{11,10} & 0 & 0 & 0 & 0 & -\rho_{11,15} \\ 0 & 0 & 0 & 0 & \rho_{12,4} & 0 & 0 & 0 & -\rho_{12,8} & 0 & 0 & 0 & 0 & -\rho_{12,13} & -\rho_{12,14} & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_{13,5} & 0 & 0 & 0 & -\rho_{13,9} & 0 & 0 & -\rho_{13,12} & 0 & 0 & -\rho_{13,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho_{14,6} & 0 & 0 & 0 & -\rho_{14,10} & 0 & -\rho_{14,12} & 0 & 0 & -\rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{15,7} & 0 & 0 & 0 & -\rho_{15,11} & 0 & -\rho_{15,13} & -\rho_{15,14} & 0 \end{pmatrix}$$

(252)

$$M_{0+} = \langle I_{0+} \rangle = tr (I_{0+} \rho_5) \quad (253)$$

$$= -(\rho_{1,0} + \rho_{3,2} + \rho_{5,2} + \rho_{7,6} + \rho_{9,8} + \rho_{11,10} + \rho_{13,12} + \rho_{15,14})$$

maka dengan demikian nilai magnetisasi total dari sistem adalah sebagai berikut:

$$M_{tot} = M_{3+} + M_{2+} + M_{1+} + M_{0+}$$

$$= \rho_{8,0} + \rho_{9,1} + \rho_{10,2} + \rho_{11,3} + \rho_{12,4} + \rho_{13,5} + \rho_{14,6} + \rho_{15,7}$$

$$- \rho_{4,0} - \rho_{5,1} - \rho_{6,2} - \rho_{7,3} - \rho_{12,8} - \rho_{13,9} - \rho_{14,10} - \rho_{15,11}$$

$$- (\rho_{2,0} + \rho_{3,1} + \rho_{6,4} + \rho_{7,5} + \rho_{10,8} + \rho_{11,9} + \rho_{14,12} + \rho_{15,13})$$

$$- (\rho_{1,0} + \rho_{3,2} + \rho_{5,4} + \rho_{7,6} + \rho_{9,8} + \rho_{11,10} + \rho_{13,12} + \rho_{15,14})$$

$$= e^{i(\omega_3 + \pi J_{30} + \pi J_{31} + \pi J_{32})t} + e^{i(\omega_3 - \pi J_{30} + \pi J_{31} + \pi J_{32})t}$$

$$+ e^{i(\omega_3 + \pi J_{30} - \pi J_{31} + \pi J_{32})t} + e^{i(\omega_3 - \pi J_{30} - \pi J_{31} + \pi J_{32})t}$$

$$+ e^{i(\omega_3 + \pi J_{30} + \pi J_{31} - \pi J_{32})t} + e^{i(\omega_3 - \pi J_{30} + \pi J_{31} - \pi J_{32})t}$$

$$+ e^{i(\omega_3 + \pi J_{30} - \pi J_{31} - \pi J_{32})t} + e^{i(\omega_3 - \pi J_{30} - \pi J_{31} - \pi J_{32})t}$$

$$- e^{i(\omega_2 + \pi J_{20} + \pi J_{21} + \pi J_{32})t} - e^{i(\omega_2 - \pi J_{20} + \pi J_{21} + \pi J_{32})t}$$

$$- e^{i(\omega_2 + \pi J_{20} - \pi J_{21} + \pi J_{32})t} - e^{i(\omega_2 - \pi J_{20} - \pi J_{21} + \pi J_{32})t} \quad (254)$$

$$- e^{i(\omega_2 + \pi J_{20} + \pi J_{21} - \pi J_{32})t} - e^{i(\omega_2 - \pi J_{20} + \pi J_{21} - \pi J_{32})t}$$

$$- e^{i(\omega_2 + \pi J_{20} - \pi J_{21} - \pi J_{32})t} - e^{i(\omega_2 - \pi J_{20} - \pi J_{21} - \pi J_{32})t}$$

$$- e^{i(\omega_1 + \pi J_{10} + \pi J_{21} + \pi J_{31})t} - e^{i(\omega_1 - \pi J_{10} + \pi J_{21} + \pi J_{31})t}$$

$$- e^{i(\omega_1 + \pi J_{10} - \pi J_{21} + \pi J_{31})t} - e^{i(\omega_1 - \pi J_{10} - \pi J_{21} + \pi J_{31})t}$$

$$- e^{i(\omega_1 + \pi J_{21} - \pi J_{31} + \pi J_{30})t} - e^{i(\omega_1 - \pi J_{10} + \pi J_{21} - \pi J_{32})t}$$

$$- e^{i(\omega_1 + \pi J_{10} - \pi J_{21} - \pi J_{31})t} - e^{i(\omega_1 - \pi J_{10} - \pi J_{21} - \pi J_{31})t}$$

$$- e^{i(\omega_0 + \pi J_{10} + \pi J_{20} + \pi J_{30})t} - e^{i(\omega_0 - \pi J_{10} + \pi J_{20} + \pi J_{30})t}$$

$$- e^{i(\omega_0 + \pi J_{10} - \pi J_{20} + \pi J_{30})t} - e^{i(\omega_0 - \pi J_{10} - \pi J_{20} + \pi J_{30})t}$$

$$- e^{i(\omega_0 + \pi J_{10} + \pi J_{20} - \pi J_{30})t} - e^{i(\omega_0 - \pi J_{10} + \pi J_{20} - \pi J_{30})t}$$

$$- e^{i(\omega_0 + \pi J_{10} - \pi J_{20} - \pi J_{30})t} - e^{i(\omega_0 - \pi J_{10} - \pi J_{20} - \pi J_{30})t}$$

kemudian didapatkan juga untuk sinyal FID sebagai berikut:

$$\begin{aligned}
S = & M_{tot} e^{-R_2 t} \\
= & e^{-(R_2 - i(\omega_3 + \pi J_{30} + \pi J_{31} + \pi J_{32}))t} + e^{-(R_2 - i(\omega_3 - \pi J_{30} + \pi J_{31} + \pi J_{32}))t} \\
+ & e^{-(R_2 - i(\omega_3 + \pi J_{30} - \pi J_{31} + \pi J_{32}))t} + e^{-(R_2 - i(\omega_3 - \pi J_{30} - \pi J_{31} + \pi J_{32}))t} \\
+ & e^{-(R_2 - i(\omega_3 + \pi J_{30} + \pi J_{31} - \pi J_{32}))t} + e^{-(R_2 - i(\omega_3 - \pi J_{30} + \pi J_{31} - \pi J_{32}))t} + e^{-(R_2 - i(\omega_3 + \pi J_{30} - \pi J_{31} - \pi J_{32}))t} \\
+ & e^{-(R_2 - i(\omega_3 - \pi J_{30} - \pi J_{31} - \pi J_{32}))t} - e^{-(R_2 - i(\omega_2 + \pi J_{20} + \pi J_{21} + \pi J_{32}))t} \\
- & e^{-(R_2 - i(\omega_2 - \pi J_{20} + \pi J_{21} + \pi J_{32}))t} - e^{-(R_2 - i(\omega_2 + \pi J_{20} - \pi J_{21} + \pi J_{32}))t} \\
- & e^{-(R_2 - i(\omega_2 - \pi J_{20} - \pi J_{21} + \pi J_{32}))t} - e^{-(R_2 - i(\omega_2 + \pi J_{20} + \pi J_{21} - \pi J_{32}))t} \\
- & e^{-(R_2 - i(\omega_2 - \pi J_{20} + \pi J_{21} - \pi J_{32}))t} - e^{-(R_2 - i(\omega_2 + \pi J_{20} - \pi J_{21} - \pi J_{32}))t} \\
- & e^{-(R_2 - i(\omega_2 - \pi J_{20} - \pi J_{21} - \pi J_{32}))t} - e^{-(R_2 - i(\omega_1 + \pi J_{10} + \pi J_{21} + \pi J_{31}))t} \\
- & e^{-(R_2 - i(\omega_1 - \pi J_{10} + \pi J_{21} + \pi J_{31}))t} - e^{-(R_2 - i(\omega_1 + \pi J_{10} - \pi J_{21} + \pi J_{31}))t} \\
- & e^{-(R_2 - i(\omega_1 - \pi J_{10} - \pi J_{21} + \pi J_{31}))t} - e^{-(R_2 - i(\omega_1 + \pi J_{10} + \pi J_{21} - \pi J_{31}))t} \\
- & e^{-(R_2 - i(\omega_1 - \pi J_{10} + \pi J_{21} - \pi J_{31}))t} - e^{-(R_2 - i(\omega_1 + \pi J_{10} - \pi J_{21} - \pi J_{31}))t} \\
- & e^{-(R_2 - i(\omega_1 - \pi J_{10} - \pi J_{21} - \pi J_{31}))t} - e^{-(R_2 - i(\omega_0 + \pi J_{10} + \pi J_{20} + \pi J_{30}))t} \\
- & e^{-(R_2 - i(\omega_0 - \pi J_{10} + \pi J_{20} + \pi J_{30}))t} - e^{-(R_2 - i(\omega_0 + \pi J_{10} - \pi J_{20} + \pi J_{30}))t} \\
- & e^{-(R_2 - i(\omega_0 - \pi J_{10} - \pi J_{20} + \pi J_{30}))t} - e^{-(R_2 - i(\omega_0 + \pi J_{10} + \pi J_{20} - \pi J_{30}))t} \\
- & e^{-(R_2 - i(\omega_0 - \pi J_{10} + \pi J_{20} - \pi J_{30}))t} - e^{-(R_2 - i(\omega_0 + \pi J_{10} - \pi J_{20} - \pi J_{30}))t} - e^{-(R_2 - i(\omega_0 - \pi J_{10} - \pi J_{20} - \pi J_{30}))t}
\end{aligned}$$

dimana $R_2 = \frac{1}{T_2}$

kemudian dapat di cari terlebih dahulu untuk perkalian antara $S(t) e^{-i\omega t}$ yaitu sebagai berikut:

$$\begin{aligned}
S(t) e^{-i\omega t} = & e^{-(R_2-i(\omega_3-\omega+\pi J_{30}+\pi J_{31}+\pi J_{32}))t} + e^{-(R_2-i(\omega_3-\omega-\pi J_{30}+\pi J_{31}+\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_3-\omega+\pi J_{30}-\pi J_{31}+\pi J_{32}))t} + e^{-(R_2-i(\omega_3-\omega-\pi J_{30}-\pi J_{31}+\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_3-\omega+\pi J_{30}+\pi J_{31}-\pi J_{32}))t} + e^{-(R_2-i(\omega_3-\omega-\pi J_{30}+\pi J_{31}-\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_3-\omega+\pi J_{30}-\pi J_{31}-\pi J_{32}))t} + e^{-(R_2-i(\omega_3-\omega-\pi J_{30}-\pi J_{31}-\pi J_{32}))t} \\
& - e^{-(R_2-i(\omega_2-\omega+\pi J_{20}+\pi J_{21}+\pi J_{32}))t} - e^{-(R_2-i(\omega_2-\omega-\pi J_{20}+\pi J_{21}+\pi J_{32}))t} \\
& - e^{-(R_2-i(\omega_2-\omega+\pi J_{20}-\pi J_{21}+\pi J_{32}))t} - e^{-(R_2-i(\omega_2-\omega-\pi J_{20}-\pi J_{21}+\pi J_{32}))t} \\
& - e^{-(R_2-i(\omega_2-\omega+\pi J_{20}+\pi J_{21}-\pi J_{32}))t} - e^{-(R_2-i(\omega_2-\omega-\pi J_{20}+\pi J_{21}-\pi J_{32}))t} \\
& - e^{-(R_2-i(\omega_2-\omega+\pi J_{20}-\pi J_{21}-\pi J_{32}))t} - e^{-(R_2-i(\omega_2-\omega-\pi J_{20}-\pi J_{21}-\pi J_{32}))t} \\
& - e^{-(R_2-i(\omega_1-\omega+\pi J_{10}+\pi J_{21}+\pi J_{31}))t} - e^{-(R_2-i(\omega_1-\omega-\pi J_{10}+\pi J_{21}+\pi J_{31}))t} \\
& - e^{-(R_2-i(\omega_1-\omega+\pi J_{10}-\pi J_{21}+\pi J_{31}))t} - e^{-(R_2-i(\omega_1-\omega-\pi J_{10}-\pi J_{21}+\pi J_{31}))t} \\
& - e^{-(R_2-i(\omega_1-\omega+\pi J_{21}-\pi J_{31}+\pi J_{30}))t} - e^{-(R_2-i(\omega_1-\omega-\pi J_{10}+\pi J_{21}-\pi J_{32}))t} \\
& - e^{-(R_2-i(\omega_1-\omega+\pi J_{10}-\pi J_{21}-\pi J_{31}))t} - e^{-(R_2-i(\omega_1-\omega-\pi J_{10}-\pi J_{21}-\pi J_{31}))t} \\
& - e^{-(R_2-i(\omega_0-\omega+\pi J_{10}+\pi J_{20}+\pi J_{30}))t} - e^{-(R_2-i(\omega_0-\omega-\pi J_{10}+\pi J_{20}+\pi J_{30}))t} \\
& - e^{-(R_2-i(\omega_0-\omega+\pi J_{10}-\pi J_{20}+\pi J_{30}))t} - e^{-(R_2-i(\omega_0-\omega-\pi J_{10}-\pi J_{20}+\pi J_{30}))t} \\
& - e^{-(R_2-i(\omega_0-\omega+\pi J_{10}+\pi J_{20}-\pi J_{30}))t} - e^{-(R_2-i(\omega_0-\omega-\pi J_{10}+\pi J_{20}-\pi J_{30}))t} \\
& - e^{-(R_2-i(\omega_0-\omega+\pi J_{10}-\pi J_{20}-\pi J_{30}))t} - e^{-(R_2-i(\omega_0-\omega-\pi J_{10}-\pi J_{20}-\pi J_{30}))t}
\end{aligned} \tag{256}$$

Transformasi Fourirer dari sinyal FiD adalah sebagai berikut:

[illegible]

$$\begin{aligned}
&= \frac{1}{(R_2 - i(\omega_3 - \omega + \pi J_{30} + \pi J_{31} + \pi J_{32}))} + \frac{1}{(R_2 - i(\omega_3 - \omega - \pi J_{30} + \pi J_{31} + \pi J_{32}))} \\
&+ \frac{1}{(R_2 - i(\omega_3 - \omega + \pi J_{30} - \pi J_{31} + \pi J_{32}))} + \frac{1}{(R_2 - i(\omega_3 - \omega - \pi J_{30} - \pi J_{31} + \pi J_{32}))} \\
&+ \frac{1}{(R_2 - i(\omega_3 - \omega - \pi J_{30} - \pi J_{31} + \pi J_{32}))} + \frac{1}{(R_2 - i(\omega_3 - \omega - \pi J_{30} + \pi J_{31} - \pi J_{32}))} \\
&+ \frac{1}{(R_2 - i(\omega_3 - \omega + \pi J_{30} - \pi J_{31} - \pi J_{32}))} + \frac{1}{(R_2 - i(\omega_3 - \omega - \pi J_{30} - \pi J_{31} - \pi J_{32}))} \\
&- \frac{1}{(R_2 - i(\omega_2 - \omega + \pi J_{20} + \pi J_{21} + \pi J_{32}))} - \frac{1}{(R_2 - i(\omega_2 - \omega - \pi J_{20} + \pi J_{21} + \pi J_{32}))} \\
&- \frac{1}{(R_2 - i(\omega_2 - \omega + \pi J_{20} - \pi J_{21} + \pi J_{32}))} - \frac{1}{(R_2 - i(\omega_2 - \omega - \pi J_{20} - \pi J_{21} + \pi J_{32}))} \\
&- \frac{1}{(R_2 - i(\omega_2 - \omega + \pi J_{20} + \pi J_{21} - \pi J_{32}))} - \frac{1}{(R_2 - i(\omega_2 - \omega - \pi J_{20} + \pi J_{21} - \pi J_{32}))} \\
&- \frac{1}{(R_2 - i(\omega_2 - \omega + \pi J_{20} - \pi J_{21} - \pi J_{32}))} - \frac{1}{(R_2 - i(\omega_2 - \omega - \pi J_{20} - \pi J_{21} - \pi J_{32}))} \\
&- \frac{1}{(R_2 - i(\omega_1 - \omega + \pi J_{10} + \pi J_{21} + \pi J_{31}))} - \frac{1}{(R_2 - i(\omega_1 - \omega - \pi J_{10} + \pi J_{21} + \pi J_{31}))} \\
&- \frac{1}{(R_2 - i(\omega_1 - \omega + \pi J_{10} - \pi J_{21} + \pi J_{31}))} - \frac{1}{(R_2 - i(\omega_1 - \omega - \pi J_{10} - \pi J_{21} + \pi J_{31}))} \\
&- \frac{1}{(R_2 - i(\omega_1 - \omega + \pi J_{21} - \pi J_{31} + \pi J_{30}))} - \frac{1}{(R_2 - i(\omega_1 - \omega - \pi J_{10} + \pi J_{21} - \pi J_{32}))} \\
&+ \frac{1}{(R_2 - i(\omega_1 - \omega + \pi J_{10} - \pi J_{21} - \pi J_{31}))} + \frac{1}{(R_2 - i(\omega_1 - \omega - \pi J_{10} - \pi J_{21} - \pi J_{31}))} \\
&- \frac{1}{(R_2 - i(\omega_0 - \omega + \pi J_{10} + \pi J_{20} + \pi J_{30}))} - \frac{1}{(R_2 - i(\omega_0 - \omega - \pi J_{10} + \pi J_{20} + \pi J_{30}))} \\
&+ \frac{1}{(R_2 - i(\omega_0 - \omega + \pi J_{10} - \pi J_{20} + \pi J_{30}))} + \frac{1}{(R_2 - i(\omega_0 - \omega - \pi J_{10} - \pi J_{20} + \pi J_{30}))} \\
&- \frac{1}{(R_2 - i(\omega_0 - \omega + \pi J_{10} + \pi J_{20} - \pi J_{30}))} - \frac{1}{(R_2 - i(\omega_0 - \omega - \pi J_{10} + \pi J_{20} - \pi J_{30}))} \\
&- \frac{1}{(R_2 - i(\omega_0 - \omega + \pi J_{10} - \pi J_{20} - \pi J_{30}))} - \frac{1}{(R_2 - i(\omega_0 - \omega - \pi J_{10} - \pi J_{20} - \pi J_{30}))}
\end{aligned}$$

maka dengan demikian diperoleh :

$$\begin{aligned}
A(\omega) = & \frac{R_2}{R_2^2 + (\omega_3 - \omega + \pi J_{30} + \pi J_{31} + \pi J_{32})^2} + \frac{R_2}{R_2^2 + (\omega_3 - \omega - \pi J_{30} + \pi J_{31} + \pi J_{32})^2} \\
& + \frac{R_2}{R_2^2 - (\omega_3 - \omega + \pi J_{30} - \pi J_{31} + \pi J_{32})^2} + \frac{R_2}{R_2^2 + (\omega_3 - \omega - \pi J_{30} - \pi J_{31} + \pi J_{32})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_3 - \omega - \pi J_{30} - \pi J_{31} + \pi J_{32})^2} + \frac{R_2}{R_2^2 + (\omega_3 - \omega - \pi J_{30} + \pi J_{31} - \pi J_{32})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_3 - \omega + \pi J_{30} - \pi J_{31} - \pi J_{32})^2} + \frac{R_2}{R_2^2 + (\omega_3 - \omega - \pi J_{30} - \pi J_{31} - \pi J_{32})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_2 - \omega + \pi J_{20} + \pi J_{21} + \pi J_{32})^2} - \frac{R_2}{R_2^2 + (\omega_2 - \omega - \pi J_{20} + \pi J_{21} + \pi J_{32})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_2 - \omega + \pi J_{20} - \pi J_{21} + \pi J_{32})^2} - \frac{R_2}{R_2^2 + (\omega_2 - \omega - \pi J_{20} - \pi J_{21} + \pi J_{32})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_2 - \omega + \pi J_{20} + \pi J_{21} - \pi J_{32})^2} - \frac{R_2}{R_2^2 + (\omega_2 - \omega - \pi J_{20} + \pi J_{21} - \pi J_{32})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_2 - \omega + \pi J_{20} - \pi J_{21} - \pi J_{32})^2} - \frac{R_2}{R_2^2 + (\omega_2 - \omega - \pi J_{20} - \pi J_{21} - \pi J_{32})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_1 - \omega + \pi J_{10} + \pi J_{21} + \pi J_{31})^2} - \frac{R_2}{R_2^2 + (\omega_1 - \omega - \pi J_{10} + \pi J_{21} + \pi J_{31})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_1 - \omega + \pi J_{10} - \pi J_{21} + \pi J_{31})^2} - \frac{R_2}{R_2^2 + (\omega_1 - \omega - \pi J_{10} - \pi J_{21} + \pi J_{31})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_1 - \omega + \pi J_{21} - \pi J_{31} + \pi J_{30})^2} - \frac{R_2}{R_2^2 + (\omega_1 - \omega - \pi J_{10} + \pi J_{21} - \pi J_{32})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_1 - \omega + \pi J_{10} - \pi J_{21} - \pi J_{31})^2} - \frac{R_2}{R_2^2 + (\omega_1 - \omega - \pi J_{10} - \pi J_{21} - \pi J_{31})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_0 - \omega + \pi J_{10} + \pi J_{20} + \pi J_{30})^2} - \frac{R_2}{R_2^2 + (\omega_0 - \omega - \pi J_{10} + \pi J_{20} + \pi J_{30})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_0 - \omega + \pi J_{10} - \pi J_{20} + \pi J_{30})^2} - \frac{R_2}{R_2^2 + (\omega_0 - \omega - \pi J_{10} - \pi J_{20} + \pi J_{30})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_0 - \omega + \pi J_{10} + \pi J_{20} - \pi J_{30})^2} - \frac{R_2}{R_2^2 + (\omega_0 - \omega - \pi J_{10} + \pi J_{20} - \pi J_{30})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_0 - \omega + \pi J_{10} - \pi J_{20} - \pi J_{30})^2} - \frac{R_2}{R_2^2 + (\omega_0 - \omega - \pi J_{10} - \pi J_{20} - \pi J_{30})^2}
\end{aligned} \tag{257}$$

dan diperoleh juga:

[illegible]

7.3 Penerapan Algoritma Deutsch jozsa 4 qubit untuk $U_2 = U_3$

apabila diambil kasus untuk $U_2 = \sigma_z \otimes \sigma_z \otimes \sigma_z \otimes \sigma_z$ maka diperoleh untuk bentuk operator uniter totalnya adalah sebagai berikut:

$$\begin{aligned}
 U &= U_5 U_2 U_1 \\
 &= U_5 (\sigma_z \otimes I \otimes I \otimes I) U_1 \\
 &= \begin{pmatrix} e^{\beta_0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & e^{\beta_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{\beta_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{\beta_3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{\beta_4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{\beta_5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_9} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_{10}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_{11}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_{12}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_{13}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_{14}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_{15}} \end{pmatrix}
 \end{aligned}$$

[illegible]

[illegible]

(260)

maka dengan demikian kita dapatkan untuk nilai ρ_5 adalah

[illegible]

×

[illegible]

$$= \frac{1}{16} \begin{pmatrix} 0 & -8\rho_{0,1} & -8\rho_{0,2} & 0 & -8\rho_{0,4} & 0 & 0 & 0 & -8\rho_{0,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -8\rho_{1,0} & 0 & 0 & -8\rho_{1,3} & 0 & -8\rho_{1,5} & 0 & 0 & 0 & -8\rho_{1,9} & 0 & 0 & 0 & 0 & 0 & 0 \\ -8\rho_{2,0} & 0 & 0 & -8\rho_{2,3} & 0 & 0 & -8\rho_{2,6} & 0 & 0 & 0 & -8\rho_{2,10} & 0 & 0 & 0 & 0 & 0 \\ 0 & -8\rho_{3,1} & -8\rho_{3,2} & 0 & 0 & 0 & 0 & -8\rho_{3,7} & 0 & 0 & 0 & -8\rho_{3,11} & 0 & 0 & 0 & 0 \\ -8\rho_{4,0} & 0 & 0 & 0 & 0 & -8\rho_{4,5} & -8\rho_{4,6} & 0 & 0 & 0 & 0 & 0 & -8\rho_{4,12} & 0 & 0 & 0 \\ 0 & -8\rho_{4,1} & 0 & 0 & -8\rho_{5,4} & 0 & 0 & -8\rho_{5,7} & 0 & 0 & 0 & 0 & 0 & -8\rho_{5,13} & 0 & 0 \\ 0 & 0 & -8\rho_{6,2} & 0 & -8\rho_{6,4} & 0 & 0 & -8\rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & -8\rho_{6,14} & 0 \\ 0 & 0 & 0 & -8\rho_{7,3} & 0 & -8\rho_{7,5} & -8\rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -8\rho_{7,15} \\ -8\rho_{8,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -8\rho_{8,9} & -8\rho_{8,10} & 0 & -8\rho_{8,12} & 0 & 0 & 0 \\ 0 & -8\rho_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & -8\rho_{9,8} & 0 & 0 & -8\rho_{9,11} & 0 & -8\rho_{9,13} & 0 & 0 \\ 0 & 0 & -8\rho_{10,2} & 0 & 0 & 0 & 0 & 0 & -8\rho_{10,8} & 0 & 0 & -8\rho_{10,11} & 0 & 0 & -8\rho_{10,14} & 0 \\ 0 & 0 & 0 & -8\rho_{11,3} & 0 & 0 & 0 & 0 & 0 & -8\rho_{11,9} & -8\rho_{11,10} & 0 & 0 & 0 & 0 & -8\rho_{11,15} \\ 0 & 0 & 0 & 0 & -8\rho_{12,4} & 0 & 0 & 0 & -8\rho_{12,8} & 0 & 0 & 0 & 0 & -8\rho_{12,13} & -8\rho_{12,14} & 0 \\ 0 & 0 & 0 & 0 & 0 & -8\rho_{13,5} & 0 & 0 & 0 & -8\rho_{13,9} & 0 & 0 & -8\rho_{13,12} & 0 & 0 & -8\rho_{13,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & -8\rho_{14,6} & 0 & 0 & 0 & -8\rho_{14,10} & 0 & -8\rho_{14,12} & 0 & 0 & -8\rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -8\rho_{15,7} & 0 & 0 & 0 & -8\rho_{15,11} & 0 & -8\rho_{15,13} & -8\rho_{15,14} & 0 \end{pmatrix} \quad (26)$$

$$= \frac{1}{2} \begin{pmatrix} 0 & -\rho_{0,1} & -\rho_{0,2} & 0 & -\rho_{0,4} & 0 & 0 & 0 & -\rho_{0,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\rho_{1,0} & 0 & 0 & -\rho_{1,3} & 0 & -\rho_{1,5} & 0 & 0 & 0 & -\rho_{1,9} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\rho_{2,0} & 0 & 0 & -\rho_{2,3} & 0 & 0 & -\rho_{2,6} & 0 & 0 & 0 & \rho_{2,10} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\rho_{3,1} & -\rho_{3,2} & 0 & 0 & 0 & 0 & -\rho_{3,7} & 0 & 0 & 0 & -\rho_{3,11} & 0 & 0 & 0 & 0 \\ -\rho_{4,0} & 0 & 0 & 0 & 0 & -\rho_{4,5} & -\rho_{4,6} & 0 & 0 & 0 & 0 & 0 & -\rho_{4,12} & 0 & 0 & 0 \\ 0 & -\rho_{4,1} & 0 & 0 & -\rho_{5,4} & 0 & 0 & -\rho_{5,7} & 0 & 0 & 0 & 0 & 0 & -\rho_{5,13} & 0 & 0 \\ 0 & 0 & -\rho_{6,2} & 0 & -\rho_{6,4} & 0 & 0 & -\rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{6,14} & 0 \\ 0 & 0 & 0 & -\rho_{7,3} & 0 & -\rho_{7,5} & -\rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{7,15} \\ -\rho_{8,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{8,9} & -\rho_{8,10} & 0 & -\rho_{8,12} & 0 & 0 & 0 \\ 0 & -\rho_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{9,8} & 0 & 0 & -\rho_{9,11} & 0 & -\rho_{9,13} & 0 & 0 \\ 0 & 0 & -\rho_{10,2} & 0 & 0 & 0 & 0 & 0 & -\rho_{10,8} & 0 & 0 & -\rho_{10,11} & 0 & 0 & -\rho_{10,14} & 0 \\ 0 & 0 & 0 & -\rho_{11,3} & 0 & 0 & 0 & 0 & 0 & -\rho_{11,9} & -\rho_{11,10} & 0 & 0 & 0 & 0 & -\rho_{11,15} \\ 0 & 0 & 0 & 0 & -\rho_{12,4} & 0 & 0 & 0 & -\rho_{12,8} & 0 & 0 & 0 & 0 & -\rho_{12,13} & -\rho_{12,14} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\rho_{13,5} & 0 & 0 & 0 & -\rho_{13,9} & 0 & 0 & -\rho_{13,12} & 0 & 0 & -\rho_{13,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{14,6} & 0 & 0 & 0 & -\rho_{14,10} & 0 & -\rho_{14,12} & 0 & 0 & -\rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{15,7} & 0 & 0 & 0 & -\rho_{15,11} & 0 & -\rho_{15,13} & -\rho_{15,14} & 0 \end{pmatrix} \quad (262)$$

dengan

$$\begin{aligned}
\rho_{0,1} &= \rho_{1,0}^* = e^{\beta_0 - \beta_1} & \rho_{6,7} &= \rho_{7,6}^* = e^{\beta_6 - \beta_7} \\
\rho_{0,2} &= \rho_{2,0}^* = e^{\beta_0 - \beta_2} & \rho_{6,14} &= \rho_{14,6}^* = e^{\beta_6 - \beta_{14}} \\
\rho_{0,8} &= \rho_{8,0}^* = e^{\beta_8 - \beta_1} & \rho_{8,9} &= \rho_{9,8}^* = e^{\beta_8 - \beta_9} \\
\rho_{1,3} &= \rho_{3,1}^* = e^{\beta_1 - \beta_3} & \rho_{8,10} &= \rho_{10,8}^* = e^{\beta_8 - \beta_{10}} \\
\rho_{1,5} &= \rho_{5,1}^* = e^{\beta_1 - \beta_5} & \rho_{8,12} &= \rho_{12,8}^* = e^{\beta_8 - \beta_{12}} \\
\rho_{1,9} &= \rho_{9,1}^* = e^{\beta_1 - \beta_9} & \rho_{9,11} &= \rho_{11,9}^* = e^{\beta_9 - \beta_{11}} \\
\rho_{2,3} &= \rho_{3,2}^* = e^{\beta_2 - \beta_3} & \rho_{9,13} &= \rho_{13,9}^* = e^{\beta_9 - \beta_{13}} \\
\rho_{2,6} &= \rho_{6,2}^* = e^{\beta_2 - \beta_6} & \rho_{10,11} &= \rho_{11,10}^* = e^{\beta_{10} - \beta_{11}} \\
\rho_{2,10} &= \rho_{10,2}^* = e^{\beta_2 - \beta_{10}} & \rho_{10,14} &= \rho_{14,10}^* = e^{\beta_{10} - \beta_{14}} \\
\rho_{3,7} &= \rho_{7,3}^* = e^{\beta_3 - \beta_7} & \rho_{12,13} &= \rho_{13,12}^* = e^{\beta_{12} - \beta_{13}} \\
\rho_{3,11} &= \rho_{11,3}^* = e^{\beta_3 - \beta_{11}} & \rho_{12,14} &= \rho_{14,12}^* = e^{\beta_{12} - \beta_{14}} \\
\rho_{4,5} &= \rho_{5,4}^* = e^{\beta_4 - \beta_5} & \rho_{11,15} &= \rho_{15,11}^* = e^{\beta_{11} - \beta_{15}} \\
\rho_{4,6} &= \rho_{6,4}^* = e^{\beta_4 - \beta_6} & \rho_{13,15} &= \rho_{15,13}^* = e^{\beta_{13} - \beta_{15}} \\
\rho_{4,12} &= \rho_{12,4}^* = e^{\beta_4 - \beta_{12}} & \rho_{14,15} &= \rho_{15,14}^* = e^{\beta_{14} - \beta_{15}} \\
\rho_{5,7} &= \rho_{7,5}^* = e^{\beta_5 - \beta_7} & \rho_{15,7} &= \rho_{7,15}^* = e^{\beta_7 - \beta_{15}} \\
\rho_{5,13} &= \rho_{13,5}^* = e^{\beta_5 - \beta_{13}} & \rho_{0,4} &= \rho_{4,0}^* = e^{\beta_0 - \beta_4}
\end{aligned} \tag{263}$$

dan

$$\begin{array}{ll}
\beta_0 - \beta_1 = -i(E_{4,0} - E_{4,1})t & \beta_0 - \beta_2 = -i(E_{4,0} - E_{4,2})t \\
\beta_0 - \beta_4 = -i(E_{4,0} - E_{4,4})t & \beta_0 - \beta_8 = -i(E_{4,0} - E_{4,8})t \\
\beta_1 - \beta_3 = -i(E_{4,1} - E_{4,3})t & \beta_1 - \beta_5 = -i(E_{4,1} - E_{4,5})t \\
\beta_1 - \beta_9 = -i(E_{4,1} - E_{4,9})t & \beta_2 - \beta_3 = -i(E_{4,2} - E_{4,3})t \\
\beta_2 - \beta_6 = -i(E_{4,2} - E_{4,6})t & \beta_2 - \beta_{10} = -i(E_{4,2} - E_{4,10})t \\
\beta_3 - \beta_7 = -i(E_{4,3} - E_{4,7})t & \beta_3 - \beta_{11} = -i(E_{4,3} - E_{4,11})t \\
\beta_4 - \beta_5 = -i(E_{4,4} - E_{4,5})t & \beta_4 - \beta_6 = -i(E_{4,4} - E_{4,6})t \\
\beta_4 - \beta_{12} = -i(E_{4,4} - E_{4,12})t & \beta_5 - \beta_7 = -i(E_{4,5} - E_{4,7})t \\
\beta_5 - \beta_{13} = -i(E_{4,5} - E_{4,13})t & \beta_6 - \beta_7 = -i(E_{4,6} - E_{4,7})t \\
\beta_6 - \beta_{14} = -i(E_{4,6} - E_{4,14})t & \beta_8 - \beta_9 = -i(E_{4,8} - E_{4,9})t \\
\beta_8 - \beta_{10} = -i(E_{4,8} - E_{4,10})t & \beta_8 - \beta_{12} = -i(E_{4,8} - E_{4,12})t \\
\beta_9 - \beta_{11} = -i(E_{4,9} - E_{4,11})t & \beta_9 - \beta_{13} = -i(E_{4,9} - E_{4,13})t \\
\beta_{10} - \beta_{11} = -i(E_{4,10} - E_{4,11})t & \beta_{10} - \beta_{14} = -i(E_{4,10} - E_{4,14})t \\
\beta_{12} - \beta_{13} = -i(E_{4,12} - E_{4,13})t & \beta_{12} - \beta_{14} = -i(E_{4,12} - E_{4,14})t \\
\beta_{11} - \beta_{15} = -i(E_{4,11} - E_{4,15})t & \beta_{13} - \beta_{15} = -i(E_{4,13} - E_{4,15})t \\
\beta_{14} - \beta_{15} = -i(E_{4,14} - E_{4,15})t & \beta_{15} - \beta_7 = -i(E_{4,15} - E_{4,7})t
\end{array} \quad (264)$$

dan

$$\begin{aligned}
E_{4,0} - E_{4,1} &= \omega_0 + \pi J_{10} + \pi J_{20} + \pi J_{30} & E_{4,0} - E_{4,2} &= \omega_1 + \pi J_{10} + \pi J_{21} + \pi J_{31} \\
E_{4,0} - E_{4,4} &= \omega_2 + \pi J_{20} + \pi J_{21} + \pi J_{32} & E_{4,0} - E_{4,8} &= \omega_3 + \pi J_{30} + \pi J_{31} + \pi J_{32} \\
E_{4,1} - E_{4,3} &= \omega_1 - \pi J_{10} + \pi J_{21} + \pi J_{31} & E_{4,1} - E_{4,5} &= \omega_2 - \pi J_{20} + \pi J_{21} + \pi J_{32} \\
E_{4,1} - E_{4,9} &= \omega_3 - \pi J_{30} + \pi J_{310} + \pi J_{32} & E_{4,2} - E_{4,3} &= \omega_0 - \pi J_{10} + \pi J_{20} + \pi J_{30} \\
E_{4,2} - E_{4,6} &= \omega_2 + \pi J_{20} - \pi J_{21} + \pi J_{32} & E_{4,2} - E_{4,10} &= \omega_3 + \pi J_{30} - \pi J_{31} + \pi J_{32} \\
E_{4,3} - E_{4,7} &= \omega_2 - \pi J_{20} - \pi J_{21} + \pi J_{32} & E_{4,3} - E_{4,11} &= \omega_3 - \pi J_{30} - \pi J_{31} + \pi J_{32} \\
E_{4,4} - E_{4,5} &= \omega_0 + \pi J_{10} - \pi J_{20} + \pi J_{30} & E_{4,4} - E_{4,6} &= \omega_1 + \pi J_{10} - \pi J_{21} + \pi J_{31} \\
E_{4,4} - E_{4,12} &= \omega_3 + \pi J_{30} + \pi J_{31} - \pi J_{32} & E_{4,5} - E_{4,7} &= \omega_1 - \pi J_{10} - \pi J_{21} + \pi J_{31} \\
E_{4,5} - E_{4,13} &= \omega_3 - \pi J_{30} + \pi J_{31} - \pi J_{32} & E_{4,6} - E_{4,7} &= \omega_0 - \pi J_{10} - \pi J_{20} + \pi J_{30} \\
E_{4,6} - E_{4,14} &= \omega_3 + \pi J_{30} - \pi J_{31} - \pi J_{32} & E_{4,8} - E_{4,9} &= \omega_0 + \pi J_{10} + \pi J_{20} - \pi J_{30} \\
E_{4,8} - E_{4,10} &= \omega_1 + \pi J_{21} - \pi J_{31} + \pi J_{30} & E_{4,8} - E_{4,12} &= \omega_2 + \pi J_{20} + \pi J_{21} - \pi J_{32} \\
E_{4,9} - E_{4,11} &= \omega_1 - \pi J_{10} + \pi J_{21} - \pi J_{32} & E_{4,10} - E_{4,11} &= \omega_0 - \pi J_{10} + \pi J_{20} - \pi J_{30} \\
E_{4,10} - E_{4,14} &= \omega_2 + \pi J_{20} - \pi J_{21} - \pi J_{32} & E_{4,12} - E_{4,13} &= \omega_0 + \pi J_{10} - \pi J_{20} - \pi J_{30} \\
E_{4,12} - E_{4,14} &= \omega_1 + \pi J_{10} - \pi J_{21} - \pi J_{31} & E_{4,11} - E_{4,15} &= \omega_2 - \pi J_{20} - \pi J_{21} - \pi J_{32} \\
E_{4,13} - E_{4,15} &= \omega_1 - \pi J_{10} - \pi J_{21} - \pi J_{31} & E_{4,14} - E_{4,15} &= \omega_0 - \pi J_{10} - \pi J_{20} - \pi J_{30} \\
E_{4,9} - E_{4,13} &= \omega_2 - \pi J_{20} + \pi J_{21} - \pi J_{32} & E_{4,15} - E_{4,7} &= \omega_3 + \pi J_{30} + \pi J_{31} + \pi J_{32}
\end{aligned} \tag{265}$$

maka dapat ditentukan nilai magnetisasi dari masing-masing qubitnya yaitu sebagai berikut:

$$(I_{3+}\rho_5) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\times \frac{1}{2} \begin{pmatrix} 0 & -\rho_{0,1} & -\rho_{0,2} & 0 & -\rho_{0,4} & 0 & 0 & 0 & -\rho_{0,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\rho_{1,0} & 0 & 0 & -\rho_{1,3} & 0 & -\rho_{1,5} & 0 & 0 & 0 & -\rho_{1,9} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\rho_{2,0} & 0 & 0 & -\rho_{2,3} & 0 & 0 & -\rho_{2,6} & 0 & 0 & 0 & \rho_{2,10} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\rho_{3,1} & -\rho_{3,2} & 0 & 0 & 0 & 0 & -\rho_{3,7} & 0 & 0 & 0 & -\rho_{3,11} & 0 & 0 & 0 & 0 \\ -\rho_{4,0} & 0 & 0 & 0 & 0 & -\rho_{4,5} & -\rho_{4,6} & 0 & 0 & 0 & 0 & 0 & -\rho_{4,12} & 0 & 0 & 0 \\ 0 & -\rho_{4,1} & 0 & 0 & -\rho_{5,4} & 0 & 0 & -\rho_{5,7} & 0 & 0 & 0 & 0 & 0 & -\rho_{5,13} & 0 & 0 \\ 0 & 0 & -\rho_{6,2} & 0 & -\rho_{6,4} & 0 & 0 & -\rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{6,14} & 0 \\ 0 & 0 & 0 & -\rho_{7,3} & 0 & -\rho_{7,5} & -\rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{7,15} \\ -\rho_{8,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{8,9} & -\rho_{8,10} & 0 & -\rho_{8,12} & 0 & 0 & 0 \\ 0 & -\rho_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{9,8} & 0 & 0 & -\rho_{9,11} & 0 & -\rho_{9,13} & 0 & 0 \\ 0 & 0 & -\rho_{10,2} & 0 & 0 & 0 & 0 & 0 & -\rho_{10,8} & 0 & 0 & -\rho_{10,11} & 0 & 0 & -\rho_{10,14} & 0 \\ 0 & 0 & 0 & -\rho_{11,3} & 0 & 0 & 0 & 0 & 0 & -\rho_{11,9} & -\rho_{11,10} & 0 & 0 & 0 & 0 & -\rho_{11,15} \\ 0 & 0 & 0 & 0 & -\rho_{12,4} & 0 & 0 & 0 & -\rho_{12,8} & 0 & 0 & 0 & 0 & -\rho_{12,13} & -\rho_{12,14} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\rho_{13,5} & 0 & 0 & 0 & -\rho_{13,9} & 0 & 0 & -\rho_{13,12} & 0 & 0 & -\rho_{13,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{14,6} & 0 & 0 & 0 & -\rho_{14,10} & 0 & -\rho_{14,12} & 0 & 0 & -\rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{15,7} & 0 & 0 & 0 & -\rho_{15,11} & 0 & -\rho_{15,13} & -\rho_{15,14} & 0 \end{pmatrix}$$

[illegible]

$$\begin{aligned}
M_{3+} &= \langle I_{3+} \rangle = tr (I_{3+} \rho_5) \\
&= -(\rho_{8,0} + \rho_{9,1} + \rho_{10,2} + \rho_{11,3} + \rho_{12,4} + \rho_{13,5} + \rho_{14,6} + \rho_{15,7}) \quad (266)
\end{aligned}$$

dan

$$(I_{2+} \rho_5) = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\times \frac{1}{2} \begin{pmatrix} 0 & -\rho_{0,1} & -\rho_{0,2} & 0 & -\rho_{0,4} & 0 & 0 & 0 & -\rho_{0,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\rho_{1,0} & 0 & 0 & -\rho_{1,3} & 0 & -\rho_{1,5} & 0 & 0 & 0 & -\rho_{1,9} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\rho_{2,0} & 0 & 0 & -\rho_{2,3} & 0 & 0 & -\rho_{2,6} & 0 & 0 & 0 & \rho_{2,10} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\rho_{3,1} & -\rho_{3,2} & 0 & 0 & 0 & 0 & -\rho_{3,7} & 0 & 0 & 0 & -\rho_{3,11} & 0 & 0 & 0 & 0 \\ -\rho_{4,0} & 0 & 0 & 0 & 0 & -\rho_{4,5} & -\rho_{4,6} & 0 & 0 & 0 & 0 & 0 & -\rho_{4,12} & 0 & 0 & 0 \\ 0 & -\rho_{4,1} & 0 & 0 & -\rho_{5,4} & 0 & 0 & -\rho_{5,7} & 0 & 0 & 0 & 0 & 0 & -\rho_{5,13} & 0 & 0 \\ 0 & 0 & -\rho_{6,2} & 0 & -\rho_{6,4} & 0 & 0 & -\rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{6,14} & 0 \\ 0 & 0 & 0 & -\rho_{7,3} & 0 & -\rho_{7,5} & -\rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{7,15} \\ -\rho_{8,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{8,9} & -\rho_{8,10} & 0 & -\rho_{8,12} & 0 & 0 & 0 \\ 0 & -\rho_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{9,8} & 0 & 0 & -\rho_{9,11} & 0 & -\rho_{9,13} & 0 & 0 \\ 0 & 0 & -\rho_{10,2} & 0 & 0 & 0 & 0 & 0 & -\rho_{10,8} & 0 & 0 & -\rho_{10,11} & 0 & 0 & -\rho_{10,14} & 0 \\ 0 & 0 & 0 & -\rho_{11,3} & 0 & 0 & 0 & 0 & 0 & -\rho_{11,9} & -\rho_{11,10} & 0 & 0 & 0 & 0 & -\rho_{11,15} \\ 0 & 0 & 0 & 0 & -\rho_{12,4} & 0 & 0 & 0 & -\rho_{12,8} & 0 & 0 & 0 & 0 & -\rho_{12,13} & -\rho_{12,14} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\rho_{13,5} & 0 & 0 & 0 & -\rho_{13,9} & 0 & 0 & -\rho_{13,12} & 0 & 0 & -\rho_{13,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{14,6} & 0 & 0 & 0 & -\rho_{14,10} & 0 & -\rho_{14,12} & 0 & 0 & -\rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{15,7} & 0 & 0 & 0 & -\rho_{15,11} & 0 & -\rho_{15,13} & -\rho_{15,14} & 0 \end{pmatrix}$$

(267)

$$\begin{aligned}
M_{2+} &= \langle I_{2+} \rangle = tr(I_{2+} \rho_5) \\
&= -(\rho_{4,0} + \rho_{5,1} + \rho_{6,2} + \rho_{7,3} + \rho_{12,8} + \rho_{13,9} + \rho_{14,10} + \rho_{15,11}) \quad (268)
\end{aligned}$$

dan

$$(I_{1+} \rho_5) = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\times \frac{1}{2} \begin{pmatrix} 0 & -\rho_{0,1} & -\rho_{0,2} & 0 & -\rho_{0,4} & 0 & 0 & 0 & -\rho_{0,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\rho_{1,0} & 0 & 0 & -\rho_{1,3} & 0 & -\rho_{1,5} & 0 & 0 & 0 & -\rho_{1,9} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\rho_{2,0} & 0 & 0 & -\rho_{2,3} & 0 & 0 & -\rho_{2,6} & 0 & 0 & 0 & \rho_{2,10} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\rho_{3,1} & -\rho_{3,2} & 0 & 0 & 0 & 0 & -\rho_{3,7} & 0 & 0 & 0 & -\rho_{3,11} & 0 & 0 & 0 & 0 \\ -\rho_{4,0} & 0 & 0 & 0 & 0 & -\rho_{4,5} & -\rho_{4,6} & 0 & 0 & 0 & 0 & 0 & -\rho_{4,12} & 0 & 0 & 0 \\ 0 & -\rho_{4,1} & 0 & 0 & -\rho_{5,4} & 0 & 0 & -\rho_{5,7} & 0 & 0 & 0 & 0 & 0 & -\rho_{5,13} & 0 & 0 \\ 0 & 0 & -\rho_{6,2} & 0 & -\rho_{6,4} & 0 & 0 & -\rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{6,14} & 0 \\ 0 & 0 & 0 & -\rho_{7,3} & 0 & -\rho_{7,5} & -\rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{7,15} \\ -\rho_{8,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{8,9} & -\rho_{8,10} & 0 & -\rho_{8,12} & 0 & 0 & 0 \\ 0 & -\rho_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{9,8} & 0 & 0 & -\rho_{9,11} & 0 & -\rho_{9,13} & 0 & 0 \\ 0 & 0 & -\rho_{10,2} & 0 & 0 & 0 & 0 & 0 & -\rho_{10,8} & 0 & 0 & -\rho_{10,11} & 0 & 0 & -\rho_{10,14} & 0 \\ 0 & 0 & 0 & -\rho_{11,3} & 0 & 0 & 0 & 0 & 0 & -\rho_{11,9} & -\rho_{11,10} & 0 & 0 & 0 & 0 & -\rho_{11,15} \\ 0 & 0 & 0 & 0 & -\rho_{12,4} & 0 & 0 & 0 & -\rho_{12,8} & 0 & 0 & 0 & 0 & -\rho_{12,13} & -\rho_{12,14} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\rho_{13,5} & 0 & 0 & 0 & -\rho_{13,9} & 0 & 0 & -\rho_{13,12} & 0 & 0 & -\rho_{13,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{14,6} & 0 & 0 & 0 & -\rho_{14,10} & 0 & -\rho_{14,12} & 0 & 0 & -\rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{15,7} & 0 & 0 & 0 & -\rho_{15,11} & 0 & -\rho_{15,13} & -\rho_{15,14} & 0 \end{pmatrix}$$

[illegible]

$$\begin{aligned}
M_{1+} &= \langle I_{1+} \rangle = tr(I_{1+} \rho_5) \\
&= -(\rho_{2,0} + \rho_{3,1} + \rho_{6,4} + \rho_{7,5} + \rho_{10,8} + \rho_{11,9} + \rho_{14,12} + \rho_{15,13}) \quad (270)
\end{aligned}$$

dan

$$(I_{0+} \rho_5) = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\times \frac{1}{2} \begin{pmatrix} 0 & -\rho_{0,1} & -\rho_{0,2} & 0 & -\rho_{0,4} & 0 & 0 & 0 & -\rho_{0,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\rho_{1,0} & 0 & 0 & -\rho_{1,3} & 0 & -\rho_{1,5} & 0 & 0 & 0 & -\rho_{1,9} & 0 & 0 & 0 & 0 & 0 & 0 \\ -\rho_{2,0} & 0 & 0 & -\rho_{2,3} & 0 & 0 & -\rho_{2,6} & 0 & 0 & 0 & \rho_{2,10} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\rho_{3,1} & -\rho_{3,2} & 0 & 0 & 0 & 0 & -\rho_{3,7} & 0 & 0 & 0 & -\rho_{3,11} & 0 & 0 & 0 & 0 \\ -\rho_{4,0} & 0 & 0 & 0 & 0 & -\rho_{4,5} & -\rho_{4,6} & 0 & 0 & 0 & 0 & 0 & -\rho_{4,12} & 0 & 0 & 0 \\ 0 & -\rho_{4,1} & 0 & 0 & -\rho_{5,4} & 0 & 0 & -\rho_{5,7} & 0 & 0 & 0 & 0 & 0 & -\rho_{5,13} & 0 & 0 \\ 0 & 0 & -\rho_{6,2} & 0 & -\rho_{6,4} & 0 & 0 & -\rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{6,14} & 0 \\ 0 & 0 & 0 & -\rho_{7,3} & 0 & -\rho_{7,5} & -\rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{7,15} \\ -\rho_{8,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{8,9} & -\rho_{8,10} & 0 & -\rho_{8,12} & 0 & 0 & 0 \\ 0 & -\rho_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{9,8} & 0 & 0 & -\rho_{9,11} & 0 & -\rho_{9,13} & 0 & 0 \\ 0 & 0 & -\rho_{10,2} & 0 & 0 & 0 & 0 & 0 & -\rho_{10,8} & 0 & 0 & -\rho_{10,11} & 0 & 0 & -\rho_{10,14} & 0 \\ 0 & 0 & 0 & -\rho_{11,3} & 0 & 0 & 0 & 0 & 0 & -\rho_{11,9} & -\rho_{11,10} & 0 & 0 & 0 & 0 & -\rho_{11,15} \\ 0 & 0 & 0 & 0 & -\rho_{12,4} & 0 & 0 & 0 & -\rho_{12,8} & 0 & 0 & 0 & 0 & -\rho_{12,13} & -\rho_{12,14} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\rho_{13,5} & 0 & 0 & 0 & -\rho_{13,9} & 0 & 0 & -\rho_{13,12} & 0 & 0 & -\rho_{13,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{14,6} & 0 & 0 & 0 & -\rho_{14,10} & 0 & -\rho_{14,12} & 0 & 0 & -\rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{15,7} & 0 & 0 & 0 & -\rho_{15,11} & 0 & -\rho_{15,13} & -\rho_{15,14} & 0 \end{pmatrix}$$

(271)

$$M_{0+} = \langle I_{0+} \rangle = tr (I_{0+} \rho_5) \quad (272)$$

$$= -(\rho_{1,0} + \rho_{3,2} + \rho_{5,2} + \rho_{7,6} + \rho_{9,8} + \rho_{11,10} + \rho_{13,12} + \rho_{15,14})$$

maka dengan demikian nilai magnetisasi total dari sistem adalah sebagai berikut:

$$M_{tot} = M_{3+} + M_{2+} + M_{1+} + M_{0+}$$

$$= -(\rho_{8,0} + \rho_{9,1} + \rho_{10,2} + \rho_{11,3} + \rho_{12,4} + \rho_{13,5} + \rho_{14,6} + \rho_{15,7})$$

$$- \rho_{4,0} - \rho_{5,1} - \rho_{6,2} - \rho_{7,3} - \rho_{12,8} - \rho_{13,9} - \rho_{14,10} - \rho_{15,11}$$

$$- (\rho_{2,0} + \rho_{3,1} + \rho_{6,4} + \rho_{7,5} + \rho_{10,8} + \rho_{11,9} + \rho_{14,12} + \rho_{15,13})$$

$$- (\rho_{1,0} + \rho_{3,2} + \rho_{5,4} + \rho_{7,6} + \rho_{9,8} + \rho_{11,10} + \rho_{13,12} + \rho_{15,14})$$

$$= e^{i(\omega_3 + \pi J_{30} + \pi J_{31} + \pi J_{32})t} - e^{i(\omega_3 - \pi J_{30} + \pi J_{31} + \pi J_{32})t}$$

$$- e^{i(\omega_3 + \pi J_{30} - \pi J_{31} + \pi J_{32})t} - e^{i(\omega_3 - \pi J_{30} - \pi J_{31} + \pi J_{32})t}$$

$$- e^{i(\omega_3 + \pi J_{30} + \pi J_{31} - \pi J_{32})t} - e^{i(\omega_3 - \pi J_{30} + \pi J_{31} - \pi J_{32})t}$$

$$- e^{i(\omega_3 + \pi J_{30} - \pi J_{31} - \pi J_{32})t} - e^{i(\omega_3 - \pi J_{30} - \pi J_{31} - \pi J_{32})t}$$

$$- e^{i(\omega_2 + \pi J_{20} + \pi J_{21} + \pi J_{32})t} - e^{i(\omega_2 - \pi J_{20} + \pi J_{21} + \pi J_{32})t}$$

$$- e^{i(\omega_2 + \pi J_{20} - \pi J_{21} + \pi J_{32})t} - e^{i(\omega_2 - \pi J_{20} - \pi J_{21} + \pi J_{32})t} \quad (273)$$

$$- e^{i(\omega_2 + \pi J_{20} + \pi J_{21} - \pi J_{32})t} - e^{i(\omega_2 - \pi J_{20} + \pi J_{21} - \pi J_{32})t}$$

$$- e^{i(\omega_2 + \pi J_{20} - \pi J_{21} - \pi J_{32})t} - e^{i(\omega_2 - \pi J_{20} - \pi J_{21} - \pi J_{32})t}$$

$$- e^{i(\omega_1 + \pi J_{10} + \pi J_{21} + \pi J_{31})t} - e^{i(\omega_1 - \pi J_{10} + \pi J_{21} + \pi J_{31})t}$$

$$- e^{i(\omega_1 + \pi J_{10} - \pi J_{21} + \pi J_{31})t} - e^{i(\omega_1 - \pi J_{10} - \pi J_{21} + \pi J_{31})t}$$

$$- e^{i(\omega_1 + \pi J_{21} - \pi J_{31} + \pi J_{30})t} - e^{i(\omega_1 - \pi J_{10} + \pi J_{21} - \pi J_{32})t}$$

$$- e^{i(\omega_1 + \pi J_{10} - \pi J_{21} - \pi J_{31})t} - e^{i(\omega_1 - \pi J_{10} - \pi J_{21} - \pi J_{31})t}$$

$$- e^{i(\omega_0 + \pi J_{10} + \pi J_{20} + \pi J_{30})t} - e^{i(\omega_0 - \pi J_{10} + \pi J_{20} + \pi J_{30})t}$$

$$- e^{i(\omega_0 + \pi J_{10} - \pi J_{20} + \pi J_{30})t} - e^{i(\omega_0 - \pi J_{10} - \pi J_{20} + \pi J_{30})t}$$

$$- e^{i(\omega_0 + \pi J_{10} + \pi J_{20} - \pi J_{30})t} - e^{i(\omega_0 - \pi J_{10} + \pi J_{20} - \pi J_{30})t}$$

$$- e^{i(\omega_0 + \pi J_{10} - \pi J_{20} - \pi J_{30})t} - e^{i(\omega_0 - \pi J_{10} - \pi J_{20} - \pi J_{30})t}$$

kemudian didapatkan juga untuk sinyal FID sebagai berikut:

$$\begin{aligned}
S &= M_{tot} e^{-R_2 t} \\
&= e^{-(R_2 - i(\omega_3 + \pi J_{30} + \pi J_{31} + \pi J_{32}))t} - e^{-(R_2 - i(\omega_3 - \pi J_{30} + \pi J_{31} + \pi J_{32}))t} \\
&- e^{-(R_2 - i(\omega_3 + \pi J_{30} - \pi J_{31} + \pi J_{32}))t} - e^{-(R_2 - i(\omega_3 - \pi J_{30} - \pi J_{31} + \pi J_{32}))t} \\
&- e^{-(R_2 - i(\omega_3 + \pi J_{30} + \pi J_{31} - \pi J_{32}))t} - e^{-(R_2 - i(\omega_3 - \pi J_{30} + \pi J_{31} - \pi J_{32}))t} \\
&- e^{-(R_2 - i(\omega_3 + \pi J_{30} - \pi J_{31} - \pi J_{32}))t} - e^{-(R_2 - i(\omega_3 - \pi J_{30} - \pi J_{31} - \pi J_{32}))t} \\
&- e^{-(R_2 - i(\omega_2 + \pi J_{20} + \pi J_{21} + \pi J_{32}))t} - e^{-(R_2 - i(\omega_2 - \pi J_{20} + \pi J_{21} + \pi J_{32}))t} \\
&- e^{-(R_2 - i(\omega_2 + \pi J_{20} - \pi J_{21} + \pi J_{32}))t} - e^{-(R_2 - i(\omega_2 - \pi J_{20} - \pi J_{21} + \pi J_{32}))t} \\
&- e^{-(R_2 - i(\omega_2 + \pi J_{20} + \pi J_{21} - \pi J_{32}))t} - e^{-(R_2 - i(\omega_2 - \pi J_{20} + \pi J_{21} - \pi J_{32}))t} \\
&- e^{-(R_2 - i(\omega_2 + \pi J_{20} - \pi J_{21} - \pi J_{32}))t} - e^{-(R_2 - i(\omega_2 - \pi J_{20} - \pi J_{21} - \pi J_{32}))t} \quad (274) \\
&- e^{-(R_2 - i(\omega_1 + \pi J_{10} + \pi J_{21} + \pi J_{31}))t} - e^{-(R_2 - i(\omega_1 - \pi J_{10} + \pi J_{21} + \pi J_{31}))t} \\
&- e^{-(R_2 - i(\omega_1 + \pi J_{10} - \pi J_{21} + \pi J_{31}))t} - e^{-(R_2 - i(\omega_1 - \pi J_{10} - \pi J_{21} + \pi J_{31}))t} \\
&- e^{-(R_2 - i(\omega_1 + \pi J_{21} - \pi J_{31} + \pi J_{30}))t} - e^{-(R_2 - i(\omega_1 - \pi J_{10} + \pi J_{21} - \pi J_{32}))t} \\
&- e^{-(R_2 - i(\omega_1 + \pi J_{10} - \pi J_{21} - \pi J_{31}))t} - e^{-(R_2 - i(\omega_1 - \pi J_{10} - \pi J_{21} - \pi J_{31}))t} \\
&- e^{-(R_2 - i(\omega_0 + \pi J_{10} + \pi J_{20} + \pi J_{30}))t} - e^{-(R_2 - i(\omega_0 - \pi J_{10} + \pi J_{20} + \pi J_{30}))t} \\
&- e^{-(R_2 - i(\omega_0 + \pi J_{10} - \pi J_{20} + \pi J_{30}))t} - e^{-(R_2 - i(\omega_0 - \pi J_{10} - \pi J_{20} + \pi J_{30}))t} \\
&- e^{-(R_2 - i(\omega_0 + \pi J_{10} + \pi J_{20} - \pi J_{30}))t} - e^{-(R_2 - i(\omega_0 - \pi J_{10} + \pi J_{20} - \pi J_{30}))t} \\
&- e^{-(R_2 - i(\omega_0 + \pi J_{10} - \pi J_{20} - \pi J_{30}))t} - e^{-(R_2 - i(\omega_0 - \pi J_{10} - \pi J_{20} - \pi J_{30}))t}
\end{aligned}$$

dimana $R_2 = \frac{1}{T_2}$

kemudian dapat di cari terlebih dahulu untuk perkalian antara $S(t) e^{-i\omega t}$ yaitu sebagai berikut:

$$\begin{aligned}
S(t) e^{-i\omega t} = & e^{-(R_2-i(\omega_3-\omega+\pi J_{30}+\pi J_{31}+\pi J_{32}))t} - e^{-(R_2-i(\omega_3-\omega-\pi J_{30}+\pi J_{31}+\pi J_{32}))t} \\
& - e^{-(R_2-i(\omega_3-\omega+\pi J_{30}-\pi J_{31}+\pi J_{32}))t} - e^{-(R_2-i(\omega_3-\omega-\pi J_{30}-\pi J_{31}+\pi J_{32}))t} \\
& - e^{-(R_2-i(\omega_3-\omega+\pi J_{30}+\pi J_{31}-\pi J_{32}))t} - e^{-(R_2-i(\omega_3-\omega-\pi J_{30}+\pi J_{31}-\pi J_{32}))t} \\
& - e^{-(R_2-i(\omega_3-\omega+\pi J_{30}-\pi J_{31}-\pi J_{32}))t} - e^{-(R_2-i(\omega_3-\omega-\pi J_{30}-\pi J_{31}-\pi J_{32}))t} \\
& - e^{-(R_2-i(\omega_2-\omega+\pi J_{20}+\pi J_{21}+\pi J_{32}))t} - e^{-(R_2-i(\omega_2-\omega-\pi J_{20}+\pi J_{21}+\pi J_{32}))t} \\
& - e^{-(R_2-i(\omega_2-\omega+\pi J_{20}-\pi J_{21}+\pi J_{32}))t} - e^{-(R_2-i(\omega_2-\omega-\pi J_{20}-\pi J_{21}+\pi J_{32}))t} \\
& - e^{-(R_2-i(\omega_2-\omega+\pi J_{20}+\pi J_{21}-\pi J_{32}))t} - e^{-(R_2-i(\omega_2-\omega-\pi J_{20}+\pi J_{21}-\pi J_{32}))t} \\
& - e^{-(R_2-i(\omega_2-\omega+\pi J_{20}-\pi J_{21}-\pi J_{32}))t} - e^{-(R_2-i(\omega_2-\omega-\pi J_{20}-\pi J_{21}-\pi J_{32}))t} \\
& - e^{-(R_2-i(\omega_1-\omega+\pi J_{10}+\pi J_{21}+\pi J_{31}))t} - e^{-(R_2-i(\omega_1-\omega-\pi J_{10}+\pi J_{21}+\pi J_{31}))t} \\
& - e^{-(R_2-i(\omega_1-\omega+\pi J_{10}-\pi J_{21}+\pi J_{31}))t} - e^{-(R_2-i(\omega_1-\omega-\pi J_{10}-\pi J_{21}+\pi J_{31}))t} \\
& - e^{-(R_2-i(\omega_1-\omega+\pi J_{21}-\pi J_{31}+\pi J_{30}))t} - e^{-(R_2-i(\omega_1-\omega-\pi J_{10}+\pi J_{21}-\pi J_{32}))t} \\
& - e^{-(R_2-i(\omega_1-\omega+\pi J_{10}-\pi J_{21}-\pi J_{31}))t} - e^{-(R_2-i(\omega_1-\omega-\pi J_{10}-\pi J_{21}-\pi J_{31}))t} \\
& - e^{-(R_2-i(\omega_0-\omega+\pi J_{10}+\pi J_{20}+\pi J_{30}))t} - e^{-(R_2-i(\omega_0-\omega-\pi J_{10}+\pi J_{20}+\pi J_{30}))t} \\
& - e^{-(R_2-i(\omega_0-\omega+\pi J_{10}-\pi J_{20}+\pi J_{30}))t} - e^{-(R_2-i(\omega_0-\omega-\pi J_{10}-\pi J_{20}+\pi J_{30}))t} \\
& - e^{-(R_2-i(\omega_0-\omega+\pi J_{10}+\pi J_{20}-\pi J_{30}))t} - e^{-(R_2-i(\omega_0-\omega-\pi J_{10}+\pi J_{20}-\pi J_{30}))t} \\
& - e^{-(R_2-i(\omega_0-\omega+\pi J_{10}-\pi J_{20}-\pi J_{30}))t} - e^{-(R_2-i(\omega_0-\omega-\pi J_{10}-\pi J_{20}-\pi J_{30}))t}
\end{aligned} \tag{275}$$

Transformasi Fourirer dari sinyal FiD adalah sebagai berikut:

[illegible]

$$\begin{aligned}
&= \frac{-1}{(R_2 - i(\omega_3 - \omega + \pi J_{30} + \pi J_{31} + \pi J_{32}))} - \frac{1}{(R_2 - i(\omega_3 - \omega - \pi J_{30} + \pi J_{31} + \pi J_{32}))} \\
&\quad - \frac{1}{(R_2 - i(\omega_3 - \omega + \pi J_{30} - \pi J_{31} + \pi J_{32}))} - \frac{1}{(R_2 - i(\omega_3 - \omega - \pi J_{30} - \pi J_{31} + \pi J_{32}))} \\
&\quad - \frac{1}{(R_2 - i(\omega_3 - \omega - \pi J_{30} + \pi J_{31} + \pi J_{32}))} - \frac{1}{(R_2 - i(\omega_3 - \omega - \pi J_{30} + \pi J_{31} - \pi J_{32}))} \\
&\quad - \frac{1}{(R_2 - i(\omega_3 - \omega + \pi J_{30} - \pi J_{31} - \pi J_{32}))} - \frac{1}{(R_2 - i(\omega_3 - \omega - \pi J_{30} - \pi J_{31} - \pi J_{32}))} \\
&\quad - \frac{1}{(R_2 - i(\omega_2 - \omega + \pi J_{20} + \pi J_{21} + \pi J_{32}))} - \frac{1}{(R_2 - i(\omega_2 - \omega - \pi J_{20} + \pi J_{21} + \pi J_{32}))} \\
&\quad - \frac{1}{(R_2 - i(\omega_2 - \omega + \pi J_{20} - \pi J_{21} + \pi J_{32}))} - \frac{1}{(R_2 - i(\omega_2 - \omega - \pi J_{20} - \pi J_{21} + \pi J_{32}))} \\
&\quad - \frac{1}{(R_2 - i(\omega_2 - \omega + \pi J_{20} + \pi J_{21} - \pi J_{32}))} - \frac{1}{(R_2 - i(\omega_2 - \omega - \pi J_{20} + \pi J_{21} - \pi J_{32}))} \\
&\quad - \frac{1}{(R_2 - i(\omega_2 - \omega + \pi J_{20} - \pi J_{21} - \pi J_{32}))} - \frac{1}{(R_2 - i(\omega_2 - \omega - \pi J_{20} - \pi J_{21} - \pi J_{32}))} \\
&\quad - \frac{1}{(R_2 - i(\omega_1 - \omega + \pi J_{10} + \pi J_{21} + \pi J_{31}))} - \frac{1}{(R_2 - i(\omega_1 - \omega - \pi J_{10} + \pi J_{21} + \pi J_{31}))} \\
&\quad - \frac{1}{(R_2 - i(\omega_1 - \omega + \pi J_{10} - \pi J_{21} + \pi J_{31}))} - \frac{1}{(R_2 - i(\omega_1 - \omega - \pi J_{10} - \pi J_{21} + \pi J_{31}))} \\
&\quad - \frac{1}{(R_2 - i(\omega_1 - \omega + \pi J_{21} - \pi J_{31} + \pi J_{30}))} - \frac{1}{(R_2 - i(\omega_1 - \omega - \pi J_{10} + \pi J_{21} - \pi J_{32}))} \\
&\quad - \frac{1}{(R_2 - i(\omega_1 - \omega + \pi J_{10} - \pi J_{21} - \pi J_{31}))} - \frac{1}{(R_2 - i(\omega_1 - \omega - \pi J_{10} - \pi J_{21} - \pi J_{31}))} \\
&\quad - \frac{1}{(R_2 - i(\omega_0 - \omega + \pi J_{10} + \pi J_{20} + \pi J_{30}))} - \frac{1}{(R_2 - i(\omega_0 - \omega - \pi J_{10} + \pi J_{20} + \pi J_{30}))} \\
&\quad - \frac{1}{(R_2 - i(\omega_0 - \omega + \pi J_{10} - \pi J_{20} + \pi J_{30}))} - \frac{1}{(R_2 - i(\omega_0 - \omega - \pi J_{10} - \pi J_{20} + \pi J_{30}))} \\
&\quad - \frac{1}{(R_2 - i(\omega_0 - \omega + \pi J_{10} + \pi J_{20} - \pi J_{30}))} - \frac{1}{(R_2 - i(\omega_0 - \omega - \pi J_{10} + \pi J_{20} - \pi J_{30}))} \\
&\quad - \frac{1}{(R_2 - i(\omega_0 - \omega + \pi J_{10} - \pi J_{20} - \pi J_{30}))} - \frac{1}{(R_2 - i(\omega_0 - \omega - \pi J_{10} - \pi J_{20} - \pi J_{30}))}
\end{aligned}$$

maka dengan demikian diperoleh :

$$\begin{aligned}
A(\omega) = & \frac{-R_2}{R_2^2 + (\omega_3 - \omega + \pi J_{30} + \pi J_{31} + \pi J_{32})^2} - \frac{R_2}{R_2^2 + (\omega_3 - \omega - \pi J_{30} + \pi J_{31} + \pi J_{32})^2} \\
& - \frac{R_2}{R_2^2 - (\omega_3 - \omega + \pi J_{30} - \pi J_{31} + \pi J_{32})^2} - \frac{R_2}{R_2^2 + (\omega_3 - \omega - \pi J_{30} - \pi J_{31} + \pi J_{32})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_3 - \omega - \pi J_{30} + \pi J_{31} - \pi J_{32})^2} - \frac{R_2}{R_2^2 + (\omega_3 - \omega - \pi J_{30} - \pi J_{31} - \pi J_{32})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_3 - \omega + \pi J_{30} - \pi J_{31} - \pi J_{32})^2} - \frac{R_2}{R_2^2 + (\omega_3 - \omega - \pi J_{30} - \pi J_{31} - \pi J_{32})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_2 - \omega + \pi J_{20} + \pi J_{21} + \pi J_{32})^2} - \frac{R_2}{R_2^2 + (\omega_2 - \omega - \pi J_{20} + \pi J_{21} + \pi J_{32})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_2 - \omega + \pi J_{20} - \pi J_{21} + \pi J_{32})^2} - \frac{R_2}{R_2^2 + (\omega_2 - \omega - \pi J_{20} - \pi J_{21} + \pi J_{32})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_2 - \omega + \pi J_{20} + \pi J_{21} - \pi J_{32})^2} - \frac{R_2}{R_2^2 + (\omega_2 - \omega - \pi J_{20} + \pi J_{21} - \pi J_{32})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_2 - \omega + \pi J_{20} - \pi J_{21} - \pi J_{32})^2} - \frac{R_2}{R_2^2 + (\omega_2 - \omega - \pi J_{20} - \pi J_{21} - \pi J_{32})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_1 - \omega + \pi J_{10} + \pi J_{21} + \pi J_{31})^2} - \frac{R_2}{R_2^2 + (\omega_1 - \omega - \pi J_{10} + \pi J_{21} + \pi J_{31})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_1 - \omega + \pi J_{10} - \pi J_{21} + \pi J_{31})^2} - \frac{R_2}{R_2^2 + (\omega_1 - \omega - \pi J_{10} - \pi J_{21} + \pi J_{31})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_1 - \omega + \pi J_{21} - \pi J_{31} + \pi J_{30})^2} - \frac{R_2}{R_2^2 + (\omega_1 - \omega - \pi J_{10} + \pi J_{21} - \pi J_{32})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_1 - \omega + \pi J_{10} - \pi J_{21} - \pi J_{31})^2} - \frac{R_2}{R_2^2 + (\omega_1 - \omega - \pi J_{10} - \pi J_{21} - \pi J_{31})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_0 - \omega + \pi J_{10} + \pi J_{20} + \pi J_{30})^2} - \frac{R_2}{R_2^2 + (\omega_0 - \omega - \pi J_{10} + \pi J_{20} + \pi J_{30})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_0 - \omega + \pi J_{10} - \pi J_{20} + \pi J_{30})^2} - \frac{R_2}{R_2^2 + (\omega_0 - \omega - \pi J_{10} - \pi J_{20} + \pi J_{30})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_0 - \omega + \pi J_{10} + \pi J_{20} - \pi J_{30})^2} - \frac{R_2}{R_2^2 + (\omega_0 - \omega - \pi J_{10} + \pi J_{20} - \pi J_{30})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_0 - \omega + \pi J_{10} - \pi J_{20} - \pi J_{30})^2} - \frac{R_2}{R_2^2 + (\omega_0 - \omega - \pi J_{10} - \pi J_{20} - \pi J_{30})^2}
\end{aligned} \tag{276}$$

dan diperoleh juga:

[illegible]

7.4 Penerapan Algoritma Deutsch jozsa 4 qubit untuk $U_2 = U_5$

apabila diambil kasus untuk $U_2 = I \otimes ([I \otimes \sigma_z] \oplus [\sigma_z \otimes \sigma_z])$ maka diperoleh untuk bentuk operator uniter totalnya adalah sebagai berikut:

$$\begin{aligned}
 U &= U_5 U_2 U_1 \\
 &= U_5 (\sigma_z \otimes I \otimes I \otimes I) U_1 \\
 &= \begin{pmatrix} e^{\beta_0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & e^{\beta_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{\beta_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{\beta_3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{\beta_4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{\beta_5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_9} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_{10}} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_{11}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_{12}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_{13}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_{14}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\beta_{15}} \end{pmatrix}
 \end{aligned}$$

[illegible]

[illegible]

[illegible]

×

[illegible]

$$= \frac{1}{16} \begin{pmatrix} 0 & 8\rho_{0,1} & -8\rho_{0,2} & 0 & 8\rho_{0,4} & 0 & 0 & 0 & 8\rho_{0,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -8\rho_{1,0} & 0 & 0 & 8\rho_{1,3} & 0 & 8\rho_{1,5} & 0 & 0 & 0 & 8\rho_{1,9} & 0 & 0 & 0 & 0 & 0 & 0 \\ 8\rho_{2,0} & 0 & 0 & -8\rho_{2,3} & 0 & 0 & -8\rho_{2,6} & 0 & 0 & 0 & 8\rho_{2,10} & 0 & 0 & 0 & 0 & 0 \\ 0 & 8\rho_{3,1} & -8\rho_{3,2} & 0 & 0 & 0 & 0 & -8\rho_{3,7} & 0 & 0 & 0 & 8\rho_{3,11} & 0 & 0 & 0 & 0 \\ 8\rho_{4,0} & 0 & 0 & 0 & 0 & -8\rho_{4,5} & -8\rho_{4,6} & 0 & 0 & 0 & 0 & 0 & 8\rho_{4,12} & 0 & 0 & 0 \\ 0 & 8\rho_{5,1} & 0 & 0 & -8\rho_{5,4} & 0 & 0 & -8\rho_{5,7} & 0 & 0 & 0 & 0 & 0 & 8\rho_{5,13} & 0 & 0 \\ 0 & 0 & -8\rho_{6,2} & 0 & -8\rho_{6,4} & 0 & 0 & -8\rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & 8\rho_{6,14} & 0 \\ 0 & 0 & 0 & -8\rho_{7,3} & 0 & -8\rho_{7,5} & -8\rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8\rho_{7,15} \\ 8\rho_{8,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -8\rho_{8,9} & 8\rho_{8,10} & 0 & 8\rho_{8,12} & 0 & 0 & 0 \\ 0 & 8\rho_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & -8\rho_{9,8} & 0 & 0 & 8\rho_{9,11} & 0 & 8\rho_{9,13} & 0 & 0 \\ 0 & 0 & 8\rho_{10,2} & 0 & 0 & 0 & 0 & 0 & 8\rho_{10,8} & 0 & 0 & -8\rho_{10,11} & 0 & 0 & -8\rho_{10,14} & 0 \\ 0 & 0 & 0 & 8\rho_{11,3} & 0 & 0 & 0 & 0 & 0 & 8\rho_{11,9} & -8\rho_{11,10} & 0 & 0 & 0 & 0 & -8\rho_{11,15} \\ 0 & 0 & 0 & 0 & 8\rho_{12,4} & 0 & 0 & 0 & 8\rho_{12,8} & 0 & 0 & 0 & 0 & -8\rho_{12,13} & -8\rho_{12,14} & 0 \\ 0 & 0 & 0 & 0 & 0 & 8\rho_{13,5} & 0 & 0 & 0 & -8\rho_{13,9} & 0 & 0 & 8\rho_{13,12} & 0 & 0 & -8\rho_{13,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 8\rho_{14,6} & 0 & 0 & 0 & -8\rho_{14,10} & 0 & -8\rho_{14,12} & 0 & 0 & -8\rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8\rho_{15,7} & 0 & 0 & 0 & -8\rho_{15,11} & 0 & -8\rho_{15,13} & -8\rho_{15,14} & 0 \end{pmatrix} \quad (280)$$

$$= \frac{1}{2} \begin{pmatrix} 0 & \rho_{0,1} & -\rho_{0,2} & 0 & \rho_{0,4} & 0 & 0 & 0 & \rho_{0,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\rho_{1,0} & 0 & 0 & \rho_{1,3} & 0 & \rho_{1,5} & 0 & 0 & 0 & \rho_{1,9} & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho_{2,0} & 0 & 0 & -\rho_{2,3} & 0 & 0 & -\rho_{2,6} & 0 & 0 & 0 & \rho_{2,10} & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_{3,1} & -\rho_{3,2} & 0 & 0 & 0 & 0 & -\rho_{3,7} & 0 & 0 & 0 & \rho_{3,11} & 0 & 0 & 0 & 0 \\ \rho_{4,0} & 0 & 0 & 0 & 0 & -\rho_{4,5} & -\rho_{4,6} & 0 & 0 & 0 & 0 & 0 & \rho_{4,12} & 0 & 0 & 0 \\ 0 & \rho_{5,1} & 0 & 0 & -\rho_{5,4} & 0 & 0 & -\rho_{5,7} & 0 & 0 & 0 & 0 & 0 & \rho_{5,13} & 0 & 0 \\ 0 & 0 & -\rho_{6,2} & 0 & -\rho_{6,4} & 0 & 0 & -\rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{6,14} & 0 \\ 0 & 0 & 0 & -\rho_{7,3} & 0 & -\rho_{7,5} & -\rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{7,15} \\ \rho_{8,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{8,9} & \rho_{8,10} & 0 & \rho_{8,12} & 0 & 0 & 0 \\ 0 & \rho_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{9,8} & 0 & 0 & \rho_{9,11} & 0 & \rho_{9,13} & 0 & 0 \\ 0 & 0 & \rho_{10,2} & 0 & 0 & 0 & 0 & 0 & \rho_{10,8} & 0 & 0 & -\rho_{10,11} & 0 & 0 & -\rho_{10,14} & 0 \\ 0 & 0 & 0 & \rho_{11,3} & 0 & 0 & 0 & 0 & 0 & \rho_{11,9} & -\rho_{11,10} & 0 & 0 & 0 & 0 & -\rho_{11,15} \\ 0 & 0 & 0 & 0 & \rho_{12,4} & 0 & 0 & 0 & \rho_{12,8} & 0 & 0 & 0 & 0 & -\rho_{12,13} & -\rho_{12,14} & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_{13,5} & 0 & 0 & 0 & -\rho_{13,9} & 0 & 0 & \rho_{13,12} & 0 & 0 & -\rho_{13,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho_{14,6} & 0 & 0 & 0 & -\rho_{14,10} & 0 & -\rho_{14,12} & 0 & 0 & -\rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{15,7} & 0 & 0 & 0 & -\rho_{15,11} & 0 & -\rho_{15,13} & -\rho_{15,14} & 0 \end{pmatrix} \quad (281)$$

dengan

$$\begin{aligned}
\rho_{0,1} &= \rho_{1,0}^* = e^{\beta_0 - \beta_1} & \rho_{6,7} &= \rho_{7,6}^* = e^{\beta_6 - \beta_7} \\
\rho_{0,2} &= \rho_{2,0}^* = e^{\beta_0 - \beta_2} & \rho_{6,14} &= \rho_{14,6}^* = e^{\beta_6 - \beta_{14}} \\
\rho_{0,8} &= \rho_{8,0}^* = e^{\beta_8 - \beta_1} & \rho_{8,9} &= \rho_{9,8}^* = e^{\beta_8 - \beta_9} \\
\rho_{1,3} &= \rho_{3,1}^* = e^{\beta_1 - \beta_3} & \rho_{8,10} &= \rho_{10,8}^* = e^{\beta_8 - \beta_{10}} \\
\rho_{1,5} &= \rho_{5,1}^* = e^{\beta_1 - \beta_5} & \rho_{8,12} &= \rho_{12,8}^* = e^{\beta_8 - \beta_{12}} \\
\rho_{1,9} &= \rho_{9,1}^* = e^{\beta_1 - \beta_9} & \rho_{9,11} &= \rho_{11,9}^* = e^{\beta_9 - \beta_{11}} \\
\rho_{2,3} &= \rho_{3,2}^* = e^{\beta_2 - \beta_3} & \rho_{9,13} &= \rho_{13,9}^* = e^{\beta_9 - \beta_{13}} \\
\rho_{2,6} &= \rho_{6,2}^* = e^{\beta_2 - \beta_6} & \rho_{10,11} &= \rho_{11,10}^* = e^{\beta_{10} - \beta_{11}} \\
\rho_{2,10} &= \rho_{10,2}^* = e^{\beta_2 - \beta_{10}} & \rho_{10,14} &= \rho_{14,10}^* = e^{\beta_{10} - \beta_{14}} \\
\rho_{3,7} &= \rho_{7,3}^* = e^{\beta_3 - \beta_7} & \rho_{12,13} &= \rho_{13,12}^* = e^{\beta_{12} - \beta_{13}} \\
\rho_{3,11} &= \rho_{11,3}^* = e^{\beta_3 - \beta_{11}} & \rho_{12,14} &= \rho_{14,12}^* = e^{\beta_{12} - \beta_{14}} \\
\rho_{4,5} &= \rho_{5,4}^* = e^{\beta_4 - \beta_5} & \rho_{11,15} &= \rho_{15,11}^* = e^{\beta_{11} - \beta_{15}} \\
\rho_{4,6} &= \rho_{6,4}^* = e^{\beta_4 - \beta_6} & \rho_{13,15} &= \rho_{15,13}^* = e^{\beta_{13} - \beta_{15}} \\
\rho_{4,12} &= \rho_{12,4}^* = e^{\beta_4 - \beta_{12}} & \rho_{14,15} &= \rho_{15,14}^* = e^{\beta_{14} - \beta_{15}} \\
\rho_{5,7} &= \rho_{7,5}^* = e^{\beta_5 - \beta_7} & \rho_{15,7} &= \rho_{7,15}^* = e^{\beta_7 - \beta_{15}} \\
\rho_{5,13} &= \rho_{13,5}^* = e^{\beta_5 - \beta_{13}} & \rho_{0,4} &= \rho_{4,0}^* = e^{\beta_0 - \beta_4}
\end{aligned} \tag{282}$$

dan

$$\begin{array}{ll}
\beta_0 - \beta_1 = -i(E_{4,0} - E_{4,1})t & \beta_0 - \beta_2 = -i(E_{4,0} - E_{4,2})t \\
\beta_0 - \beta_4 = -i(E_{4,0} - E_{4,4})t & \beta_0 - \beta_8 = -i(E_{4,0} - E_{4,8})t \\
\beta_1 - \beta_3 = -i(E_{4,1} - E_{4,3})t & \beta_1 - \beta_5 = -i(E_{4,1} - E_{4,5})t \\
\beta_1 - \beta_9 = -i(E_{4,1} - E_{4,9})t & \beta_2 - \beta_3 = -i(E_{4,2} - E_{4,3})t \\
\beta_2 - \beta_6 = -i(E_{4,2} - E_{4,6})t & \beta_2 - \beta_{10} = -i(E_{4,2} - E_{4,10})t \\
\beta_3 - \beta_7 = -i(E_{4,3} - E_{4,7})t & \beta_3 - \beta_{11} = -i(E_{4,3} - E_{4,11})t \\
\beta_4 - \beta_5 = -i(E_{4,4} - E_{4,5})t & \beta_4 - \beta_6 = -i(E_{4,4} - E_{4,6})t \\
\beta_4 - \beta_{12} = -i(E_{4,4} - E_{4,12})t & \beta_5 - \beta_7 = -i(E_{4,5} - E_{4,7})t \\
\beta_5 - \beta_{13} = -i(E_{4,5} - E_{4,13})t & \beta_6 - \beta_7 = -i(E_{4,6} - E_{4,7})t \\
\beta_6 - \beta_{14} = -i(E_{4,6} - E_{4,14})t & \beta_8 - \beta_9 = -i(E_{4,8} - E_{4,9})t \\
\beta_8 - \beta_{10} = -i(E_{4,8} - E_{4,10})t & \beta_8 - \beta_{12} = -i(E_{4,8} - E_{4,12})t \\
\beta_9 - \beta_{11} = -i(E_{4,9} - E_{4,11})t & \beta_9 - \beta_{13} = -i(E_{4,9} - E_{4,13})t \\
\beta_{10} - \beta_{11} = -i(E_{4,10} - E_{4,11})t & \beta_{10} - \beta_{14} = -i(E_{4,10} - E_{4,14})t \\
\beta_{12} - \beta_{13} = -i(E_{4,12} - E_{4,13})t & \beta_{12} - \beta_{14} = -i(E_{4,12} - E_{4,14})t \\
\beta_{11} - \beta_{15} = -i(E_{4,11} - E_{4,15})t & \beta_{13} - \beta_{15} = -i(E_{4,13} - E_{4,15})t \\
\beta_{14} - \beta_{15} = -i(E_{4,14} - E_{4,15})t & \beta_{15} - \beta_7 = -i(E_{4,15} - E_{4,7})t
\end{array} \quad (283)$$

dan

$$\begin{aligned}
E_{4,0} - E_{4,1} &= \omega_0 + \pi J_{10} + \pi J_{20} + \pi J_{30} & E_{4,0} - E_{4,2} &= \omega_1 + \pi J_{10} + \pi J_{21} + \pi J_{31} \\
E_{4,0} - E_{4,4} &= \omega_2 + \pi J_{20} + \pi J_{21} + \pi J_{32} & E_{4,0} - E_{4,8} &= \omega_3 + \pi J_{30} + \pi J_{31} + \pi J_{32} \\
E_{4,1} - E_{4,3} &= \omega_1 - \pi J_{10} + \pi J_{21} + \pi J_{31} & E_{4,1} - E_{4,5} &= \omega_2 - \pi J_{20} + \pi J_{21} + \pi J_{32} \\
E_{4,1} - E_{4,9} &= \omega_3 - \pi J_{30} + \pi J_{310} + \pi J_{32} & E_{4,2} - E_{4,3} &= \omega_0 - \pi J_{10} + \pi J_{20} + \pi J_{30} \\
E_{4,2} - E_{4,6} &= \omega_2 + \pi J_{20} - \pi J_{21} + \pi J_{32} & E_{4,2} - E_{4,10} &= \omega_3 + \pi J_{30} - \pi J_{31} + \pi J_{32} \\
E_{4,3} - E_{4,7} &= \omega_2 - \pi J_{20} - \pi J_{21} + \pi J_{32} & E_{4,3} - E_{4,11} &= \omega_3 - \pi J_{30} - \pi J_{31} + \pi J_{32} \\
E_{4,4} - E_{4,5} &= \omega_0 + \pi J_{10} - \pi J_{20} + \pi J_{30} & E_{4,4} - E_{4,6} &= \omega_1 + \pi J_{10} - \pi J_{21} + \pi J_{31} \\
E_{4,4} - E_{4,12} &= \omega_3 + \pi J_{30} + \pi J_{31} - \pi J_{32} & E_{4,5} - E_{4,7} &= \omega_1 - \pi J_{10} - \pi J_{21} + \pi J_{31} \\
E_{4,5} - E_{4,13} &= \omega_3 - \pi J_{30} + \pi J_{31} - \pi J_{32} & E_{4,6} - E_{4,7} &= \omega_0 - \pi J_{10} - \pi J_{20} + \pi J_{30} \\
E_{4,6} - E_{4,14} &= \omega_3 + \pi J_{30} - \pi J_{31} - \pi J_{32} & E_{4,8} - E_{4,9} &= \omega_0 + \pi J_{10} + \pi J_{20} - \pi J_{30} \\
E_{4,8} - E_{4,10} &= \omega_1 + \pi J_{21} - \pi J_{31} + \pi J_{30} & E_{4,8} - E_{4,12} &= \omega_2 + \pi J_{20} + \pi J_{21} - \pi J_{32} \\
E_{4,9} - E_{4,11} &= \omega_1 - \pi J_{10} + \pi J_{21} - \pi J_{32} & E_{4,10} - E_{4,11} &= \omega_0 - \pi J_{10} + \pi J_{20} - \pi J_{30} \\
E_{4,10} - E_{4,14} &= \omega_2 + \pi J_{20} - \pi J_{21} - \pi J_{32} & E_{4,12} - E_{4,13} &= \omega_0 + \pi J_{10} - \pi J_{20} - \pi J_{30} \\
E_{4,12} - E_{4,14} &= \omega_1 + \pi J_{10} - \pi J_{21} - \pi J_{31} & E_{4,11} - E_{4,15} &= \omega_2 - \pi J_{20} - \pi J_{21} - \pi J_{32} \\
E_{4,13} - E_{4,15} &= \omega_1 - \pi J_{10} - \pi J_{21} - \pi J_{31} & E_{4,14} - E_{4,15} &= \omega_0 - \pi J_{10} - \pi J_{20} - \pi J_{30} \\
E_{4,9} - E_{4,13} &= \omega_2 - \pi J_{20} + \pi J_{21} - \pi J_{32} & E_{4,15} - E_{4,7} &= \omega_3 + \pi J_{30} + \pi J_{31} + \pi J_{32}
\end{aligned} \tag{284}$$

maka dapat ditentukan nilai magnetisasi dari masing-masing qubitnya yaitu sebagai berikut:

$$(I_{3+}\rho_5) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\times \frac{1}{2} \begin{pmatrix} 0 & \rho_{0,1} & -\rho_{0,2} & 0 & \rho_{0,4} & 0 & 0 & 0 & \rho_{0,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\rho_{1,0} & 0 & 0 & \rho_{1,3} & 0 & \rho_{1,5} & 0 & 0 & 0 & \rho_{1,9} & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho_{2,0} & 0 & 0 & -\rho_{2,3} & 0 & 0 & -\rho_{2,6} & 0 & 0 & 0 & \rho_{2,10} & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_{3,1} & -\rho_{3,2} & 0 & 0 & 0 & 0 & -\rho_{3,7} & 0 & 0 & 0 & \rho_{3,11} & 0 & 0 & 0 & 0 \\ \rho_{4,0} & 0 & 0 & 0 & 0 & -\rho_{4,5} & -\rho_{4,6} & 0 & 0 & 0 & 0 & 0 & \rho_{4,12} & 0 & 0 & 0 \\ 0 & \rho_{5,1} & 0 & 0 & -\rho_{5,4} & 0 & 0 & -\rho_{5,7} & 0 & 0 & 0 & 0 & 0 & \rho_{5,13} & 0 & 0 \\ 0 & 0 & -\rho_{6,2} & 0 & -\rho_{6,4} & 0 & 0 & -\rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{6,14} & 0 \\ 0 & 0 & 0 & -\rho_{7,3} & 0 & -\rho_{7,5} & -\rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{7,15} \\ \rho_{8,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{8,9} & \rho_{8,10} & 0 & \rho_{8,12} & 0 & 0 & 0 \\ 0 & \rho_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{9,8} & 0 & 0 & \rho_{9,11} & 0 & \rho_{9,13} & 0 & 0 \\ 0 & 0 & \rho_{10,2} & 0 & 0 & 0 & 0 & 0 & \rho_{10,8} & 0 & 0 & -\rho_{10,11} & 0 & 0 & -\rho_{10,14} & 0 \\ 0 & 0 & 0 & \rho_{11,3} & 0 & 0 & 0 & 0 & 0 & \rho_{11,9} & -\rho_{11,10} & 0 & 0 & 0 & 0 & -\rho_{11,15} \\ 0 & 0 & 0 & 0 & \rho_{12,4} & 0 & 0 & 0 & \rho_{12,8} & 0 & 0 & 0 & 0 & -\rho_{12,13} & -\rho_{12,14} & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_{13,5} & 0 & 0 & 0 & -\rho_{13,9} & 0 & 0 & \rho_{13,12} & 0 & 0 & -\rho_{13,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho_{14,6} & 0 & 0 & 0 & -\rho_{14,10} & 0 & -\rho_{14,12} & 0 & 0 & -\rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{15,7} & 0 & 0 & 0 & -\rho_{15,11} & 0 & -\rho_{15,13} & -\rho_{15,14} & 0 \end{pmatrix}$$

[illegible]

$$\begin{aligned}
M_{3+} &= \langle I_{3+} \rangle = \text{tr} (I_{3+} \rho_5) \\
&= \rho_{8,0} + \rho_{9,1} + \rho_{10,2} + \rho_{11,3} + \rho_{12,4} + \rho_{13,5} + \rho_{14,6} + \rho_{15,7}
\end{aligned} \tag{285}$$

dan

$$(I_{2+} \rho_5) = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\times \frac{1}{2} \begin{pmatrix} 0 & \rho_{0,1} & -\rho_{0,2} & 0 & \rho_{0,4} & 0 & 0 & 0 & \rho_{0,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\rho_{1,0} & 0 & 0 & \rho_{1,3} & 0 & \rho_{1,5} & 0 & 0 & 0 & \rho_{1,9} & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho_{2,0} & 0 & 0 & -\rho_{2,3} & 0 & 0 & -\rho_{2,6} & 0 & 0 & 0 & \rho_{2,10} & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_{3,1} & -\rho_{3,2} & 0 & 0 & 0 & 0 & -\rho_{3,7} & 0 & 0 & 0 & \rho_{3,11} & 0 & 0 & 0 & 0 \\ \rho_{4,0} & 0 & 0 & 0 & 0 & -\rho_{4,5} & -\rho_{4,6} & 0 & 0 & 0 & 0 & 0 & \rho_{4,12} & 0 & 0 & 0 \\ 0 & \rho_{5,1} & 0 & 0 & -\rho_{5,4} & 0 & 0 & -\rho_{5,7} & 0 & 0 & 0 & 0 & 0 & \rho_{5,13} & 0 & 0 \\ 0 & 0 & -\rho_{6,2} & 0 & -\rho_{6,4} & 0 & 0 & -\rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{6,14} & 0 \\ 0 & 0 & 0 & -\rho_{7,3} & 0 & -\rho_{7,5} & -\rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{7,15} \\ \rho_{8,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{8,9} & \rho_{8,10} & 0 & \rho_{8,12} & 0 & 0 & 0 \\ 0 & \rho_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{9,8} & 0 & 0 & \rho_{9,11} & 0 & \rho_{9,13} & 0 & 0 \\ 0 & 0 & \rho_{10,2} & 0 & 0 & 0 & 0 & 0 & \rho_{10,8} & 0 & 0 & -\rho_{10,11} & 0 & 0 & -\rho_{10,14} & 0 \\ 0 & 0 & 0 & \rho_{11,3} & 0 & 0 & 0 & 0 & 0 & \rho_{11,9} & -\rho_{11,10} & 0 & 0 & 0 & 0 & -\rho_{11,15} \\ 0 & 0 & 0 & 0 & \rho_{12,4} & 0 & 0 & 0 & \rho_{12,8} & 0 & 0 & 0 & 0 & -\rho_{12,13} & -\rho_{12,14} & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_{13,5} & 0 & 0 & 0 & -\rho_{13,9} & 0 & 0 & \rho_{13,12} & 0 & 0 & -\rho_{13,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho_{14,6} & 0 & 0 & 0 & -\rho_{14,10} & 0 & -\rho_{14,12} & 0 & 0 & -\rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{15,7} & 0 & 0 & 0 & -\rho_{15,11} & 0 & -\rho_{15,13} & -\rho_{15,14} & 0 \end{pmatrix}$$

(286)

$$\begin{aligned}
M_{2+} &= \langle I_{2+} \rangle = tr(I_{2+} \rho_5) \\
&= \rho_{4,0} + \rho_{5,1} - \rho_{6,2} - \rho_{7,3} + \rho_{12,8} + \rho_{13,9} - \rho_{14,10} - \rho_{15,11}
\end{aligned} \tag{287}$$

dan

$$(I_{1+} \rho_5) = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\times \frac{1}{2} \begin{pmatrix} 0 & \rho_{0,1} & -\rho_{0,2} & 0 & \rho_{0,4} & 0 & 0 & 0 & \rho_{0,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\rho_{1,0} & 0 & 0 & \rho_{1,3} & 0 & \rho_{1,5} & 0 & 0 & 0 & \rho_{1,9} & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho_{2,0} & 0 & 0 & -\rho_{2,3} & 0 & 0 & -\rho_{2,6} & 0 & 0 & 0 & \rho_{2,10} & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_{3,1} & -\rho_{3,2} & 0 & 0 & 0 & 0 & -\rho_{3,7} & 0 & 0 & 0 & \rho_{3,11} & 0 & 0 & 0 & 0 \\ \rho_{4,0} & 0 & 0 & 0 & 0 & -\rho_{4,5} & -\rho_{4,6} & 0 & 0 & 0 & 0 & 0 & \rho_{4,12} & 0 & 0 & 0 \\ 0 & \rho_{5,1} & 0 & 0 & -\rho_{5,4} & 0 & 0 & -\rho_{5,7} & 0 & 0 & 0 & 0 & 0 & \rho_{5,13} & 0 & 0 \\ 0 & 0 & -\rho_{6,2} & 0 & -\rho_{6,4} & 0 & 0 & -\rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{6,14} & 0 \\ 0 & 0 & 0 & -\rho_{7,3} & 0 & -\rho_{7,5} & -\rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{7,15} \\ \rho_{8,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{8,9} & \rho_{8,10} & 0 & \rho_{8,12} & 0 & 0 & 0 \\ 0 & \rho_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{9,8} & 0 & 0 & \rho_{9,11} & 0 & \rho_{9,13} & 0 & 0 \\ 0 & 0 & \rho_{10,2} & 0 & 0 & 0 & 0 & 0 & \rho_{10,8} & 0 & 0 & -\rho_{10,11} & 0 & 0 & -\rho_{10,14} & 0 \\ 0 & 0 & 0 & \rho_{11,3} & 0 & 0 & 0 & 0 & 0 & \rho_{11,9} & -\rho_{11,10} & 0 & 0 & 0 & 0 & -\rho_{11,15} \\ 0 & 0 & 0 & 0 & \rho_{12,4} & 0 & 0 & 0 & \rho_{12,8} & 0 & 0 & 0 & 0 & -\rho_{12,13} & -\rho_{12,14} & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_{13,5} & 0 & 0 & 0 & -\rho_{13,9} & 0 & 0 & \rho_{13,12} & 0 & 0 & -\rho_{13,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho_{14,6} & 0 & 0 & 0 & -\rho_{14,10} & 0 & -\rho_{14,12} & 0 & 0 & -\rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{15,7} & 0 & 0 & 0 & -\rho_{15,11} & 0 & -\rho_{15,13} & -\rho_{15,14} & 0 \end{pmatrix}$$

(288)

$$\begin{aligned}
M_{1+} &= \langle I_{1+} \rangle = tr(I_{1+}\rho_5) \\
&= \rho_{2,0} + \rho_{3,1} - \rho_{6,4} - \rho_{7,5} + \rho_{10,8} + \rho_{11,9} - \rho_{14,12} - \rho_{15,13}
\end{aligned} \tag{289}$$

dan

$$(I_{0+}\rho_5) = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\times \frac{1}{2} \begin{pmatrix} 0 & \rho_{0,1} & -\rho_{0,2} & 0 & \rho_{0,4} & 0 & 0 & 0 & \rho_{0,8} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\rho_{1,0} & 0 & 0 & \rho_{1,3} & 0 & \rho_{1,5} & 0 & 0 & 0 & \rho_{1,9} & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho_{2,0} & 0 & 0 & -\rho_{2,3} & 0 & 0 & -\rho_{2,6} & 0 & 0 & 0 & \rho_{2,10} & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_{3,1} & -\rho_{3,2} & 0 & 0 & 0 & 0 & -\rho_{3,7} & 0 & 0 & 0 & \rho_{3,11} & 0 & 0 & 0 & 0 \\ \rho_{4,0} & 0 & 0 & 0 & 0 & -\rho_{4,5} & -\rho_{4,6} & 0 & 0 & 0 & 0 & 0 & \rho_{4,12} & 0 & 0 & 0 \\ 0 & \rho_{5,1} & 0 & 0 & -\rho_{5,4} & 0 & 0 & -\rho_{5,7} & 0 & 0 & 0 & 0 & 0 & \rho_{5,13} & 0 & 0 \\ 0 & 0 & -\rho_{6,2} & 0 & -\rho_{6,4} & 0 & 0 & -\rho_{6,7} & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{6,14} & 0 \\ 0 & 0 & 0 & -\rho_{7,3} & 0 & -\rho_{7,5} & -\rho_{7,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{7,15} \\ \rho_{8,0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{8,9} & \rho_{8,10} & 0 & \rho_{8,12} & 0 & 0 & 0 \\ 0 & \rho_{9,1} & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_{9,8} & 0 & 0 & \rho_{9,11} & 0 & \rho_{9,13} & 0 & 0 \\ 0 & 0 & \rho_{10,2} & 0 & 0 & 0 & 0 & 0 & \rho_{10,8} & 0 & 0 & -\rho_{10,11} & 0 & 0 & -\rho_{10,14} & 0 \\ 0 & 0 & 0 & \rho_{11,3} & 0 & 0 & 0 & 0 & 0 & \rho_{11,9} & -\rho_{11,10} & 0 & 0 & 0 & 0 & -\rho_{11,15} \\ 0 & 0 & 0 & 0 & \rho_{12,4} & 0 & 0 & 0 & \rho_{12,8} & 0 & 0 & 0 & 0 & -\rho_{12,13} & -\rho_{12,14} & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_{13,5} & 0 & 0 & 0 & -\rho_{13,9} & 0 & 0 & \rho_{13,12} & 0 & 0 & -\rho_{13,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho_{14,6} & 0 & 0 & 0 & -\rho_{14,10} & 0 & -\rho_{14,12} & 0 & 0 & -\rho_{14,15} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{15,7} & 0 & 0 & 0 & -\rho_{15,11} & 0 & -\rho_{15,13} & -\rho_{15,14} & 0 \end{pmatrix}$$

(290)

$$\begin{aligned}
M_{0+} &= \langle I_{0+} \rangle = tr (I_{0+} \rho_5) \\
&= -(\rho_{1,0} + \rho_{3,2} + \rho_{5,2} + \rho_{7,6} + \rho_{9,8} + \rho_{11,10} + \rho_{13,12} + \rho_{15,14}) \quad (291)
\end{aligned}$$

maka dengan demikian nilai magnetisasi total dari sistem adalah sebagai berikut:

$$\begin{aligned}
M_{tot} &= M_{3+} + M_{2+} + M_{1+} + M_{0+} \\
&= \rho_{8,0} + \rho_{9,1} + \rho_{10,2} + \rho_{11,3} + \rho_{12,4} + \rho_{13,5} + \rho_{14,6} + \rho_{15,7} \\
&\quad + \rho_{4,0} + \rho_{5,1} - \rho_{6,2} - \rho_{7,3} + \rho_{12,8} + \rho_{13,9} - \rho_{14,10} - \rho_{15,11} \\
&\quad + \rho_{2,0} + \rho_{3,1} - \rho_{6,4} - \rho_{7,5} + \rho_{10,8} + \rho_{11,9} - \rho_{14,12} - \rho_{15,13} \\
&\quad - (\rho_{1,0} + \rho_{3,2} + \rho_{5,4} + \rho_{7,6} + \rho_{9,8} + \rho_{11,10} + \rho_{13,12} + \rho_{15,14}) \\
&= e^{i(\omega_3 + \pi J_{30} + \pi J_{31} + \pi J_{32})t} + e^{i(\omega_3 - \pi J_{30} + \pi J_{31} + \pi J_{32})t} \\
&\quad + e^{i(\omega_3 + \pi J_{30} - \pi J_{31} + \pi J_{32})t} + e^{i(\omega_3 - \pi J_{30} - \pi J_{31} + \pi J_{32})t} \\
&\quad + e^{i(\omega_3 + \pi J_{30} + \pi J_{31} - \pi J_{32})t} + e^{i(\omega_3 - \pi J_{30} + \pi J_{31} - \pi J_{32})t} \\
&\quad + e^{i(\omega_3 + \pi J_{30} - \pi J_{31} - \pi J_{32})t} + e^{i(\omega_3 - \pi J_{30} - \pi J_{31} - \pi J_{32})t} \\
&\quad + e^{i(\omega_2 + \pi J_{20} + \pi J_{21} + \pi J_{32})t} + e^{i(\omega_2 - \pi J_{20} + \pi J_{21} + \pi J_{32})t} \\
&\quad - e^{i(\omega_2 + \pi J_{20} - \pi J_{21} + \pi J_{32})t} - e^{i(\omega_2 - \pi J_{20} - \pi J_{21} + \pi J_{32})t} \quad (292) \\
&\quad + e^{i(\omega_2 + \pi J_{20} + \pi J_{21} - \pi J_{32})t} + e^{i(\omega_2 - \pi J_{20} + \pi J_{21} - \pi J_{32})t} \\
&\quad - e^{i(\omega_2 + \pi J_{20} - \pi J_{21} - \pi J_{32})t} - e^{i(\omega_2 - \pi J_{20} - \pi J_{21} - \pi J_{32})t} \\
&\quad + e^{i(\omega_1 + \pi J_{10} + \pi J_{21} + \pi J_{31})t} + e^{i(\omega_1 - \pi J_{10} + \pi J_{21} + \pi J_{31})t} \\
&\quad - e^{i(\omega_1 + \pi J_{10} - \pi J_{21} + \pi J_{31})t} - e^{i(\omega_1 - \pi J_{10} - \pi J_{21} + \pi J_{31})t} \\
&\quad + e^{i(\omega_1 + \pi J_{21} - \pi J_{31} + \pi J_{30})t} + e^{i(\omega_1 - \pi J_{10} + \pi J_{21} - \pi J_{32})t} \\
&\quad - e^{i(\omega_1 + \pi J_{10} - \pi J_{21} - \pi J_{31})t} - e^{i(\omega_1 - \pi J_{10} - \pi J_{21} - \pi J_{31})t} \\
&\quad - e^{i(\omega_0 + \pi J_{10} + \pi J_{20} + \pi J_{30})t} - e^{i(\omega_0 - \pi J_{10} + \pi J_{20} + \pi J_{30})t} \\
&\quad - e^{i(\omega_0 + \pi J_{10} - \pi J_{20} + \pi J_{30})t} - e^{i(\omega_0 - \pi J_{10} - \pi J_{20} + \pi J_{30})t} \\
&\quad - e^{i(\omega_0 + \pi J_{10} + \pi J_{20} - \pi J_{30})t} - e^{i(\omega_0 - \pi J_{10} + \pi J_{20} - \pi J_{30})t} \\
&\quad - e^{i(\omega_0 + \pi J_{10} - \pi J_{20} - \pi J_{30})t} - e^{i(\omega_0 - \pi J_{10} - \pi J_{20} - \pi J_{30})t}
\end{aligned}$$

kemudian didapatkan juga untuk sinyal FID sebagai berikut:

$$\begin{aligned}
S &= M_{tot} e^{-R_2 t} \\
&= e^{-(R_2 - i(\omega_3 + \pi J_{30} + \pi J_{31} + \pi J_{32}))t} + e^{-(R_2 - i(\omega_3 - \pi J_{30} + \pi J_{31} + \pi J_{32}))t} \\
&+ e^{-(R_2 - i(\omega_3 + \pi J_{30} - \pi J_{31} + \pi J_{32}))t} + e^{-(R_2 - i(\omega_3 - \pi J_{30} - \pi J_{31} + \pi J_{32}))t} \\
&+ e^{-(R_2 - i(\omega_3 + \pi J_{30} + \pi J_{31} - \pi J_{32}))t} + e^{-(R_2 - i(\omega_3 - \pi J_{30} + \pi J_{31} - \pi J_{32}))t} \\
&+ e^{-(R_2 - i(\omega_3 + \pi J_{30} - \pi J_{31} - \pi J_{32}))t} + e^{-(R_2 - i(\omega_3 - \pi J_{30} - \pi J_{31} - \pi J_{32}))t} \\
&+ e^{-(R_2 - i(\omega_2 + \pi J_{20} + \pi J_{21} + \pi J_{32}))t} + e^{-(R_2 - i(\omega_2 - \pi J_{20} + \pi J_{21} + \pi J_{32}))t} \\
&- e^{-(R_2 - i(\omega_2 + \pi J_{20} - \pi J_{21} + \pi J_{32}))t} - e^{-(R_2 - i(\omega_2 - \pi J_{20} - \pi J_{21} + \pi J_{32}))t} \\
&+ e^{-(R_2 - i(\omega_2 + \pi J_{20} + \pi J_{21} - \pi J_{32}))t} + e^{-(R_2 - i(\omega_2 - \pi J_{20} + \pi J_{21} - \pi J_{32}))t} \\
&- e^{-(R_2 - i(\omega_2 + \pi J_{20} - \pi J_{21} - \pi J_{32}))t} - e^{-(R_2 - i(\omega_2 - \pi J_{20} - \pi J_{21} - \pi J_{32}))t} \\
&+ e^{-(R_2 - i(\omega_1 + \pi J_{10} + \pi J_{21} + \pi J_{31}))t} + e^{-(R_2 - i(\omega_1 - \pi J_{10} + \pi J_{21} + \pi J_{31}))t} \\
&- e^{-(R_2 - i(\omega_1 + \pi J_{10} - \pi J_{21} + \pi J_{31}))t} - e^{-(R_2 - i(\omega_1 - \pi J_{10} - \pi J_{21} + \pi J_{31}))t} \\
&+ e^{-(R_2 - i(\omega_1 + \pi J_{21} - \pi J_{31} + \pi J_{30}))t} + e^{-(R_2 - i(\omega_1 - \pi J_{10} + \pi J_{21} - \pi J_{32}))t} \\
&- e^{-(R_2 - i(\omega_1 + \pi J_{10} - \pi J_{21} - \pi J_{31}))t} - e^{-(R_2 - i(\omega_1 - \pi J_{10} - \pi J_{21} - \pi J_{31}))t} \\
&- e^{-(R_2 - i(\omega_0 + \pi J_{10} + \pi J_{20} + \pi J_{30}))t} - e^{-(R_2 - i(\omega_0 - \pi J_{10} + \pi J_{20} + \pi J_{30}))t} \\
&- e^{-(R_2 - i(\omega_0 + \pi J_{10} - \pi J_{20} + \pi J_{30}))t} - e^{-(R_2 - i(\omega_0 - \pi J_{10} - \pi J_{20} + \pi J_{30}))t} \\
&- e^{-(R_2 - i(\omega_0 + \pi J_{10} + \pi J_{20} - \pi J_{30}))t} - e^{-(R_2 - i(\omega_0 - \pi J_{10} + \pi J_{20} - \pi J_{30}))t} \\
&- e^{-(R_2 - i(\omega_0 + \pi J_{10} - \pi J_{20} - \pi J_{30}))t} - e^{-(R_2 - i(\omega_0 - \pi J_{10} - \pi J_{20} - \pi J_{30}))t}
\end{aligned} \tag{293}$$

dimana $R_2 = \frac{1}{T_2}$

kemudian dapat di cari terlebih dahulu untuk perkalian antara $S(t) e^{-i\omega t}$ yaitu sebagai berikut:

$$\begin{aligned}
S(t) e^{-i\omega t} = & e^{-(R_2-i(\omega_3-\omega+\pi J_{30}+\pi J_{31}+\pi J_{32}))t} + e^{-(R_2-i(\omega_3-\omega-\pi J_{30}+\pi J_{31}+\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_3-\omega+\pi J_{30}-\pi J_{31}+\pi J_{32}))t} + e^{-(R_2-i(\omega_3-\omega-\pi J_{30}-\pi J_{31}+\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_3-\omega+\pi J_{30}+\pi J_{31}-\pi J_{32}))t} + e^{-(R_2-i(\omega_3-\omega-\pi J_{30}+\pi J_{31}-\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_3-\omega+\pi J_{30}-\pi J_{31}-\pi J_{32}))t} + e^{-(R_2-i(\omega_3-\omega-\pi J_{30}-\pi J_{31}-\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_2-\omega+\pi J_{20}+\pi J_{21}+\pi J_{32}))t} + e^{-(R_2-i(\omega_2-\omega-\pi J_{20}+\pi J_{21}+\pi J_{32}))t} \\
& - e^{-(R_2-i(\omega_2-\omega+\pi J_{20}-\pi J_{21}+\pi J_{32}))t} - e^{-(R_2-i(\omega_2-\omega-\pi J_{20}-\pi J_{21}+\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_2-\omega+\pi J_{20}+\pi J_{21}-\pi J_{32}))t} + e^{-(R_2-i(\omega_2-\omega-\pi J_{20}+\pi J_{21}-\pi J_{32}))t} \\
& - e^{-(R_2-i(\omega_2-\omega+\pi J_{20}-\pi J_{21}-\pi J_{32}))t} - e^{-(R_2-i(\omega_2-\omega-\pi J_{20}-\pi J_{21}-\pi J_{32}))t} \\
& + e^{-(R_2-i(\omega_1-\omega+\pi J_{10}+\pi J_{21}+\pi J_{31}))t} + e^{-(R_2-i(\omega_1-\omega-\pi J_{10}+\pi J_{21}+\pi J_{31}))t} \\
& - e^{-(R_2-i(\omega_1-\omega+\pi J_{10}-\pi J_{21}+\pi J_{31}))t} - e^{-(R_2-i(\omega_1-\omega-\pi J_{10}-\pi J_{21}+\pi J_{31}))t} \\
& + e^{-(R_2-i(\omega_1-\omega+\pi J_{21}-\pi J_{31}+\pi J_{30}))t} + e^{-(R_2-i(\omega_1-\omega-\pi J_{10}+\pi J_{21}-\pi J_{32}))t} \\
& - e^{-(R_2-i(\omega_1-\omega+\pi J_{10}-\pi J_{21}-\pi J_{31}))t} - e^{-(R_2-i(\omega_1-\omega-\pi J_{10}-\pi J_{21}-\pi J_{31}))t} \\
& - e^{-(R_2-i(\omega_0-\omega+\pi J_{10}+\pi J_{20}+\pi J_{30}))t} - e^{-(R_2-i(\omega_0-\omega-\pi J_{10}+\pi J_{20}+\pi J_{30}))t} \\
& - e^{-(R_2-i(\omega_0-\omega+\pi J_{10}-\pi J_{20}+\pi J_{30}))t} - e^{-(R_2-i(\omega_0-\omega-\pi J_{10}-\pi J_{20}+\pi J_{30}))t} \\
& - e^{-(R_2-i(\omega_0-\omega+\pi J_{10}+\pi J_{20}-\pi J_{30}))t} - e^{-(R_2-i(\omega_0-\omega-\pi J_{10}+\pi J_{20}-\pi J_{30}))t} \\
& - e^{-(R_2-i(\omega_0-\omega+\pi J_{10}-\pi J_{20}-\pi J_{30}))t} - e^{-(R_2-i(\omega_0-\omega-\pi J_{10}-\pi J_{20}-\pi J_{30}))t}
\end{aligned} \tag{294}$$

Transformasi Fourirer dari sinyal FID adalah sebagai berikut:

$$\begin{aligned}
S(\omega) &= \int_0^{\infty} S(t) e^{-i\omega t} dt \\
&= \frac{e^{-(R_2-i(\omega_3-\omega+\pi J_{30}+\pi J_{31}+\pi J_{32}))t}}{-(R_2-i(\omega_3-\omega+\pi J_{30}+\pi J_{31}+\pi J_{32}))} + \frac{e^{-(R_2-i(\omega_3-\omega-\pi J_{30}+\pi J_{31}+\pi J_{32}))t}}{-(R_2-i(\omega_3-\omega-\pi J_{30}+\pi J_{31}+\pi J_{32}))} \\
&+ \frac{e^{-(R_2-i(\omega_3-\omega+\pi J_{30}-\pi J_{31}+\pi J_{32}))t}}{-(R_2-i(\omega_3-\omega+\pi J_{30}-\pi J_{31}+\pi J_{32}))} + \frac{e^{-(R_2-i(\omega_3-\omega-\pi J_{30}-\pi J_{31}+\pi J_{32}))t}}{-(R_2-i(\omega_3-\omega-\pi J_{30}-\pi J_{31}+\pi J_{32}))} \\
&+ \frac{e^{-(R_2-i(\omega_3-\omega-\pi J_{30}+\pi J_{31}-\pi J_{32}))t}}{-(R_2-i(\omega_3-\omega-\pi J_{30}+\pi J_{31}-\pi J_{32}))} + \frac{e^{-(R_2-i(\omega_3-\omega-\pi J_{30}-\pi J_{31}-\pi J_{32}))t}}{-(R_2-i(\omega_3-\omega-\pi J_{30}-\pi J_{31}-\pi J_{32}))} \\
&+ \frac{e^{-(R_2-i(\omega_2-\omega+\pi J_{20}+\pi J_{21}+\pi J_{32}))t}}{-(R_2-i(\omega_2-\omega+\pi J_{20}+\pi J_{21}+\pi J_{32}))} + \frac{e^{-(R_2-i(\omega_2-\omega-\pi J_{20}+\pi J_{21}+\pi J_{32}))t}}{-(R_2-i(\omega_2-\omega-\pi J_{20}+\pi J_{21}+\pi J_{32}))} \\
&+ \frac{e^{-(R_2-i(\omega_2-\omega+\pi J_{20}-\pi J_{21}+\pi J_{32}))t}}{-(R_2-i(\omega_2-\omega+\pi J_{20}-\pi J_{21}+\pi J_{32}))} + \frac{e^{-(R_2-i(\omega_2-\omega-\pi J_{20}-\pi J_{21}+\pi J_{32}))t}}{-(R_2-i(\omega_2-\omega-\pi J_{20}-\pi J_{21}+\pi J_{32}))} \\
&+ \frac{e^{-(R_2-i(\omega_2-\omega+\pi J_{20}+\pi J_{21}-\pi J_{32}))t}}{-(R_2-i(\omega_2-\omega+\pi J_{20}+\pi J_{21}-\pi J_{32}))} + \frac{e^{-(R_2-i(\omega_2-\omega-\pi J_{20}+\pi J_{21}-\pi J_{32}))t}}{-(R_2-i(\omega_2-\omega-\pi J_{20}+\pi J_{21}-\pi J_{32}))} \\
&+ \frac{e^{-(R_2-i(\omega_2-\omega+\pi J_{20}-\pi J_{21}-\pi J_{32}))t}}{-(R_2-i(\omega_2-\omega+\pi J_{20}-\pi J_{21}-\pi J_{32}))} + \frac{e^{-(R_2-i(\omega_2-\omega-\pi J_{20}-\pi J_{21}-\pi J_{32}))t}}{-(R_2-i(\omega_2-\omega-\pi J_{20}-\pi J_{21}-\pi J_{32}))} \\
&+ \frac{e^{-(R_2-i(\omega_1-\omega+\pi J_{10}+\pi J_{21}+\pi J_{31}))t}}{-(R_2-i(\omega_1-\omega+\pi J_{10}+\pi J_{21}+\pi J_{31}))} + \frac{e^{-(R_2-i(\omega_1-\omega-\pi J_{10}+\pi J_{21}+\pi J_{31}))t}}{-(R_2-i(\omega_1-\omega-\pi J_{10}+\pi J_{21}+\pi J_{31}))} \\
&+ \frac{e^{-(R_2-i(\omega_1-\omega+\pi J_{10}-\pi J_{21}+\pi J_{31}))t}}{(R_2-i(\omega_1-\omega+\pi J_{10}-\pi J_{21}+\pi J_{31}))} + \frac{e^{-(R_2-i(\omega_1-\omega-\pi J_{10}-\pi J_{21}+\pi J_{31}))t}}{(R_2-i(\omega_1-\omega-\pi J_{10}-\pi J_{21}+\pi J_{31}))} \\
&+ \frac{e^{-(R_2-i(\omega_1-\omega+\pi J_{21}-\pi J_{31}+\pi J_{30}))t}}{-(R_2-i(\omega_1-\omega+\pi J_{21}-\pi J_{31}+\pi J_{30}))} + \frac{e^{-(R_2-i(\omega_1-\omega-\pi J_{10}+\pi J_{21}-\pi J_{32}))t}}{-(R_2-i(\omega_1-\omega-\pi J_{10}+\pi J_{21}-\pi J_{32}))} \\
&+ \frac{e^{-(R_2-i(\omega_1-\omega+\pi J_{10}-\pi J_{21}-\pi J_{31}))t}}{(R_2-i(\omega_1-\omega+\pi J_{10}-\pi J_{21}-\pi J_{31}))} + \frac{e^{-(R_2-i(\omega_1-\omega-\pi J_{10}-\pi J_{21}-\pi J_{31}))t}}{(R_2-i(\omega_1-\omega-\pi J_{10}-\pi J_{21}-\pi J_{31}))} \\
&+ \frac{e^{-(R_2-i(\omega_0-\omega+\pi J_{10}+\pi J_{20}+\pi J_{30}))t}}{(R_2-i(\omega_0-\omega+\pi J_{10}+\pi J_{20}+\pi J_{30}))} + \frac{e^{-(R_2-i(\omega_0-\omega-\pi J_{10}+\pi J_{20}+\pi J_{30}))t}}{(R_2-i(\omega_0-\omega-\pi J_{10}+\pi J_{20}+\pi J_{30}))} \\
&+ \frac{e^{-(R_2-i(\omega_0-\omega+\pi J_{10}-\pi J_{20}+\pi J_{30}))t}}{(R_2-i(\omega_0-\omega+\pi J_{10}-\pi J_{20}+\pi J_{30}))} + \frac{e^{-(R_2-i(\omega_0-\omega-\pi J_{10}-\pi J_{20}+\pi J_{30}))t}}{(R_2-i(\omega_0-\omega-\pi J_{10}-\pi J_{20}+\pi J_{30}))} \\
&+ \frac{e^{-(R_2-i(\omega_0-\omega+\pi J_{10}+\pi J_{20}-\pi J_{30}))t}}{(R_2-i(\omega_0-\omega+\pi J_{10}+\pi J_{20}-\pi J_{30}))} + \frac{e^{-(R_2-i(\omega_0-\omega-\pi J_{10}+\pi J_{20}-\pi J_{30}))t}}{(R_2-i(\omega_0-\omega-\pi J_{10}+\pi J_{20}-\pi J_{30}))} \\
&+ \frac{e^{-(R_2-i(\omega_0-\omega+\pi J_{10}-\pi J_{20}-\pi J_{30}))t}}{(R_2-i(\omega_0-\omega+\pi J_{10}-\pi J_{20}-\pi J_{30}))} + \frac{e^{-(R_2-i(\omega_0-\omega-\pi J_{10}-\pi J_{20}-\pi J_{30}))t}}{(R_2-i(\omega_0-\omega-\pi J_{10}-\pi J_{20}-\pi J_{30}))}
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{(R_2 - i(\omega_3 - \omega + \pi J_{30} + \pi J_{31} + \pi J_{32}))} + \frac{1}{(R_2 - i(\omega_3 - \omega - \pi J_{30} + \pi J_{31} + \pi J_{32}))} \\
&+ \frac{1}{(R_2 - i(\omega_3 - \omega + \pi J_{30} - \pi J_{31} + \pi J_{32}))} + \frac{1}{(R_2 - i(\omega_3 - \omega - \pi J_{30} - \pi J_{31} + \pi J_{32}))} \\
&+ \frac{1}{(R_2 - i(\omega_3 - \omega - \pi J_{30} - \pi J_{31} + \pi J_{32}))} + \frac{1}{(R_2 - i(\omega_3 - \omega - \pi J_{30} + \pi J_{31} - \pi J_{32}))} \\
&+ \frac{1}{(R_2 - i(\omega_3 - \omega + \pi J_{30} - \pi J_{31} - \pi J_{32}))} + \frac{1}{(R_2 - i(\omega_3 - \omega - \pi J_{30} - \pi J_{31} - \pi J_{32}))} \\
&+ \frac{1}{(R_2 - i(\omega_2 - \omega + \pi J_{20} + \pi J_{21} + \pi J_{32}))} + \frac{1}{(R_2 - i(\omega_2 - \omega - \pi J_{20} + \pi J_{21} + \pi J_{32}))} \\
&- \frac{1}{(R_2 - i(\omega_2 - \omega + \pi J_{20} - \pi J_{21} + \pi J_{32}))} - \frac{1}{(R_2 - i(\omega_2 - \omega - \pi J_{20} - \pi J_{21} + \pi J_{32}))} \\
&+ \frac{1}{(R_2 - i(\omega_2 - \omega + \pi J_{20} + \pi J_{21} - \pi J_{32}))} + \frac{1}{(R_2 - i(\omega_2 - \omega - \pi J_{20} + \pi J_{21} - \pi J_{32}))} \\
&- \frac{1}{(R_2 - i(\omega_2 - \omega + \pi J_{20} - \pi J_{21} - \pi J_{32}))} - \frac{1}{(R_2 - i(\omega_2 - \omega - \pi J_{20} - \pi J_{21} - \pi J_{32}))} \\
&+ \frac{1}{(R_2 - i(\omega_1 - \omega + \pi J_{10} + \pi J_{21} + \pi J_{31}))} + \frac{1}{(R_2 - i(\omega_1 - \omega - \pi J_{10} + \pi J_{21} + \pi J_{31}))} \\
&- \frac{1}{(R_2 - i(\omega_1 - \omega + \pi J_{10} - \pi J_{21} + \pi J_{31}))} - \frac{1}{(R_2 - i(\omega_1 - \omega - \pi J_{10} - \pi J_{21} + \pi J_{31}))} \\
&+ \frac{1}{(R_2 - i(\omega_1 - \omega + \pi J_{21} - \pi J_{31} + \pi J_{30}))} + \frac{1}{(R_2 - i(\omega_1 - \omega - \pi J_{10} + \pi J_{21} - \pi J_{32}))} \\
&- \frac{1}{(R_2 - i(\omega_1 - \omega + \pi J_{10} - \pi J_{21} - \pi J_{31}))} - \frac{1}{(R_2 - i(\omega_1 - \omega - \pi J_{10} - \pi J_{21} - \pi J_{31}))} \\
&- \frac{1}{(R_2 - i(\omega_0 - \omega + \pi J_{10} + \pi J_{20} + \pi J_{30}))} - \frac{1}{(R_2 - i(\omega_0 - \omega - \pi J_{10} + \pi J_{20} + \pi J_{30}))} \\
&+ \frac{1}{(R_2 - i(\omega_0 - \omega + \pi J_{10} - \pi J_{20} + \pi J_{30}))} + \frac{1}{(R_2 - i(\omega_0 - \omega - \pi J_{10} - \pi J_{20} + \pi J_{30}))} \\
&- \frac{1}{(R_2 - i(\omega_0 - \omega + \pi J_{10} + \pi J_{20} - \pi J_{30}))} - \frac{1}{(R_2 - i(\omega_0 - \omega - \pi J_{10} + \pi J_{20} - \pi J_{30}))} \\
&- \frac{1}{(R_2 - i(\omega_0 - \omega + \pi J_{10} - \pi J_{20} - \pi J_{30}))} - \frac{1}{(R_2 - i(\omega_0 - \omega - \pi J_{10} - \pi J_{20} - \pi J_{30}))}
\end{aligned}$$

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maka dengan demikian diperoleh :

$$\begin{aligned}
A(\omega) = & \frac{R_2}{R_2^2 + (\omega_3 - \omega + \pi J_{30} + \pi J_{31} + \pi J_{32})^2} + \frac{R_2}{R_2^2 + (\omega_3 - \omega - \pi J_{30} + \pi J_{31} + \pi J_{32})^2} \\
& + \frac{R_2}{R_2^2 - (\omega_3 - \omega + \pi J_{30} - \pi J_{31} + \pi J_{32})^2} + \frac{R_2}{R_2^2 + (\omega_3 - \omega - \pi J_{30} - \pi J_{31} + \pi J_{32})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_3 - \omega - \pi J_{30} - \pi J_{31} + \pi J_{32})^2} + \frac{R_2}{R_2^2 + (\omega_3 - \omega - \pi J_{30} + \pi J_{31} - \pi J_{32})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_3 - \omega + \pi J_{30} - \pi J_{31} - \pi J_{32})^2} + \frac{R_2}{R_2^2 + (\omega_3 - \omega - \pi J_{30} - \pi J_{31} - \pi J_{32})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_2 - \omega + \pi J_{20} + \pi J_{21} + \pi J_{32})^2} + \frac{R_2}{R_2^2 + (\omega_2 - \omega - \pi J_{20} + \pi J_{21} + \pi J_{32})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_2 - \omega + \pi J_{20} - \pi J_{21} + \pi J_{32})^2} - \frac{R_2}{R_2^2 + (\omega_2 - \omega - \pi J_{20} - \pi J_{21} + \pi J_{32})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_2 - \omega + \pi J_{20} + \pi J_{21} - \pi J_{32})^2} + \frac{R_2}{R_2^2 + (\omega_2 - \omega - \pi J_{20} + \pi J_{21} - \pi J_{32})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_2 - \omega + \pi J_{20} - \pi J_{21} - \pi J_{32})^2} - \frac{R_2}{R_2^2 + (\omega_2 - \omega - \pi J_{20} - \pi J_{21} - \pi J_{32})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_1 - \omega + \pi J_{10} + \pi J_{21} + \pi J_{31})^2} + \frac{R_2}{R_2^2 + (\omega_1 - \omega - \pi J_{10} + \pi J_{21} + \pi J_{31})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_1 - \omega + \pi J_{10} - \pi J_{21} + \pi J_{31})^2} - \frac{R_2}{R_2^2 + (\omega_1 - \omega - \pi J_{10} - \pi J_{21} + \pi J_{31})^2} \\
& + \frac{R_2}{R_2^2 + (\omega_1 - \omega + \pi J_{21} - \pi J_{31} + \pi J_{30})^2} + \frac{R_2}{R_2^2 + (\omega_1 - \omega - \pi J_{10} + \pi J_{21} - \pi J_{32})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_1 - \omega + \pi J_{10} - \pi J_{21} - \pi J_{31})^2} - \frac{R_2}{R_2^2 + (\omega_1 - \omega - \pi J_{10} - \pi J_{21} - \pi J_{31})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_0 - \omega + \pi J_{10} + \pi J_{20} + \pi J_{30})^2} - \frac{R_2}{R_2^2 + (\omega_0 - \omega - \pi J_{10} + \pi J_{20} + \pi J_{30})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_0 - \omega + \pi J_{10} - \pi J_{20} + \pi J_{30})^2} - \frac{R_2}{R_2^2 + (\omega_0 - \omega - \pi J_{10} - \pi J_{20} + \pi J_{30})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_0 - \omega + \pi J_{10} + \pi J_{20} - \pi J_{30})^2} - \frac{R_2}{R_2^2 + (\omega_0 - \omega - \pi J_{10} + \pi J_{20} - \pi J_{30})^2} \\
& - \frac{R_2}{R_2^2 + (\omega_0 - \omega + \pi J_{10} - \pi J_{20} - \pi J_{30})^2} - \frac{R_2}{R_2^2 + (\omega_0 - \omega - \pi J_{10} - \pi J_{20} - \pi J_{30})^2}
\end{aligned} \tag{295}$$

dan diperoleh juga:

[illegible]

Profil Penulis



Bayu Dwi Hatmoko atau yang lebih akrab dipanggil Bayu dilahirkan di Kota Bojonegoro pada 22 April 1994. Penulis merupakan anak kedua dari tiga bersaudara dari pasangan Bapak Sadiman dan Ibu Yukeshi Y.K. Penulis mengawali

pendidikan formal di TK Dharma Wanita Jatiblimbing yang kemudian melanjutkan ke SDN Jatiblimbing II. Penulis menamatkan pendidikan menengah pertama di SMPN 3 Bojonegoro dan melanjutkan di SMAN 4 Bojonegoro. Pada tahun 2012 penulis diterima di jurusan Fisika ITS melalui jalur SNMPTN Tulis yang terdaftar dengan NRP 11 12 100 060. Penulis menggeluti bidang minat Fisika Teori selama berkuliah di Fisika ITS. Sejak SMA hingga masa perkuliahan, penulis aktif dalam kegiatan Karya Ilmiah dan ikut aktif berpartisipasi dalam kompetisi karya ilmiah. Selain itu, penulis juga aktif sebagai asisten dosen Fisika dasar dan asisten laboratorium Fisika Modern dan Gelombang. Harapan

besar penulis adalah karya ini bisa bermanfaat bagi orang lain dan diri sendiri sebagai sarana pengembangan potensi diri, serta mampu menjadi pribadi yang lebih beruntung yang berlandaskan ajaran Allah SWT. Penulis lebih suka mengisi waktu luang dengan membaca. Penulis bisa dihubungi melalui email bayuhatmoko49@gmail.com.